Enhancing the Power Grid Small-Signal Stability via Optimally Coordinating Inverter-Based Renewables

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October 31, 2023

Abstract

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Index Terms—inverter-based renewables (IBRs), operation adjustment, small-signal stability, grid strength.

I. INTRODUCTION

The growing penetration of renewable resources (e.g., wind and solar) into power grids through power inverters is challenging secure and stable grid operations [1]-[2]. Particularly, Inverter-Based Renewables (IBRs) that commonly synchronize with power grids through phase-lock loops (PLLs) have caused a set of small-signal stability issues due to strong interaction between IBRs and network, especially in low short-circuit grids [3]-[7]. For instance, in July, 2015, a wind-farm-related sub-synchronous resonance event occurred in North China Power Grid, resulting in drop out of generators and a large active-power deficit [5]; In 2017, three similar oscillation issues were captured in South Texas, USA [7].

In the recent literature, different methods were reported to tackle with IBR-induced oscillation issues. These methods mainly focused on controller design, which can be divided into two main categories: (1) control design of IBRs [8],[9] and (2) control design of auxiliary devices (e.g., static synchronous compensator, STATCOM [10]). For the former, Ref. [8] proposed a PLL-reshaped method to suppress IBR-induced oscillation issues in a single-IBR infinite-bus system (SIIBS). To consider the interaction among interconnected IBRs, Ref. [9] proposed an impedance-participation-factor-based method to guide control parameter design of IBRs for enhancing small-signal stability in a multi-machine system, which can consider the scenario that IBRs are “grey-boxed”. However, IBRs are usually “black-boxed” due to manufacturers’ protection of intellectual property. Thus, it may be difficult to modify control strategies or control parameters of IBRs in practical operations. For control design of auxiliary devices, Ref. [10] used gain margin and phase margin criteria to modify control parameters of STATCOMs for damping oscillations caused by wind farms. The proposed method in Ref. [10] assumed that the reduced single-input single-output open-loop transfer function of the system has no right-half-plane poles. However, this assumption may not be satisfied in practice [11]. Moreover, these previous control methods are based on a presumed operating condition, which may be invalid under critical operating conditions [12].

Coordinating active power outputs (or operation adjustments) [12]-[16] can be an effective control strategy for IBRs to improve small-signal stability while accommodating critical operation conditions. Moreover, such a control strategy does not need to modify IBRs’ control configuration and install auxiliary devices for enhancing system’s damping, which can avoid additional investment increments. In previous related works [12]-[16], the operation adjustment method has been used for synchronous generators [12]-[14], multi-terminal direct current (MTDC) [15], and IBRs in microgrids [16] to satisfy system’s small-signal stability demand. These previous efforts are all based on eigenvalue analysis, which needs to establish linearized system’s state-space model. For instance, Ref.[12]-[15] used the sensitivity of active power outputs for damping ratio of dominant eigenvalues to guide the direction of operation adjustment of generators. However, the eigenvalue analysis is difficult for power systems with a large-scale (or many) IBRs because: 1) linearized state matrix is high-dimensional, so it causes a high demand of modeling and computational power; 2) it is hard to obtain accurate state matrix due to unknown parameters of “black-boxed” IBRs. Thus, the eigenvalue-analysis-based operation-adjustment method may be inapplicable for IBRs in power systems with a large-scale IBRs.

This paper proposes a novel operation-adjustment method for coordinating active power outputs of IBRs with small-signal stability constraints (SSSCs) in a multi-IBR system. To avoid the challenges of performing eigenvalue analysis, we introduce generalized operating short-circuit ratio (gOSCR) [17] to formulate SSSCs in the concerned operation-adjustment problems. The gOSCR-analysis method was our previous work for evaluating small-signal stability margin in the multi-IBR system under varying operating conditions from the viewpoint of grid strength, which can avoid establishing system’s state-space models and is suitable for “black-boxed” IBRs. However, the established gOSCR-based optimization problem is nonconvex, nonlinear and discontinuous, since it contains the

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constraints of nonlinear power flow equations and gOSCR-related eigenvalue inequality. Due to the discontinuity of gOSCR-related eigenvalue constraint, the existing algorithms for nonlinear optimization problems, e.g., interior point method, may be invalid. To deal with this issue, a sequential solution approach is proposed to decompose the optimization problems into a sequence of sub-optimization problems (SOPs). In each SOP, the gOSCR-based constraint is linearized locally with bounded region. By this way, the original optimization problems are converted as multiple quadratic programming problems, which can be conveniently solved by commercial solvers, e.g., IPOPT [18] and CPLEX [19]. In the established SOPs, improper bounded region for linearized gOSCR-based constraint may lead to infeasible solutions of SOPs, and improper initial starting operating point may cause slow convergence of established SOPs and thus a high computation demand. To address these issues, an adaptive method is proposed to dynamically update the region size in each SOP and a convex-relaxing method is proposed to find a proper initial starting operating point.

The remainder of this paper is organized as follows. Section II states the problem for coordinating active power outputs of IBRs with SSSCs. In Section III, gOSCR-based operation adjustment optimization problems are formulated for coordinating active power outputs of IBRs with SSSCs. Section IV proposes the algorithm for solving formulated optimization problems. In section V, the proposed method is validated on a 11-IBR system. Section VI draws the conclusions.

II. OPERATION ADJUSTMENT OF IBRS GUIDED WITH SSSCS

A. Features of Multi-IBR Systems

Power grids face IBR-induced small-signal stability issues with the increasing penetration of IBRs [7]. To deal with these issues, one effective method can be operation adjustment of IBRs’ active power outputs, which can consider all operating conditions [16]. In conventional power systems, similar operation adjustment methods of generators’ active power outputs have been investigated for enhancing the small-signal stability, which uses eigenvalue analysis based on system’s state-space model [12]-[15]. However, unlike synchronous generators (SGs), IBRs have different features, which causes that conventional eigenvalue-analysis-based operation adjustment methods are unsuitable for IBRs to enhance small-signal stability. The detailed discussion will be given in the following. Firstly, we will introduce the operation adjustment issues of IBRs with small-signal stability constraints:

$$\min f(x, y)$$
$$s.t. \begin{cases}
g(x, y) = 0 \\
h_{\min} \leq h(x, y) \leq h_{\max}
\end{cases}$$

Small-Signal Stability Constraint

where $f$ is objective function, e.g., economic cost$^{[12]}$ and adjustment amount of IBRs’ active power outputs$^{[13]}$, which are aimed to satisfy economic optimization and minimal adjustment of IBRs’ active power outputs; $x$ is vector of independent variables, i.e., IBRs’ active power outputs $P_i$ $(i=1, \ldots, n)$; $y$ is vector of dependent variables, e.g., amplitude of terminal voltage; $g$ is vector of equality constraints, e.g., power balance and power flows; $h$ is vector of inequality constraints, e.g., allowable intervals of IBRs’ active power outputs and voltage amplitude; $h_{\max}$ and $h_{\min}$ are vectors of upper and lower limits.

In (1), how to formulate SSSCs is important for efficiently solving the optimization problem. In previous works related with power systems’ optimization problems with SSSCs, the small-signal stability is commonly quantified by damping ratio of dominant eigenvalues. Thus, the sensitivity of system’s concerned variables (i.e., generators’ active power output) for minimal damping ratios was used to formulate SSSCs for operation-adjustment problems in previous works. However, the eigenvalue-sensitivity-based method needs to establish linearized state-space models, which rely on detailed knowledge of system’s parameters. Such knowledge may not be available for IBRs since it is not usually disclosed by manufacturers. Moreover, the dynamics of IBRs are complex, which leads to a high-order state matrix for a system with hundreds of IBRs and thus difficulty for efficient calculation of dominant eigenvalues’ damping ratio and sensitivity.

To address this challenge, this paper will next formulate SSSCs in (1) by gOSCR. The gOSCR-analysis method converted small-signal stability analysis of the multi-IBR system under non-rated operating conditions into that of an equivalent SIIBS from the viewpoint of grid strength$^{[17]}$. This avoids establishing detailed state-space models, of which detailed introduction is given in next subsection. Sections III and IV further propose gOSCR-based operation adjustment optimization models and algorithm.

B. Stability Analysis Indices of Multi-IBR Systems

Grid strength analysis is a widely-used stability analysis method for IBR-induced oscillation issues $^{[6]}$, $^{[17]}$, $^{[20]}$-$^{[22]}$, which can avoid system’s detailed modelling for conventional eigenvalue analysis method. In Ref. $^{[21]}$, short-circuit ratio (SCR) was proposed to evaluate small signal stability of SIIBS. Ref. $^{[6]}$, $^{[22]}$ extended SCR from SIIBS to multiple homogeneous (or heterogeneous)-IBR system, wherein the grid strength indicator is named as generalized SCR (gSCR). However, in Ref. $^{[6]}$, $^{[22]}$, gSCR is only suitable for rated operating conditions, but not non-rated operating conditions. To

Consider a typical structure-preserving multi-IBR system with $n$ IBRs (synchronizing with grids through PLL) in Fig. 1, wherein IBRs represent renewables (e.g., wind farms and photovoltaic plants), and nodes $n+1$- are connected to IBRs, node $n+m+1$ is connected to external grids (simplified as ideal voltage source) and remaining nodes represent passive nodes. It is assumed that control parameters, initial value and upper limit of IBRs’ active power outputs, and network parameters are given. Under these assumptions, solving optimization problem (1) (i.e., operation-adjustment issues) can coordinate active power outputs of IBRs while satisfying SSSCs:
deal with this issue, Ref. [17] proposed generalized operating SCR (gOSCR) to evaluate small-signal stability for multi-IBR system under non-rated operations. In the following, we will use gOSCR to represent system’s small-signal stability margin instead of damping ratio.

Let us consider a multi-IBR system as shown in Fig. 1, where small-signal stability issues are mainly caused by PLL-dominated oscillations. According to Ref. [17], this multi-IBR system can be represented by an equivalent SIIBS for small-signal stability analysis, wherein the IBR’s dynamics are the weighted sum of all IBRs and SCR is smallest eigenvalue of a weighted Laplace matrix, i.e., gOSCR. The closed-loop characteristic equation of the equivalent SIIBS is given as [17]

$$\det \left( Y_d(s) \gamma(s) + gOSCR \cdot I_s \right) = 0,$$

where $Y_d(s)$ is admittance matrix of equivalent IBR; $Y_e(s)$ is admittance matrix of IBR in the multi-IBR system; $\gamma(s)$ is grid line’s dynamics [6]; gOSCR is smallest eigenvalue of $U^T P_n B_{red}$, wherein $U = \text{diag} \{ U_i \}$, $P_i = \text{diag} \{ P_{ei} \}$, diag() is a diagonal matrix; $B_{red} \in \mathbb{R}^{n \times n}$ is node-reduced susceptance matrix of network, wherein passive nodes are eliminated; $p_{ij}$ is participation factor; $v_{ij}$ and $u_{ij}$ are $i$th elements of $u^T$ and $v$, $u^T$ and $v$ are normalized left and right eigenvectors of $U^T P_n B_{red}$ for smallest eigenvalue, i.e., gOSCR; $P_{ei}$ and $U_i$ are active power output and terminal voltage of IBR; $H_d(s)$ and $H_{ph}(s)$ are transfer functions of current control loop and PLL; $L_d$ is filter inductor; here we ignore dynamics of power control loops.

It is noteworthy that for the equivalent SIIBS with given IBRs’ control parameters, the system’s stability becomes worse with the decrease of gOSCR. Due to this, Ref. [17] defined the critical value of gOSCR (named as CgOSCR) for stability margin evaluation. CgOSCR is the value of gOSCR, when the system is critically stable (i.e., dominant eigenvalues located as imaginary axis), given as

$$CgOSCR = \arg \left\{ \text{det} \left( \tilde{Y}_d(s) \gamma(s) + gOSCR \cdot I_s \right) = 0 \right\}$$

where $\arg\{\}$ represents eigenvalue calculation for CgOSCR; $s = j \omega_o$ is dominant eigenvalue; $\omega_o$ is oscillation frequency.

Based on gOSCR and CgOSCR, the system’s stability margin can be described as [17]: if gOSCR $>$ CgOSCR, the system is stable; if gOSCR $<$ CgOSCR, the system is unstable; the larger (gOSCR - CgOSCR) is, the more stable the system is.

Moreover, we can extend gOSCR to represent the condition when the stability margin reaches given damping ratio (named as gOSCR$_{\text{min}}$). This will be described in the next section.

### III. OPTIMIZATION MODEL

In this section, we use gOSCR to formulate SSSCs in optimization problem (1), instead of damping ratio constraints, which can avoid establishing system’s detailed models. Besides, a method is provided to evaluate upper limit of gOSCR$_{\text{min}}$, (i.e., gOSCR$_{\text{U min}}$), where gOSCR$_{\text{min}}$ represents the value of gOSCR, when system’s stability margin reaches given value, i.e., system’s damping ratio demand in this paper. By replacing gOSCR$_{\text{min}}$ with gOSCR$_{\text{U min}}$, we simplify the formulated gOSCR-based optimization problems.

#### A. Optimization Modelling Based on gOSCR

As discussed in Section III, gOSCR can evaluate small-signal stability of the multi-IBR system under non-rated operating conditions. On this basis, gOSCR is used to formulate SSSCs in operation adjustment optimization problem (1) described as:

$$\min \{ \sum_{i=1}^{n} (P_i - P_{ei})^2 \}$$

subject to:

$$\begin{align*}
\sum_{i=1}^{n} P_i &= \sum_{i=1}^{n} P_{ei} = P_{set} \\
0 &\leq P_i \leq P_{i,\text{max}}, \ i = 1, \ldots, n \\
U_{min} &\leq U_i \leq U_{max}, \ i = 1, \ldots, n \\
gOSCR &\geq gOSCR_{\text{min}}
\end{align*}$$

subject to:

$$\begin{align*}
P_i + jQ_i &= \text{diag} \{ U \} \left( jB_{\text{loc}} \tilde{U} + jB_i E \right)
\end{align*}$$

where $P_{ei}$, $P_{set}$, and $P_{\text{max}}$ are variables of active power output, its initial order, and its upper limit of IBR; note that IBRs are commonly required to have active power reserves [20]; first equation in (6) represents constraint of power balance; $U_i$, $U_{\text{min}}$, and $U_{\text{max}}$ are terminal voltage, required lower and upper limits; gOSCR$_{\text{min}}$ is required small-signal stability limit; “$\cdots$” denotes conjugated vector; last equality constraint in (6) is power flow equations, wherein italic bold letter and bold letter denote a vector and a matrix; $P_i = [P_{ei}, \ldots, P_{n0}]^T$ and $Q_i = [Q_{ei}, \ldots, Q_{n0}]^T$, wherein “$T$” denotes transposed vector, and $Q_{ei}$ is IBR’s reactive power output and equal to zero; $U = [U_i + j U_j]$; $U_i = [U_{i1}, \ldots, U_{in}]^T$, $U_j = [U_{j1}, \ldots, U_{jn}]^T$; $U_{i0}$ and $U_{j0}$ are real and imaginary components of voltage at node $i$; $j$ is current vector; $B_i = [B_{1(i+1)}, \ldots, B_{n(i+1)}]^T$ is admittance vector and $B_{i(j+1)}$, $(i=1, \ldots ,n)$ is mutual admittance between nodes $i$ and $n+m+1$; $E$ is voltage of infinite node and its angle is set as 0.

Objective function in (6) is designed to satisfy SSSCs with minimal adjustment amounts of IBRs’ active power outputs. However, optimization problem (6) may have no solutions. In other words, the system may not reach given stability margin by operation adjustments with constraint of $\sum_{i=1}^{n} P_i = P_{set}$ in (6), which will be demonstrated in Section V. To deal with this issue, we relax total active power output of all IBRs, since the decrease of IBRs’ active power outputs can improve small-signal stability. In this case, the objective function of concerned operation-adjustment optimization problem (6) is minimal decrement of total active power outputs of IBRs with SSSCs. This objective function can also be described as maximizing total active power outputs of IBRs, formulated as

$$\max \{ \sum_{i=1}^{n} P_i \}$$

subject to:

$$\begin{align*}
0 &\leq P_i \leq P_{\text{max}}, \ i = 1, \ldots, n \\
U_{min} &\leq U_i \leq U_{max}, \ i = 1, \ldots, n \\
gOSCR &\geq gOSCR_{\text{min}}
\end{align*}$$

subject to:

$$\begin{align*}
P_i + jQ_i &= \text{diag} \{ U \} \left( jB_{\text{loc}} \tilde{U} + jB_i E \right)
\end{align*}$$

Based on optimization problems (6) and (7), calculation
process of optimal operation adjustments of IBRs with SSSCs can be described as: 1) calculate optimal solution of optimization problem (7); 2) during the iterations, if the feasible solution $P_{nl} = \sum_{i=1}^{n} P_{nl,i}$ is larger than $P_{all}$, i.e., $P_{nl} \geq P_{all}$, this means the system can reach given stability limit by operation adjustments without reducing IBRs’ total active power outputs. That is, optimization problem (6) has solutions, and we can obtain optimal operation adjustments for IBRs by solving optimization problem (6). In this case, we can use the last feasible solution of optimization problem (7) as initial starting operating point for solving optimization problem (6) to reduce solution iterations. Otherwise, the optimal solution of optimization problem (6) is desired operation adjustment.

B. Evaluating Upper Limit of $gOSCR_{\text{min}}$ (i.e., $gOSCR_{\text{min}}^{u}$)

To solve optimization problems (6) and (7), one needs to determine the value of $gOSCR_{\text{min}}$, i.e., the value of $gOSCR$ when system’s stability margin reaches given limit. Typically, the small-signal stability limit of power systems is described by damping ratio, defined as $\zeta_{\text{min}}$. Thus, $gOSCR_{\text{min}}$ can be described as the corresponding $gOSCR$, when the system damping ratio reaches $\zeta_{\text{min}}$, given as

$$gOSCR_{\text{min}} = \arg \left\{ \det \left( \tilde{Y}_g(s_n) \right) \gamma^{-1}(s_n) + gOSCR \ I_2 \right\} = 0 \right\}$$

where $s_n = \sigma_n + j \omega_n$ is system’s dominant eigenvalues, $\zeta_{\text{min}} = -\frac{\sigma_n}{\sqrt{\sigma_n^2 + \omega_n^2}}$ is required damping ratio. However, it may be hard to calculate $gOSCR_{\text{min}}$ by (8), since it is related with all IBRs’ dynamics referring to (2) and (3). To simplify evaluation of $gOSCR_{\text{min}}$, an upper limit of $gOSCR_{\text{min}}$ (named as $gOSCR_{\text{min}}^{u}$) is evaluated in the following:

Note that varying voltage amplitude has little impact on $\tilde{Y}_{g,22}(s)$ in (4) by experiment. Due to this, the equivalent device’s dynamic $\tilde{Y}_g(s)$ in (8) can be rewritten as

$$\tilde{Y}_g(s) = \sum_{i=1}^{m} p_{io} \tilde{Y}_{io}(s), \quad \tilde{Y}_{io}(s) = \begin{bmatrix} 0 & 0 \\ 0 & \tilde{Y}_{g,22}(s) \end{bmatrix}$$

(9)

$$\tilde{Y}_{g,22}(s) = \frac{H_{g,22}(s)}{(H_{g,22}(s) + s \ell_k) \left\{ 1 + H_{g,22}(s) \right\}}$$

(10)

We can see from (9) that $\tilde{Y}_g(s)$ is the weighted sum of $\tilde{Y}_{io}(s)$. According to Ref. [25] and (10), small-signal stability of the system in (8) can be considered to be bounded by $n$ SIIBSs, wherein IBR’s dynamic is one of $\tilde{Y}_{io}(s)$ and SCR is equal to $gOSCR$. In other words, we can consider maximal value of $gOSCR_{\text{min}}$, $(i=1,...,n)$ in these $n$ SIIBSs as upper limit of $gOSCR_{\text{min}}$, i.e., $gOSCR_{\text{min}}^{u}$. $gOSCR_{\text{min}}$ is the value of SCR in these $n$ SIIBSs, when system’s damping ratio reaches $\zeta_{\text{min}}$. To avoid establishing detailed dynamics $\tilde{Y}_g(s)$ in (9) for evaluating $gOSCR_{\text{min}}$, one can evaluate $gOSCR_{\text{min}}^{u}$, since $gOSCR > gOSCR_{\text{min}}^{u}$ means $gOSCR > gOSCR_{\text{min}}$. $gOSCR_{\text{min}}^{u}$ can be evaluated by following steps:

1) For an $n$-IBR system, $n$ small-signal stability limits (defined as $gOSCR_{\text{min,}i}$, ..., $gOSCR_{\text{min,n}}$) is evaluated by $n$ characteristic equations $\det \left[ \tilde{Y}_{io}(s_n) \gamma^{-1}(s_n) + gOSCR_{\text{min,i}} \ I_2 \right] = 0$ with $\zeta = \zeta_{\text{min}}$, $(i=1,...,n)$. If IBRs are “black-boxed” models, $gOSCR_{\text{min}}$ can be evaluated by experiment [25];

2) Sort $gOSCR_{\text{min},i}$, $(i=1,...,n)$ from the largest to smallest. $gOSCR_{\text{min}}^{u}$ is the largest one.

To simplify the difficulty of solving optimization problems (6) and (7), we replace $gOSCR_{\text{min}}$ with $gOSCR_{\text{min}}^{u}$ in SSSCs. Note that if the multi-IBR system is homogeneous, the optimal solutions of optimization problems (6) and (7) are the same for the scenarios whether replacing $gOSCR_{\text{min}}$ with $gOSCR_{\text{min}}^{u}$ or not, since $gOSCR_{\text{min}} = gOSCR_{\text{min}}^{u}$; if the system is heterogeneous, optimal solutions of optimization problems (6) and (7) with constraints of $gOSCR > gOSCR_{\text{min}}^{u}$ can be considered as near-optimal solutions for optimization problems (6) and (7) with constraints of $gOSCR > gOSCR_{\text{min}}$.

IV. SOLUTION ALGORITHM

In this section, we first propose a sequential approach to deal with the discontinuity of formulated $gOSCR$-based optimization problems, which decomposes $gOSCR$-based optimization problems into sequential sub-optimization problems (SOPs). Then, an adaptive method is proposed to dynamically adjust the region of proposed SOPs for ensuring the existence of feasible solutions. Besides, a convex-relaxing method is proposed to choose the proper initial starting operating point, which can enhance calculating efficiency for solving SOPs. Finally, the algorithm for solving optimal operation adjustment of IBRs is provided.

A. Sequential Approach for Solving Optimization Problems

Optimization problems (6) and (7) with constraints of $gOSCR \geq gOSCR_{\text{min}}^{u}$ are discontinuous and nonlinear, since they contain constraints of $gOSCR$-based eigenvalue inequalities and nonlinear power flow equations. To solve these nonlinear optimization problems, a sequential approach is proposed. In the proposed approach, the first-order sensitivity of $gOSCR$ with respect to IBRs’ active power outputs (defined as $dgOSCR/dP_e$) are used to decide the direction of active power adjustments. Since $gOSCR$ of $U^T \cdot P_e B_{red}$ is related with nonlinear power flow equations, the sensitivity $dgOSCR/dP_e$ is only meaningful in neighborhood of a certain operating point and should be limited within a reasonable region. Hence, optimization problems (6) and (7) with constraints of $gOSCR \geq gOSCR_{\text{min}}^{u}$ are decomposed into a sequence of sub-optimization problems (SOPs) with different starting operating points, as shown in Fig. 2. The starting operating point for each SOP is optimal solution of the last SOP. Based on the solutions of sequential SOPs, we extract a path of operating points to optimal solutions of optimization problems (6) and (7). These SOPs are similar to optimization problems (6) and (7) with constraints of $gOSCR \geq gOSCR_{\text{min}}^{u}$ (named as $P_1$ and $P_2$), and the only difference is that $gOSCR$-based inequality is replaced by

$$\sum_{i=1}^{n} (P_{nl,i} - P_{nl,i}) \frac{dgOSCR}{dP_{nl}} \geq \zeta_0$$

(11)
where $P'_{e0} (i=1,\ldots,n)$ are values of starting operating point in each SOP; $dgOSCR/dP_{e0}$ is sensitivity of gOSCR for $P_{e0}$; $\zeta_0$ is required gOSCR’s increment in each SOP; second inequation in (11) represents reasonable region, wherein $D$ is the upper limit of adjustable deviation ($P_e - P_{e0}$).

The deviation of $dgOSCR/dP_{e0}$ in (11) is given as follows: we demonstrate that $gOSCR^{1\leq} = \max \{U^T P Z_{red}\}$, since $U^T P Z_{red}$ is a positive semi-definite matrix. Here, $\max \{ \}$ is maximal eigenvalue. Note that $gOSCR^{1\leq} = \max \{U^T P Z_{red}\}$ can consider the case that IBRs’ active power output $P_{e0}$ is zero. On this basis, we obtain that

$$d_gOSCR = -gOSCR^2 u^T [dP_e - 2U^T (dU) P Z_{red}]$$

where “$d$” denotes partial derivative.

According to (12), we should calculate sensitivity $dU/dP_e$ by linearizing power flow equations in (6) at the operating point:

$$\frac{dU}{dP_e} = \begin{bmatrix} A_{1i} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} dU \\ dP_e \end{bmatrix}$$

where $A_{1i}, A_{12}, A_{21}, A_{22}$ are coefficient matrices, e.g., $A_{1i} = dP_i/dU_i$; subscript “0” represents the value at the operating point; $U_{0i} = \text{diag} \{ U_{0i} \}$, $U_{0s} = \text{diag} \{ U_{0s} \}$, $U_{0b} = \text{diag} \{ U_{0b} \}$.

By schur complement, Eqn. (13) can be converted as

$$\frac{dU}{dP_e} = (U_0 - U_i A_{1i}^{-1} A_{12})(A_{1i} - A_{12} A_{22}^{-1} A_{21})^{-1}$$

Combining (14) with (12), $dgOSCR/dP_{e0}$ can be obtained.

Fig. 2 Obtained optimal operating point by proposed sequential approach.

B. Evaluating $D, \zeta_0$ and Initial Starting Operating Point

As discussed in Section IV.A, $dgOSCR/dP_{e0}$ is only meaningful in the neighborhood of a certain operating point. Therefore, $\zeta_0$ and $D$ in (11) should be properly chosen. To be specific, if $\zeta_0$ is too big, established SOPs (i.e., $P_1$ and $P_2$) may be infeasible; but too small $\zeta_0$ may establish too many $P_1$ and $P_2$, which highly increases computation demand. Similarly, if $D$ is too big, eigenvalue sensitivity $dgOSCR/dP_{e0}$ in (11) may be invalid and thus the solutions of $P_1$ and $P_2$ may be invalid; otherwise, if $D$ is too small, $\zeta_0$ should be very small to ensure existence of solutions for $P_1$ and $P_2$, which will need much computation power. Moreover, an improper initial operating point may cause slow convergence of established SOPs and thus a high computation demand. Here, we propose methods to adaptively adjust $\zeta_0$ and $D$ in each $P_1$ and $P_2$, and to find a proper initial starting operating point.

1) given $\zeta_0$ and $D$: We firstly establish an optimization problem (named as $P_3$) with objective function in (15) and constraints of power flow equations in (6) to find maximal $\zeta_0$ (i.e., $\zeta_{0\text{max}}$). Then, $\zeta_0$ is determined by (16) to ensure feasible solutions of $P_1$ and $P_2$.

$$\max \zeta_{0\text{max}} = \sum_{i=1}^{n} (P_{ei} - P_{e0}) \frac{dgOSCR}{dP_{e0}}$$

where $a$ is a given positive number smaller than 1, e.g., 0.9.

Similarly, when we need to decrease gOSCR in a certain SOP, we should find minimal $\zeta_0$. This can be evaluated by establishing an optimization problem, similar as $P_3$, and the only difference is the objective function is “min” not “max”.

$D$ can be determined as follows: Firstly, an initial $D_0$ is given, e.g., $D_0=0.1$; then, we update $D$ based on assessing accuracy of $\zeta_0$ sensitivity, given as (17). On this basis, $D$ can be determined as (18): 1) if $|\varepsilon| > 0.1$ meaning that the error of $\zeta_0$ sensitivity is bigger than 10%, we decrease $D$ and define $D_{k+1} = D_k + b D_k$, ($k=0,1,\ldots), where b$ is a positive number smaller than 1; then replace $D$ with $D_{k+1}$ in $P_3$, and solve $P_3$ for $\varepsilon$ in (17); 2) if $|\varepsilon| \leq 0.1$, it has that $D = D_{k+1} = D_k$ and we stop solving $P_3$.

$$\varepsilon = \frac{\zeta_{0\text{max}}}{gOSCR(P_e, U) - gOSCR_0} - 1$$

where $gOSCR(P_e, U)$ is gOSCR at operating point ($P_e, U$), which is the solution of $P_3$; $gOSCR_0$ is initial value of gOSCR for each $P_1$ and $P_2$.

$$D_{k+1} = b D_k, \; \varepsilon > 0.1,$$

$$D_{k+1} = D_k, \; \varepsilon \leq 0.1, \; k=1,2...$$

2) Initial starting operating point: Optimizing problems (6) and (7) with constraints of $gOSCR \geq gOSCR_{min}$ can be relaxed as semi-definite programming (SDP) problems to obtain a proper initial starting operating point. That is, if we omit the variation of terminal voltage of each IBR and set them as 1 p.u., these optimization problems are simplified as (19.a) and (19.b):

$$\min \sum_{i=1}^{n} (P_{ei} - P_{e0})^2$$

$$\sum_{i=1}^{n} P_{ei} = P_{e0}$$

$s.t.$

$$gOSCR \geq gOSCR_{min}$$

$$0 \leq P_{ei} \leq P_{e\text{max}}, \; i=1,\ldots,n$$

$$\max \sum_{i=1}^{n} P_{ei}$$

$s.t.$

$$gOSCR \geq gOSCR_{min}$$

$$0 \leq P_{ei} \leq P_{e\text{max}}, \; i=1,\ldots,n$$

To convert optimization problems in (19.a) and (19.b) as SDPs, a following proposition is given.

Proposition 1: Define $P_{ek} \in \mathbb{R}^{k \times k}$, ($k \in \{1,\ldots,n\}$) as a diagonal positive-definite matrix and $B_{red}$ as $B_{red} = \begin{bmatrix} B_{11k} & B_{12k} \\ B_{21k} & B_{22k} \end{bmatrix}$, where $B_{11k} \in \mathbb{R}^{k \times k}, B_{12k} \in \mathbb{R}^{k \times (k-1)}, B_{21k} \in \mathbb{R}^{(k-1) \times k}, B_{22k} \in \mathbb{R}^{(k-1) \times (k-1)}$ are submatrices of $B_{red}$. Then, it satisfies that:

$$\lambda_{\text{max}} \{ P_{ek} B_{red} \} \geq r \Leftrightarrow B_{red} - r \left[ \begin{bmatrix} P_{ek} \\ 0 \end{bmatrix} \right] \succeq 0$$

where $B_{red} = B_{11k} - B_{12k} B_{22k}^{-1} B_{21k}$ is a node-reduced susceptance matrix; $r$ is a given constant; “$\Leftrightarrow$” represents “be equivalent to”; $0$ is a proper zero matrix; “$\succeq$” denotes Löwner partial order, in which $A \succeq 0$ means that $A$ is positive semi-definite.
Proof. The right inequality in (20) can be written as:

\[
B_{\text{red}} - rP_{\text{eg}} \geq 0 \iff \begin{bmatrix}
B_{11k} - rP_{\text{eg}} & B_{12k} \\
B_{21k} & B_{22k}
\end{bmatrix} \geq 0
\]  

By Schur complement, inequation (21) can be converted to:

\[
B_{22k} \geq 0 \\
B_{\text{red}} - rP_{\text{eg}} \geq 0
\]  

Since \( B_{\text{red}} \) is a susceptance matrix of a pure-inductive network, its submatrix \( B_{22k} \) always satisfies that \( B_{22k} \geq 0 \). On this basis, inequation (22) can be simplified as:

\[
B_{\text{red}} - rP_{\text{eg}} \geq 0
\]  

Since \( P_{\text{eg}} \) is positive-definite, inequation (23) is converted to:

\[
\frac{1}{r}B_{\text{red}}P_{\text{eg}} - I \geq 0 \iff \lambda_{\min}(P_{\text{eg}}B_{\text{red}}) \geq r
\]  

where \( \frac{1}{r}B_{\text{red}}P_{\text{eg}} \) and \( P_{\text{eg}}B_{\text{red}} \) are similar (have the same eigenvalues), since it satisfies \( \frac{1}{r}P_{\text{eg}}B_{\text{red}} = P_{\text{eg}}B_{\text{red}} \).

Proposition 1 shows that inequality \( g\text{OSCR} \geq g\text{OSCR}_{\text{min}} \) in optimization problems (19.a) and (19.b) can be converted to

\[
g\text{OSCR} \geq g\text{OSCR}_{\text{min}} \iff B_{\text{red}} - g\text{OSCR}_{\text{min}}P_{\text{eg}} \geq 0
\]  

Note that we define \( P_{\text{eg}} \in \mathbb{R}^{n \times n} \) in Proposition 1, but not \( P_{\text{eg}} \in \mathbb{R}^{n \times n} \) due to that some \( P_{\text{eg}} \) (\( i = 1,\ldots,n \)) may be zero, which will be demonstrated in Section V. In this case, \( P_{\text{eg}} \) is irreversible and thus \( g\text{OSCR} \) cannot be calculated by \( P_{\text{eg}}B_{\text{red}} \). To obtain \( g\text{OSCR} \), one needs to transform \( P_{\text{eg}}B_{\text{red}} \) to \( P_{\text{eg}}B_{\text{red}} \) in Proposition 1.

With (25), optimization problems (19.a) and (19.b) can be converted as SDPs (26) and (28) [26], respectively:

\[
\min \quad tr(X) \\
\text{s.t.} \quad X \geq 0 \\
X = \begin{bmatrix}
X_1 \\
X_2 & X_3 \\
X_4 & X_5
\end{bmatrix} = \begin{bmatrix} t \\
X_2 & X_3 & X_4 & X_5
\end{bmatrix} = \begin{bmatrix} t \\
P_e^1 - P_e^0 \\
-I
\end{bmatrix}
\]  

\[
X_2 = B_{\text{red}} - g\text{OSCR}_{\text{min}} \text{diag}(P_e) \\
X_3 = \text{diag}(P_e) \\
X_4 = \text{diag}(P_{\text{max}} - P_e)
\]  

where \( P_e = [P_{e1}, P_{e2}, \ldots, P_{en}]^T \), in which \( P_{ei} \) satisfies \( p_{mi} = p_{m0} - \sum_{i=1}^{n} p_{ei} \); \( P_{e0} = [P_{e10}, P_{e20}, \ldots, P_{eon}]^T \), \( t \) is a variable.

\[
\min \quad \sum_{i=1}^{n} p_{ei}
\]  

\[
\begin{bmatrix} B_{\text{red}} & 0 \\
0 & \sum_{i=1}^{n} g\text{OSCR}_{\text{min}} \text{diag}(P_{\text{max}}) - I
\end{bmatrix} X_{\text{eg}} \geq 0
\]

By solving SDPs (26) and (28), we can obtain initial active power outputs \( P_e^* = [P_{e1}^*, P_{e2}^*, \ldots, P_{en}^*]^T \). Then, submitting \( P_e^* \) into power flow equations in (6), a proper initial starting operating point \( (P_e^*, U^*) \) is obtained. We note that this obtained operating point \( (P_e^*, U^*) \) is near optimal solutions of sequential \( \text{P}_1 \) and \( \text{P}_2 \) by experiment, which highly enhances calculating efficiency. This will be detailly discussed in Section V.

C. Algorithm of Proposed Operation-Adjustment Method

As discussed above, the detailed implementation procedure of proposed operation-adjustment method is illustrated in Fig. 3. Considering that when solving sequential \( \text{P}_1 \), the solutions may oscillate, we give Criterion 1 to avoid the oscillation. Besides, Criterion 2 is provided to stop solving sequential \( \text{P}_2 \).

Fig. 3 implementation procedure of proposed operation-adjustment method.

**Criterion 1:** Record the solution of sequential \( \text{P}_1 \) in each iteration. If the minimal value of these solutions decreases not larger than c within m iterations, then iteration stops. Here, c and m are given constants, e.g., \( c=0.001 \), \( m=6 \). The minimal solution of sequential \( \text{P}_1 \) is desired operation adjustment.

**Criterion 2:** If the increment of objective function \( \sum_{i=1}^{n} p_{ei} \) is smaller than given constants (i.e., small enough) during iterations, then stop solving sequential \( \text{P}_1 \) and the last solution of \( \text{P}_2 \) is desired optimal operation adjustment.

The main steps of proposed operation-adjustment method are summarized as follows:

1) Calculate \( g\text{OSCR} \) at initial operating point and \( g\text{OSCR}_{\text{min}} \). If \( g\text{OSCR} \) is bigger than \( g\text{OSCR}_{\text{min}} \), the system’s stability satisfies the demand, **end**; Otherwise, the system faces the risk of small-signal stability issues and needs to adjustment IBRs’ active power outputs (go to step 2).

2) Calculate proper initial starting operating point \( (P_e^*, U^*) \): solve SDP (28). If the sum of the solution is bigger than \( P_{e0} \), solve SDP (26) and its optimal solution is \( P_e^* \); otherwise, \( P_e^* \) is the optimal solution of SDP (28). Then, submit \( P_e^* \) in power flow equations in (6), and calculate \( (P_e^*, U^*) \) and corresponding...
gOSCR. If \( gOSCR \geq gOSCR_{\text{min}}^{U} \) and \( \sum_{i}^{n} P_{r}^{i} = P_{\text{min}} \), go to step 3; otherwise, go to step 4.

3) Solve \( P_{1} \) for \( \zeta_{0} \) and \( D \), and then solve sequential \( P_{1} \). If iterations of \( P_{1} \leq N_{\text{max}} \) or it satisfies Criterion 1, go to step 5; otherwise, repeat step 3.

4) Solve \( P_{3} \) for \( \zeta_{0} \) and \( D \), and then solve sequential \( P_{2} \). If the solution of \( P_{2} \) satisfies \( \sum_{i}^{n} P_{r}^{i} = P_{\text{max}} \), go to step 3; if iterations of \( P_{2} \geq N_{\text{max}} \) or it satisfies Criterion 2, go to step 5; otherwise, repeat step 4.

5) Output \( P_{r}, i = 1, \ldots, n \) and its sum of \( \sum_{i}^{n} P_{r}^{i} \), which is the desired operation adjustment of all IBRs.

V. CASE STUDIES

To verify the effectiveness of proposed operation-adjustment method, a system with 11 IBRs is created in MATLAB/Simulink as shown in Fig. 4. In this system, nodes 1-11 are connected to IBRs and node 39 is connected to external grids simplified as ideal voltage source. Network parameters of this multi-IBR system can be found in [27]. Control parameters of IBRs are given in TABLE I. To demonstrate the validity of the proposed method, we use the full-order electromagnetic model of IBRs. Rated capacities of all IBRs are given in TABLE II. Three operating conditions are considered as shown in TABLE III. Optimization problems \( P_{1}, P_{2} \) and \( P_{3} \) are solved by IPOPT [18], and SDPs (26) and (28) are solved by YALMIP [28].

In the eigenvalue analysis, several cases are created by equally increasing active power outputs of IBRs at nodes 1–3 from 0.1 to 3 p.u., while keeping the other IBRs operating at their given active power outputs based on case 1) of TABLE III. Note that the terminal voltage is nearly 1 p.u. in the equivalent subsystem (8) for solving \( gOSCR_{\text{min}}^{U} \). To ensure that the 11-IBR system can be represented by the subsystem (8) for small-signal stability analysis under these cases, we ensure IBRs’ terminal voltage is nearly 1 p.u. in the 11-IBR system by adjusting the voltage at node 39. Under these cases, we evaluate gOSCR and damping ratio of dominant eigenvalues in the 11-IBR system and the most unstable system among \( n \) SIIBSs as shown in Fig. 5. In these \( n \) SIIBSs, SCR is equal to gOSCR of 11-IBR system and the device’s dynamic is one of \( Y_{d} (s) \) in (9), as described in Section III. In Fig. 5, black (or red) curve is the loci of damping ratio of dominant eigenvalues in 11-IBR system (or most unstable SIIBS). We can see from Fig. 5 that black curve is above red curve, which suggests that small-signal stability of 11-IBR system has a lower limit. In other words, we can find an upper limit \( gOSCR_{\text{min}}^{U} \) for small-signal stability limit \( gOSCR_{\text{min}} \) by most unstable SIIBS. These results are consistent with theoretical analysis in Section III.

**TABLE I**

<table>
<thead>
<tr>
<th>IBRs</th>
<th>PLL ( H_{d}(s) )</th>
<th>DC voltage control ( \text{current} )</th>
<th>Constant ( \text{power} )</th>
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<tr>
<td>1–4</td>
<td>15+8500/s</td>
<td>/</td>
<td>1+10/s</td>
</tr>
<tr>
<td>5–8</td>
<td>14+8200/s</td>
<td>0.4+5/s</td>
<td>/</td>
</tr>
<tr>
<td>9–11</td>
<td>15+8500/s</td>
<td>0.4+5/s</td>
<td>/</td>
</tr>
</tbody>
</table>

Filter inductance \( L_{f} \), filter capacitance \( C_{d} \), dc capacitance \( C_{dc} \): 0.05, 0.05, 0.038; Transfer function of current control \( H_{d}(s) \): 0.6+10/s;

**TABLE II**

<table>
<thead>
<tr>
<th>IBR1</th>
<th>IBR2</th>
<th>IBR3</th>
<th>IBR4</th>
<th>IBR5</th>
<th>IBR6</th>
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<tr>
<td>3</td>
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<td>3</td>
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**TABLE III**

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<th>Case1</th>
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<th>IBR2</th>
<th>IBR3</th>
<th>IBR4</th>
<th>IBR5</th>
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<table>
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<tr>
<th>Case2</th>
<th>IBR1</th>
<th>IBR2</th>
<th>IBR3</th>
<th>IBR4</th>
<th>IBR5</th>
<th>IBR6</th>
</tr>
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<tbody>
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<table>
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<th>Case3</th>
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<th>IBR2</th>
<th>IBR3</th>
<th>IBR4</th>
<th>IBR5</th>
<th>IBR6</th>
</tr>
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<tbody>
<tr>
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<tr>
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<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

In the eigenvalue analysis, several cases are created by equally increasing active power outputs of IBRs at nodes 1–3 from 0.1 to 3 p.u., while keeping the other IBRs operating at their given active power outputs based on case 1) of TABLE III. Note that the terminal voltage is nearly 1 p.u. in the equivalent subsystem (8) for solving \( gOSCR_{\text{min}}^{U} \). To ensure that the 11-IBR system can be represented by the subsystem (8) for small-signal stability analysis under these cases, we ensure IBRs’ terminal voltage is nearly 1 p.u. in the 11-IBR system by adjusting the voltage at node 39. Under these cases, we evaluate gOSCR and damping ratio of dominant eigenvalues in the 11-IBR system and the most unstable system among \( n \) SIIBSs as shown in Fig. 5. In these \( n \) SIIBSs, SCR is equal to gOSCR of 11-IBR system and the device’s dynamic is one of \( Y_{d} (s) \) in (9), as described in Section III. In Fig. 5, black (or red) curve is the loci of damping ratio of dominant eigenvalues in 11-IBR system (or most unstable SIIBS). We can see from Fig. 5 that black curve is above red curve, which suggests that small-signal stability of 11-IBR system has a lower limit. In other words, we can find an upper limit \( gOSCR_{\text{min}}^{U} \) for small-signal stability limit \( gOSCR_{\text{min}} \) by most unstable SIIBS. These results are consistent with theoretical analysis in Section III.

**Fig. 4** One-line diagram of a 11-IBR system.

A. Verification of Upper limit of \( gOSCR_{\text{min}} \) (i.e., \( gOSCR_{\text{min}}^{U} \))

The proposed operation adjustment method for IBRs to ensure small-signal stability is based on gOSCR as described in Section III, where gOSCR can evaluate small-signal stability of the multi-IBR system under varying operating conditions. Considering that the calculation of \( gOSCR_{\text{min}} \) is related with dynamics of all IBRs, which may be hard to be solved, we evaluate upper limit of \( gOSCR_{\text{min}} \), i.e., \( gOSCR_{\text{min}}^{U} \) instead of \( gOSCR_{\text{min}} \) to simplify the difficulty of concerned operation adjustment issues. Before validating proposed operation adjustment method, we first verify that \( gOSCR_{\text{min}} \) has an upper limit as described in Section III.B in 11-IBR system.
B. Verification of Proposed Operation adjustment Method

To verify the validity of proposed operation-adjustment method for IBRs, three different operating conditions for 11-IBR system are considered, wherein IBRs’ active power outputs are given in TABLE III. To simplify the analysis, we set maximal active power outputs of all IBRs the same as their rated capacities in TABLE III. Besides, the small-signal stability limit is set as \( \zeta_{\text{min}}=0.03 \). As shown in Fig. 5, the corresponding \( g_{\text{OSCR}}^{\text{all}}=3 \).

![Fig. 6 Loci of dual gap between feasible solutions of SDPs for cases 2) and 3) when increasing iterations.](image)

According to the implementation procedure of proposed operation-adjustment method in Section IV.C, we calculate \( g_{\text{OSCR}} \) for cases 1) to 3) of 11-IBR system: case1) \( g_{\text{OSCR}}=3.07 \); case2) \( g_{\text{OSCR}}=1.91 \); case3) \( g_{\text{OSCR}}=1.8 \). By comparing \( g_{\text{OSCR}} \) with \( g_{\text{OSCR}}^{\text{U}} \), we can see that for case 1) \( g_{\text{OSCR}} > g_{\text{OSCR}}^{\text{U}} \), which means that the system satisfies small-signal stability demand (i.e., \( \zeta_{\text{min}}=0.03 \)) and does not need operation adjustments; for cases 2) and 3), \( g_{\text{OSCR}} < g_{\text{OSCR}}^{\text{U}} \), which suggests that the system does not satisfy small-signal stability demand and needs operation adjustment of IBRs.

![Fig. 7 Under No. i of sequential \( P_i \) for case2). (a) computational information when solving \( P_i \); (b) computational information when solving \( P_i \).](image)

Then, we solve SDPs (26) and (28) by YALMIP solver to obtain initial starting operating points for cases 2) and 3). We first solve SDP (28) to obtain maximal allowable sum of all IBRs’ active power outputs \( P_{\text{all}}^* \). Since each IBR’s maximal active power output is assumed to be the same (i.e., rated capacity) for cases 2) and 3), iteration process of SDP (28) is the same for cases 2) and 3). Fig. 6 shows loci of dual gap of feasible solutions for SDP (28). We can see from Fig. 6 that after 12 iterations the dual gap is nearly \( 10^{-5} \approx 0 \), which means that the SDP (28) finds global optimal solution for cases 2) and 3). And the obtained \( P_{\text{all}}^* \) is 12.605 p.u. larger than the sum of IBRs’ initial active power output \( P_{\text{all}}=12.6 \) p.u. for case 2). This demonstrates that when ignoring the change of IBRs’ terminal voltage (assumed to be 1 p.u.), we can satisfy system’s small-signal stability demand by operation adjustment without reducing the sum of IBRs’ active power output for case 2). That is, the SDP (26) has solutions. Then, we solve the SDP (26) for case 2). We can see from Fig. 6 that after 13 iterations we find global optimal solution of SDP (26) for case 2), which can be used as initial starting operating point for solving sequential \( P_1 \sim P_3 \).

![Fig. 8 Under No. i of sequential \( P_i \) for case2). (a) computational information when solving \( P_i \); (b) computational information when solving \( P_i \).](image)

<table>
<thead>
<tr>
<th>TABLE IV</th>
<th>OPTIMAL OPERATION ADJUSTMENTS FOR CASE 2 (PER-UNIT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IBR1</td>
<td>0.5840</td>
</tr>
<tr>
<td>IBR2</td>
<td>0.6083</td>
</tr>
<tr>
<td>IBR3</td>
<td>0.6716</td>
</tr>
<tr>
<td>IBR4</td>
<td>1.8326</td>
</tr>
<tr>
<td>IBR5</td>
<td>0.7621</td>
</tr>
<tr>
<td>IBR6</td>
<td>0.1015</td>
</tr>
<tr>
<td>IBR7</td>
<td>0.3199</td>
</tr>
<tr>
<td>IBR8</td>
<td>0.5133</td>
</tr>
<tr>
<td>IBR9</td>
<td>0.2816</td>
</tr>
<tr>
<td>IBR10</td>
<td>0.8652</td>
</tr>
</tbody>
</table>

On the other hand, \( P_{\text{all}}^* =12.605 \) p.u. is larger than the sum of IBRs’ active power outputs \( P_{\text{all}}=13.2 \) p.u. (\( P_{\text{all}}^* < P_{\text{all}} \)) for case 3). This demonstrates that the SDP (26) has no solutions for case 3). And we use the obtained IBRs’ active power outputs by solving the SDP (28) as initial starting operating point for case 3). Then, submitting IBRs’ active power outputs \( P_{\text{all}}^* \)
(obtained by solving SDPs (26) and (28)) into power flow equations in (6), we can obtain initial starting operating point \( (P_r, V) \) for cases 2) and 3). The corresponding gOSCR = 2.953 (or 2.948) for case 2) (or case 3)) smaller than gOSCR_\text{min}, but near gOSCR_\text{u}. This demonstrates that the obtained initial starting operating point is near the desired operating point. We will discuss solving sequential \( P_1 \sim P_3 \) for cases 2) and 3) in the following.

1) case 2)

According to the algorithm procedure in Section IV.C, we first solve sequential \( P_3 \) and \( P_2 \). Fig. 7 shows corresponding computational information. To demonstrate the superiority of calculating initial starting operating point, we directly use the operating point in TABLE III for solving sequential \( P_3 \) and \( P_2 \) (as a reference) as shown in Fig. 7. We can see in Fig. 7 that it only takes 4 SOPs for sequential \( P_2 \) until Criterion 2 (in Section IV) is satisfied, i.e., \( \sum_{i=1}^{n} P_r > P_{\text{sum}} \) and \( gSCR = gOSCR_\text{u} \), when using initial starting operating point; but it takes 59 SOPs for sequential \( P_1 \) when not using initial starting operating point. This demonstrates the initial starting operating point increases calculating efficiency for solving sequential \( P_2 \).

Since it satisfies that \( \sum_{i=1}^{n} P_r > P_{\text{sum}} \) for sequential \( P_2 \), the sequential \( P_1 \) has solutions. That is, we can ensure satisfying system’s stability demand by operation adjustments without reducing the sum of IBRs’ active power outputs. Fig. 8 shows corresponding computational information, when solving sequential \( P_1 \). We can see from Fig. 8 that under the two scenarios it takes eight SOPs for sequential \( P_1 \) until Criterion 1 is satisfied due to that objective function \( \sum_{n=1}^{m} (P_n - P_{\text{sum}})^2 \) of \( P_1 \) at No. 2 is minimal in the next \( m \) (set as 6) SOPs. That is, the solution of \( P_1 \) at No. 2 is the desired operating point for the concerned operation adjustment issues. The optimal operation adjustment of IBRs is given in TABLE IV. As shown in TABLE IV, the active power output of IBR10 is zero. This indicates that it is necessary to consider the scenario that active power outputs of partial IBRs are zero when solving formulated optimization problems, as discussed in Section IV.B.

Besides, for sequential \( P_1 \), we can see from Fig. 7 and Fig. 8 that \( D \) is unchanged, but \( \zeta_0 \) is varying. This demonstrates that \( D=0.1 \) is commonly reasonable and it only takes one SOP for \( P_1 \) for solving \( \zeta_0 \) in each SOP for \( P_1 \) and \( P_2 \); it is necessary to adaptively adjustment \( \zeta_0 \) to ensure sequential \( P_1 \) and \( P_2 \) have feasible solutions.

2) Case 3)

According to the algorithm procedure in Section IV.C, we solve sequential \( P_2 \) and \( P_3 \). Fig. 9 shows corresponding computational information. The scenario that we directly use initial operating point in TABLE III for solving sequential \( P_2 \) and \( P_3 \) is provided as a reference in Fig. 9. We can see from Fig. 9 that when using initial starting operating point, it takes 10 SOPs for sequential \( P_2 \) until Criterion 2 is satisfied, i.e., \( gSCR = gOSCR_\text{u} \) and the increment of deviation of \( \sum_{i=1}^{n} P_r \) is small enough; but it takes 65 SOPs for sequential \( P_2 \) when not using initial starting operating point. This demonstrates that the initial starting operating point can improve calculating efficiency for solving sequential \( P_2 \). Besides, the obtained maximal \( \sum_{n=1}^{m} P_r \) by solving sequential \( P_2 \) is 12.777 p.u. (or 12.768 p.u.) when (or not) using initial starting operating point, which is smaller than the sum of initial IBRs’ active power outputs \( P_{\text{all}} \). This demonstrates the sequential \( P_1 \) for case 3) has no solutions. In other words, we should decrease the sum of all IBRs’ active power outputs to satisfy the system’s small-signal stability demand. TABLE V shows optimal operation adjustment of IBRs by solving sequential \( P_2 \).

Besides, for sequential \( P_1 \), we can see from Fig. 9 that \( D \) is unchanged; but \( \zeta_0 \) is varying, especially in initial several SOPs for \( P_2 \) (changed in range of \([0.001, 0.1]\)). This demonstrates that \( D=0.1 \) is commonly reasonable and it only takes one \( P_1 \) for solving \( \zeta_0 \) in each SOP of \( P_2 \); and it is necessary to adaptively adjustment \( \zeta_0 \) to ensure the sequential \( P_1 \) has feasible solutions.

Moreover, TABLE VI shows time cost of solving sequential \( P_1 \sim P_3 \) for cases 2) and 3) under the scenarios whether using initial starting operating point or not. We can see from TABLE VI that the calculating efficiency can increase at least 71\%. This verifies the superiority of evaluating initial starting operating point by solving SDPs in (26) and (28). It should be mentioned that in some critical operating conditions, we may not find feasible solutions of the established sequential \( P_2 \) if not using initial starting operating points, due to the nonlinear, nonconvex and discontinuous characteristics of optimization problem (7). Due to page limit, we will not discuss this in detail. Therefore, it is necessary to find a proper initial starting operating point when solving sequential \( P_2 \).

Finally, we mention that by the proposed adjustment operation method, the system’s minimal damping ratio is increased from 0.014 to 0.034 for case 2) and from 0.012 to

\[
\sum_{i=1}^{n} P_r \geq P_{\text{sum}} \quad \text{and} \quad gSCR = gOSCR_\text{u} \quad \text{when using initial starting operating point.}
\]

This demonstrates that the initial starting operating point can improve calculating efficiency for solving sequential \( P_2 \). Besides, the obtained maximal \( \sum_{n=1}^{m} P_r \) by solving sequential \( P_2 \) is 12.777 p.u. (or 12.768 p.u.) when (or not) using initial starting operating point, which is smaller than the sum of initial IBRs’ active power outputs \( P_{\text{all}} \). This demonstrates the sequential \( P_1 \) for case 3) has no solutions. In other words, we should decrease the sum of all IBRs’ active power outputs to satisfy the system’s small-signal stability demand. TABLE V shows optimal operation adjustment of IBRs by solving sequential \( P_2 \).

Besides, for sequential \( P_1 \), we can see from Fig. 9 that \( D \) is unchanged; but \( \zeta_0 \) is varying, especially in initial several SOPs for \( P_2 \) (changed in range of \([0.001, 0.1]\)). This demonstrates that \( D=0.1 \) is commonly reasonable and it only takes one \( P_1 \) for solving \( \zeta_0 \) in each SOP of \( P_2 \); and it is necessary to adaptively adjustment \( \zeta_0 \) to ensure the sequential \( P_1 \) has feasible solutions.
0.033 for case 3), which satisfies system’s stability demand. This illustrates the validity of the proposed method. Due to page limit, time-domain responses of the system are omitted.

**TABLE V**

<table>
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<th>IBR1</th>
<th>IBR2</th>
<th>IBR3</th>
<th>IBR4</th>
<th>IBR5</th>
<th>IBR6</th>
<th>IBR7</th>
<th>IBR8</th>
<th>IBR9</th>
<th>IBR10</th>
<th>IBR11</th>
<th>IBR12</th>
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<tr>
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<td>1.6433</td>
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<td>0.1559</td>
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</tbody>
</table>

**TABLE VI**

<table>
<thead>
<tr>
<th>Case</th>
<th>Time Cost (using initial starting operating point obtained)</th>
<th>Time Cost (not using initial starting operating point)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case2</td>
<td>2.87s</td>
<td>4.91s</td>
</tr>
<tr>
<td>Case3</td>
<td>2.20s</td>
<td>4.51s</td>
</tr>
</tbody>
</table>

**VI. CONCLUSIONS**

This paper proposed a novel method for coordinating active power outputs of IBRs to address IBR-induced small-signal stability problems. It has been demonstrated that:

1) By using gOSCR to formulate SSSCs, the proposed optimization model for operation adjustment of IBRs circumvented the complexity of small-signal stability assessment based on system eigenvalue calculation, due to unknown parameters of “black-boxed” IBRs and high-dimensional state-space model of the system.

2) The proposed sequential solution approach could deal with the discontinuity of established nonlinear optimization problems, which decoupled established optimization problems into a set of sequential sub-optimization problems (SOPs).

3) In each SOP, the proposed adaptive method for adjusting region size could ensure the existence of feasible solutions. Moreover, the proposed convex-relaxing method for finding proper initial starting operating point could increase solving efficiency for established SOPs.

The proposed method can help grid operators coordinate IBRs’ active power outputs for enhancing system’s small-signal stability and the use efficiency of renewable generation. In the future research, we will explore operation adjustments of IBRs considering additional devices, including dynamical loads, energy storage and static Var compensators.

**REFERENCES**


