DD Transmission Part II: Experimental Demonstration and Analysis

Xiong Wu $^{1,1}$, Abdullah S. Karar $^{2}$, Kangping Zhong $^{2}$, Zeynep Nilhan Gürkan $^{2}$, Alan Pak Tao Lau $^{2}$, and Chao Lu $^{2}$

$^{1}$ The Hong Kong Polytechnic University
$^{2}$ Affiliation not available

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Abstract

In this article, the first experimental demonstration of a non-iterative electronic dispersion compensation (EDC) solution implemented at the transmitter using a finite impulse response (FIR) filter optimized with the Gerchberg-Saxton (GS) algorithm, is presented, for intensity-modulation and direct-detection (IM/DD) systems. The theoretical framework for the GS FIR filter and preliminary simulations have been presented in Part I of this work. Here, the performance of the GS FIR filter is compared to that of the iterative GS algorithm in the transmissions of 56-Gb/s on-off keying (OOK) over 80 km of single mode fiber (SMF) with a post feed-forward equalizer (FFE) for combating residual inter-symbol interference (ISI). Furthermore, the influence of the pulse shape (raised cosine or rectangular) and modulation format (return-to-zero (RZ) or non-return-to-zero (NRZ)) on the measured bit error ratio (BER) is experimentally investigated, while changing the number of FIR taps, the number of post-FFE taps, and the number of post-FFE samples-per-symbol. It is shown that within the range of the target digital extinction ratios (DERs) for which the original iterative GS algorithm offers benefit, both analytical and numerical methods for calculating the optimum FIR taps, outlined in Part I of this work, produce similar BER performance as predicted. To that end, the former method is extended, here, through a non-recursive frequency response formula, which offers insight into the action of the GS filter with different pulse shaping and enables the derivation of the explicit GS impulse response. It is also shown that rectangular RZ pulse shaping exploits the full benefit of GS filtering through a uniform spectrum, enabling a BER below the 7% hard-decision forward error correction (HD-FEC) limit, with a 641-tap $T/2$-spaced pre-EDC FIR filter and a 3-tap adaptive $T$-spaced post-FFE.
Gerchberg-Saxton Based FIR Filter for Electronic Dispersion Compensation in IM/DD Transmission
Part II: Experimental Demonstration and Analysis

Xiong Wu, Abdullah S. Karar, Senior Member, IEEE, Kangping Zhong, Zeynep Nilhan Gürkan, Alan Pak Tao Lau, and Chao Lu, Senior Member, IEEE, Fellow, Optica

Abstract—In this article, the first experimental demonstration of a non-iterative electronic dispersion compensation (EDC) solution implemented at the transmitter using a finite impulse response (FIR) filter optimized with the Gerchberg-Saxton (GS) algorithm, is presented, for intensity-modulation and direct-detection (IM/DD) systems. The theoretical framework for the GS FIR filter and preliminary simulations have been presented in Part I of this work. Here, the performance of the GS FIR filter is compared to that of the iterative GS algorithm in the transmissions of 56-Gb/s on-off keying (OOK) over 80 km of single mode fiber (SMF) with a post feed-forward equalizer (FFE) for combating residual inter-symbol interference (ISI). Furthermore, the influence of the pulse shape (raised cosine or rectangular) and modulation format (return-to-zero (RZ) or non-return-to-zero (NRZ)) on the measured bit error ratio (BER) is experimentally investigated, while changing the number of FIR taps, the number of post-FFE taps, and the number of post-FE samples-per-symbol. It is shown that within the range of the target digital extinction ratios (DERs) for which the original iterative GS algorithm offers benefit, both analytical and numerical methods for calculating the optimum FIR taps, outlined in Part I of this work, produce similar BER performance as predicted. To that end, the former method is extended, here, through a non-recursive frequency response formula, which offers insight into the action of the GS filter with different pulse shaping and enables the derivation of the explicit GS impulse response. It is also shown that rectangular RZ pulse shaping exploits the full benefit of GS filtering through a uniform spectrum, enabling a BER below the 7% hard-decision forward error correction (HD-FEC) limit, with a 641-tap 7/2-spaced pre-EDC FIR filter and a 3-tap adaptive T/2-spaced post-FFE.

Index Terms—Mach-Zehnder modulator, digital signal processing, digital to analog converter, electronic dispersion compensation, Gerchberg-Saxton algorithm, finite impulse response filter, intensity modulation and direct detection.

I. INTRODUCTION

Intensity modulation and direct detection (IM/DD) transmission is the most viable and cost-effective solution for short reach optical interconnects. However, due to the interaction of chromatic dispersion (CD) with square-law detection, the received signal suffers from inter-symbol-interference (ISI) due to a linear power fading contribution and a nonlinear contribution originating from the photo-current power series expansion and signal-to-signal beating interference (SSBI). Using an I/Q nested Mach-Zehnder modulator (MZM) or coherent detection, enables the complete mitigation of liner effects, including CD, by electronic dispersion compensation (EDC), as both the real (in-phase) and imaginary (quadrature) components of the electric field are either fully controlled at the transmitter or fully recovered at the receiver. However, under direct detection, the optical phase is lost and complex-valued compensation of CD used in coherent systems could not be adopted. In an effort to compensate the CD-induced linear power fading penalty in IM/DD transmission, a phase-retrieval iterative algorithm, known as Gerchberg-Saxton (GS) algorithm was introduced theoretically, and demonstrated experimentally [1]–[8]. In all prior implementations of the GS algorithm in the context of IM/DD transmission, a number of real-time iterations on the incoming information bearing waveform were required, which increase both the hardware complexity and power consumption of the EDC circuit.

In Part I of this work, the theory and simulation of a novel non-iterative implementation of GS algorithm was introduced for pre-EDC at the transmitter, by means of a feed-forward static digital finite impulse response (FIR) filter [9]. The FIR filter taps were optimized offline using either a time-domain impulse propagation approach or a frequency domain small-signal analysis approach. The FIR filter was numerically simulated using a non-recursive tapped delay line structure with 8-bit finite-precision arithmetic. In this article, we report the first experimental demonstration of the aforementioned GS-based FIR filter implementation. The transmitter-end digital signal processing (DSP) utilizes a T/2-spaced N-tap GS-based FIR...
filter for pre-EDC with an 8-bit fixed-point arithmetic, while the receiver-end employs a J-tap adaptive T-spaced or T/2-spaced feed-forward equalizer (FFE) for combating residual ISI. Furthermore, a systematic analysis of the influence of the pulse shape and modulation format on the measured bit error ratio (BER) is experimentally reported, while sweeping the number of FIR taps \( N \), the number of post-FFE taps \( J \), the number of post-FFE samples-per-symbol (sps\(_{Rx} \)) and the digital extinction ratio (DER). To that end, the post-EDC was set to operate above a minimum of 2 samples-per-symbol to allow for both raised cosine (RC) and rectangular (Rect) pulse shaping. In addition to the RC non-return-to-zero (NRZ) modulation format simulated in Part I, the rectangular NRZ and rectangular return-to-zero (RZ) are also experimentally demonstrated in this article.

The remainder of the article is structured as follows. Section II briefly reviews the key points of the proposed FIR filter optimization method. Section III presents the experimental setup with the various digital signal processing steps, performed at the transmitter and receiver, highlighted. The experimental results and discussion are presented in section IV, while a conclusion is provided in section V.

II. GS-based FIR Filter Tap Optimization

In Part I, of this work, the decoupling of the pattern-dependent and modulation format-dependent aspects of the transmission from the GS iterations was achieved for the purpose of designing a CD compensation scheme that is agnostic to pattern-dependent effects, while primarily depending on the fiber parameters, DER, and baud rate. The proposed GS FIR filter operates on a real-valued signal as shown in Fig. 1 (a) for an \( N \) tap FIR. The offline iterative optimization is shown in Fig. 1 (b), with the target amplitude at the receiver-end set to \( \delta[i] \) + d.c., with the direct current (DC) bias reflecting a predefined DER. In this work, the basic GS algorithm is executed offline for a total of \( m = 15 \) iterations. With every successive iteration the impulse response of the required electronic dispersion pre-compensating filter at iteration \( i \), denoted \( h^m_{GS}(n) \), becomes manifest. Here \( n \) is the tap index, with \( 0 < n \leq N - 1 \) and \( i \) is the iteration index with \( 1 < i \leq m \). The number of taps \( N \) is assumed to be odd, while the taps weights are quantized into an 8-bit fixed-point resolution. The aforementioned procedure, for optimizing the taps in Fig. 1 (b), uses time-domain iterative impulse propagation. The resulting GS-FIR filter will simply be referred to as FIR, in the remainder of this article.

It is possible to obtain a closed-form analytical expression for the optimized taps weights through the iterative application of the CD frequency domain small-signal (SS) transfer function to the GS algorithm [4, 9]. This method does not include the DER as a tuning parameter in offline optimization. The starting point is the frequency domain normalized power fading transfer function \( H(f) \) causing the linear ISI, which can be expressed as [10]–[12]:

\[
H(f) = \cos \left( 2\pi^2 \beta_2 L f^2 \right) \tag{1}
\]

where \( L \) is the fiber propagation length, and \( \beta_2 = -DL^2/(2\pi c) \), with \( c \) denoting the speed of light, \( D \) is the dispersion coefficient and \( \lambda \) is the wavelength of light. Under the assumption of small-signal analysis, the transfer function in (1) can be iterated through the GS algorithm yielding the GS filter transfer function required for mitigating power fading ISI, after iteration \( i \), here denoted, \( H^m_{GS}(f) \), which can be expressed using the following recursive formula [4, 9], while setting \( \theta = 2\pi^2 \beta_2 L f^2 \):

\[
H^m_{GS}(f) = \left[ \cos(\theta) + \sin^2(\theta) \right] \left[ \cos(\theta) + \sin^2(\theta) \right] \ldots \] \tag{2}
\]

In this section, we extend our previous work [4, 9] through obtaining a closed-form non-recursive expression of the GS filter \( H^m_{GS}(f) \) after \( m \) iterations. The following variable substitutions are introduced, \( x = \cos(\theta) \) and \( y = \sin^2(\theta) \), resulting in \( H^1_{GS}(f) = x + y \) after the first iteration. Accordingly, the GS filter transfer function after 2, 3 and 4 iterations are:

\[
H^2_{GS}(f) = x + yx + y^2 = x(1 + y) + y^2
\]

\[
H^3_{GS}(f) = x + yx + y^2x + y^3 = x(1 + y + y^2) + y^3
\]

\[
H^4_{GS}(f) = x + yx + y^2x + y^3x + y^4 = x(1 + y + y^2 + y^3) + y^4
\]

Fig. 1. (a) The schematic diagram for the real-valued FIR filter structure operating at samples-per-symbol (T/2-spaced) to facilitate pulse shaping and pre-EDC at the transmitter, (b) the offline optimization procedure for obtaining \( h^m_{GS}(n) \) after \( m \) iteration through time-domain impulse propagation following by 8-bit fixed-point quantization, (c) semi-analytical method of obtaining \( h^m_{GS}(n) \) after \( m \) iteration through the inverse Fourier transform of SS GS filter (5), followed by sampling, truncation and 8-bit fixed-point quantization.
In general, one can express \( H^m_{GS}(f) \) after \( m \) iterations as:

\[
H^m_{GS}(f) = x \sum_{i=0}^{m-1} y^i + y^m, \quad (3)
\]

where \( y \) is bounded for all frequencies \((0 \leq y \leq 1)\), and (3), can be expressed as:

\[
H^m_{GS}(f) = \frac{1 - y^m}{1 - y} + y^m. \quad (4)
\]

Substituting back the expressions for \( x \) and \( y \), while noting that \( x^2 = 1 - y \), yields the following closed-form expression for the GS filter after \( m \) iterations:

\[
H^m_{GS}(f) = \frac{1 - \sin^2m(\theta)}{\cos(\theta)} + \sin^2m(\theta). \quad (5)
\]

In fact, at a low enough extinction ratio where small-signal (SS) analysis is applicable, closed form semi-analytical expressions for the FIR impulse response can be obtained using the inverse Fourier transform of (5), \( h^m_{GS}(t) = F^{-1}\{H^m_{GS}(f)\} \), followed by direct sampling and truncation, as depicted in Fig. 1(c). The explicit analytical formula for \( h^m_{GS}(t) \) is:

\[
h^m_{GS}(t) = \frac{2}{\sqrt{L|\beta|^2}} \left( \frac{\Gamma(m + \frac{1}{2})}{\Gamma(m + 1)} \right) \delta(t) + \sqrt{\frac{2}{L|\beta|^2}} \sum_{k=1}^{m} \left( \frac{(-1)^k}{2^{2k}} \sum_{r=0}^{k-1} a_{r,k}(t) \right)
\]

\[
a_{r,k}(t) = \frac{-2(2k-1)}{\sqrt{|r - k + \frac{1}{2}|}} \sin \left( \frac{t^2}{4L|\beta|^2} \left| r - k + \frac{1}{2} \right| + \frac{\pi}{4} \right) + \frac{(2k)}{\sqrt{|r - k|}} \sin \left( \frac{t^2}{4L|\beta|^2} (k - r) + \frac{\pi}{4} \right), \quad (6)
\]

where, the symbol \( \Gamma \) denotes the (complete) gamma function, which extends the factorial operation to real and complex numbers. The derivation of (6) is presented in the Appendix. The FIR filter optimized using this approach is referred to as FIR SS in the remainder of this article.

In theory, it is understood that a decision feedback structure is better suited for compensating the power fading nulls, as opposed to a feed-forward structure, which requires an infinite number of taps. However, the feed-forward FIR filter optimized in this work, uplifts the frequency content of the signal around the power fading nulls, while simultaneously, sharpening the notches within their close vicinity, without actually eliminating the unbounded singularity at the null frequency [4]. Evidence for this action might be gleaned from the non-recursive formulation of \( H^m_{GS}(f) \). In particular, evaluating the limit of (5), as \( \theta \) approaches the null points at \((2k\pi + \pi/2)\), where \( k \) is an integer, yields \( H^m_{GS}(f) \) approaching 1, which is equivalent to the all-pass no-action scenario, rendering the null point uncompensated.

III. EXPERIMENTAL SET-UP

The experimental set-up for the transmission of the 56-Gb/s OOK signal over 80-km of single mode fiber (SMF) is shown in Fig. 2, with the transmitter DSP (Tx-DSP) and receiver DSP (Rx-DSP) outlined separately in insets (a) and (b), respectively. The incoming binary data stream is upsampled to 2 samples-per-symbol prior to pulse shaping and modulation formatting according to one of the following configurations: (i) bipolar Rect-NRZ (Rect for short), (ii) bipolar Rect-RZ (RZ for short), and (iii) bipolar RC-NRZ (RC for short). A representative example of the up-sampled waveform at 2 samples-per-symbol for Rect, RZ and RC is shown in inset (c) of Fig. 2. Electronic dispersion pre-compensation is then performed through the convolution of the up-sampled data stream with the GS-based FIR filter taps. The filter taps were optimized using both the time-domain impulse propagation approach, and the frequency-domain small-signal analysis approach, denoted FIR and FIR SS in Tx-DSP steps of Fig. 2 (a), respectively.

In addition, the performance of the aforementioned non-iterative GS FIR filters is compared to the performance of the basic iterative GS algorithm, where the information bearing waveform is subject to forward and backward propagation through a virtual fiber model handled in the frequency domain via fast Fourier transform (FFT) and inverse FFT operations. Although impractical to implement, the iterative GS, serves as a bench-mark and reference point and is simply referred to as GS in the remainder of this article. The bandwidth limitation of the overall electrical-to-electrical system, including the effect of all components involved in the electrical-to-optical, and optical-to-electrical conversions is depicted in the measured magnitude response of Fig. 2 (d), and is mitigated through enabling the digital pre-emphasis at 2 samples-per-symbol with a 15-tap FIR filter calibrated for the back-to-back case. Subsequently, the digital signal is resampled to a sampling rate of 120 GSa/s in preparation for generating the corresponding analog signal using the arbitrary waveform generator (AWG, Keysight M8194A, ∼ 45-GHz 3-dB bandwidth) employing a 120-GSa/s digital-to-analog converter (DAC).

After amplification by the electrical amplifier (EA, SHF S807, ∼ 50-GHz 3-dB bandwidth), the light at 1550.12 nm from an external cavity laser (ECL) is modulated by 56-GBaud OOK via a chirp-free Mach–Zehnder modulator (MZM, Fujitsu FTM7938EZ, ∼ 32-GHz 3-dB bandwidth) which is biased at the quadrature point.

The optical signal is transmitted over an unamplified 80 km SMF link. An erbium-doped fiber amplifier (EDFA) is used to boost the optical signal incident onto a ∼ 70-GHz photodetector (PD). A variable optical attenuator (VOA) is placed between the EDFA output and PD input to facilitate changing the received optical power (ROP). The resulting photocurrent is digitized by a real-time sampling oscilloscope (OSC, Keysight UXR0804A, ∼ 80-GHz 3-dB bandwidth), employing an analog-to-digital converter (ADC) operating at a sample rate of 256 GSa/s. The received signal is re-sampled to 2 samples-per-symbol for synchronization and timing recovery using Gardner’s algorithm [13]. The signal is then maintained at or down-sampled to \( sps_{Rx} \) sample-per-symbol before a \( T/sps_{Rx} \)-spaced adaptive FFE with \( J \) taps. In this study, \( sps_{Rx} \) is set at a value of 1 or a value of 2. The tap weights are optimized using the least mean square (LMS) algorithm. Finally, the bit error ratio (BER) is measured using direct error
counting. For each BER measurement, a total of 30 frames are taken into consideration with each frame consisting of $2^{17}$ symbols, for which 10,000 are allocated for training the post-FFE, and 121,072 symbols are regarded as payload data.

IV. RESULTS AND DISCUSSION

It is imperative to limit the large number of variables affecting the system performance, while isolating specific trends involving key critical parameters useful in engineering the FIR filter for pre-EDC. To that end, the optimum number of GS iterations used for the offline optimization of the FIR tap weights with either time-domain or frequency-domain approaches was found to be $m = 15$ [9]. This was also found to be the optimum number of iterations needed for the basic iterative GS algorithm to generate the pre-EDC waveform which minimizes the BER at the receiver. As such the number of iterations $m$ is fixed at 15 for all experimental results presented herein. Furthermore, to ensure a fair comparison between FIR, FIR SS, and the iterative GS, the Tx Pre-emphasis was enabled for all experimental results presented in this work, with 15-taps static FIR operating at 2 samples-per-symbol and placed after the pre-EDC waveform is generated. To accurately measure the influence of the transmitter-end EDC parameters on the effectiveness of the GS based FIR filter for CD compensation, the receiver-end equalization must be kept at a bare minimum to only combat residual ISI. Consequently, the number of post-FFE taps $J$ was fixed to 3 taps, operating at $sps_{rx} = 1$ sample-per-symbol for all subsequent BER measurements, unless otherwise specified.

The influence of the number of FIR taps $N$ on the BER performance of the non-iterative (FIR / FIR SS) filtering approaches is experimentally measured and reported in Fig. 3 (a), (b) and (c), for RC, Rect, and RZ pulse shapes, respectively. The performance of the standard iterative GS algorithm, which is not dependent on the number of taps is shown as a solid line in Fig. 3 (a), (b), and (c), while the 7% hard-decision forward error correction (HD-FEC) limit of BER = $3.8 \times 10^{-3}$, is superimposed. It is evident that changing the pulse shape from RC to Rect improves the BER performance, allowing the iterative GS to operate below the FEC limit. Further improvements can be obtained through changing the modulation format from Rect, Fig. 3 (a), to RZ, Fig. 3 (b). The primary differences between the three pulses shapes, examined in this work, are their distinct frequency envelops and peak-to-average power ratios (PAPRs). Even though all three pulse shapes enjoy the full bandwidth of the AWG, the informational content of the modulated signal is not always uniformly distributed within this bandwidth. The RZ pulse shape, in particular, stands out in comparison to Rect and RC, as it exhibits a flatter and evenly distributed frequency spectrum. Consequently, the effect of the CD-induced nulls is evenly spread out over a wide frequency range, while the remaining two pulse shapes constrain more of the signal power within low frequencies. In fact, it is possible to show that
the action of the GS filter for CD pre-compensation is not uniform over the entire frequency band. Ideally, the GS filter $H_{GS}^m(f)$ in (5) introduces its own frequency null points, that should coincide with the frequency null points of the fiber transfer function in (1). This alignment between the frequency nulls of the pre-compensating filter and those of the IM/DD channel, is not always achieved, and a mismatch occurs with the following properties: (i) the misalignment decreases with an increasing number of iterations $m$ for all frequencies, and (ii) the misalignment is larger for the first few nulls (closer to DC) and decreases with increasing frequency at fixed iteration $m$. These properties can be demonstrated analytically through evaluating and comparing the roots of the transfer functions $H(f)$ in (1) and $H_{GS}^m(f)$ in (5). The results in Fig. 4 (a), show the first 5 zero-crossing point for (1) and (5), assuming a normalized frequency axis for $m = 1, 5, 10$ and 15 iterations. The results in Fig. 4(b), show the relative error, as a percentage, between the roots of (1) and (5) as a function of the null index counted starting from 1 to 14, which corresponds to all the frequency nulls within $(0 - 35 \text{ GHz})$ (assuming $L = 80 \text{ km}$ and $\beta_2 = -21.36 \text{ ps/km}$). These findings indicate that spectral shaping may be necessary to fully exploit the benefits of the GS algorithm. For example, in the case of RC and Rect pulse shaping, most of the spectral content of the signal is exposed to the brunt of this frequency null misalignment, reducing the potential benefits of the GS filter and increasing their vulnerability to the power fading effect. This same cannot be said for the case of RZ shaping which reduces the
Fig. 4. (a) The amplitude response of $H(f)$ and $H_{GS}^2(f)$ for $m = [1, 5, 10, 15]$. (b) The relative error between the roots of $H(f)$ and $H_{GS}^2(f)$ as a function of the counted null index starting from DC for the first 14 nulls.

impact of this misalignment through a uniformly distributed spectrum, even though more of the spectrum is exposed to power fading. Additional merits of using RZ pulse shaping can also be found in work [14]. A total of $N = 641$ taps is sufficient for the GS-based FIR and FIR SS filters with RZ to operate below the FEC limit. Accordingly, all subsequent results utilize the rectangular pulse shape with the RZ modulation format. It is worth noting that the BER performance of the FIR filter and FIR SS filter is quite close, indicating that the time-domain impulse propagation approach and frequency-domain SS analysis approaches yield similar performance within the range of digital extinction ratio (DER) of interest (1 dB to 4 dB), as predicted in the simulation work of Part I. The results presented in Fig. 3 (a), (b), and (c), were obtained at a DER of 1 dB for the iterative GS and a DER of 2 dB for the non-iterative FIR implementation, while the peak-to-peak voltage prior-to the EA ($V_{pp} = 2 \times$ amplitude) was set to $2 \times 60$ mV, for both cases, after which the EA’s output drives the MZM. In fact, these DER values were found to be optimum as evidenced by the BER measurement in Fig. 3 (d), which is nearly identical to BER measurements of the FIR SS, which does not account for the DER [9]. Consequently, within the range of target DER for which the original iterative GS algorithm offers benefit, both methods for calculating the optimum FIR taps, outlined in Part I of this work, produce similar BER performance. In addition, all the PAPR values for FIR, FIR SS (fixed value), GS, and without pre-EDC (fixed value) cases shaped with RC, Rect, and RZ pulses are presented in Fig. 3 (f) under different DERs. It can be observed, that pre-EDC enhances the PAPR of the generated signal and introduces additional DAC noise compared to the no pre-EDC cases. This enhancement is fully pronounced when the iterative GS is employed, for it exhibits the largest PAPR for all three pulse shapes. This is expected, as the iterative GS does not only mitigate linear power fading ISI, but also partially mitigates nonlinear sources of ISI.

Finally, increasing the number of taps $J$ in the post-FFE from 1 to 13 at $sps_{Rx} = 1$ and from 5 to 25 at $sps_{Rx} = 2$ offers only marginal improvement in the BER performance as shown in Fig. 3 (g). Note that the case with 1 tap at $sps_{Rx} = 1$ means no post-FFE utilized after timing recovery. The results indicate that the linear portion of the ISI induced by the frequency domain selective power fading has been effectively mitigated using the transmitter-end non-iterative FIR and FIR SS filters, rendering any additional linear equalization efforts at the receiver-end inconsequential. Furthermore, the results show that the iterative GS algorithm, which takes the pattern-dependent information bearing waveform into account every iteration, offers additional gain relative to the non-iterative linear FIR solution. As a consequence, one can infer that the iterative GS algorithm does not only mitigate linear ISI, but also offers means of compensating nonlinear ISI originating from the photo-current power series expansion and receiver SSBI.

The measured BER versus the ROP is shown for the non-iterative FIR / FIR SS filtering and the iterative GS algorithm with the rectangular pulse shape and RZ modulation format ($m = 15$, $J = 3$, $sps_{Rx} = 1$) is shown in Fig. 3 (h). As a reference, the measured BER versus ROP for the rectangular RZ without pre-EDC and a 601-tap $T$-spaced post-FFE is also plotted. It is evident, the transmission is inoperable without the Tx pre-EDC enabled. As for the required ROP under the 7% HD-FEC threshold for three pre-EDC cases, the iterative GS’s < (GS-based) FIR SS’s < (GS-based) FIR’s, where the former one is at the price of much-higher complexity relative to the other two non-iterative FIR approaches. Furthermore, the sensitivity penalty between these two FIR and FIR SS implementations is approximately 1 dB. Besides, the received electrical eye-diagrams under 7-dBm ROP, plotted over a span of a 3-bit period, measured after Gardner timing recovery (first row) and after the post-FFE (second row) for the GS-based FIR / FIR SS ($J = 3$), iterative GS ($J = 3$), and rectangular RZ without pre-EDC ($J = 601$) are shown in Fig. 3 (i). It can be
.observed that with pre-EDC performed, an appreciable eye-opening is already obtained, even without post equalization (BER at $\sim 10^{-2}$ as shown in Fig. 3(g)), avoiding the demand for specially-designed CD-resisted timing recovery algorithms.

V. CONCLUSION

The first experimental demonstration of a non-iterative Gerchberg-Saxton (GS)-based FIR filter for mitigating ISI induced by linear frequency error fading. A feed-forward static digital FIR filter was designed and optimized for pre-EDC at the transmitter end. This was achieved by decoupling the information bearing waveform from the GS iterations by time-domain impulse propagation or frequency-domain small-signal analysis. An adaptive $T$-spaced feed-forward equalizer (FFE) was utilized at the receiver for compensating residual CD. The influence of the pulse shaping and modulation format on the measured BER is experimentally investigated, while sweeping the number of FIR taps $N$, the number of post-FFE taps $J$, and the number of post-FFE samples-per-symbol $sps_{Rx}$. The pulse shapes and modulation formats considered in this work are: raised-cosine non-return-to-zero (RC), rectangular non-return-to-zero (Rect), and rectangular return-to-zero (RZ). It was demonstrated that within the range of target DER for which the original iterative GS algorithm offers benefit, both methods for calculating the optimum FIR taps, outlined in Part I of this work, produced similar BER performance. Furthermore, the measured BER, using the outperformed RZ with a 641-tap $T/2$-spaced static FIR filter at the transmitter-end and only a T-tap adaptive $T$-spaced post-FFE after timing recovery at the receiver-end, was below the 7% HD-FEC limit of $3.8 \times 10^{-3}$.

The results presented in this article as Part II extend the theoretical and simulation study of the proposed GS filter, outlined in Part I, by offering the first experimental demonstration of pre-EDC using a non-iterative feed-forward FIR filter based on the GS algorithm. Furthermore, the analytical and numerical methods for obtaining the FIR tap values, detailed in Part I, were experimentally verified. Building on the work in Part I, a non-recursive formulation of the GS filter response offered insights into the filter action when different pulse shapes were used and enabled the completion of the aforementioned analytical method through the derivation of the explicit GS filter impulse response.

APPENDIX

DERIVATION OF GS FILTER IMPULSE RESPONSE

Proof. The non-recursive formula for the GS FIR frequency response in (5), can be expressed in terms of a single variable $x$, where $x = \cos(\theta)$, as follows:

$$H_{GS}^m = 1 + \sum_{k=1}^{m} (-1)^k \binom{m}{k} x^{2k}.$$

The binomial expansion of $(1 - x^2)^m$ is given by:

$$1 - (1 - x^2)^m + (1 - x^2)^m = 1 + \sum_{k=1}^{m} (-1)^k \binom{m}{k} x^{2k}.$$  (8)

Substituting (8) into (7) yields:

$$H_{GS}^m = 1 + \sum_{k=1}^{m} (-1)^k \binom{m}{k} x^{2k}.$$

Since (9) is expressed in terms of powers of $x = \cos(\theta)$, the binomial expansion of even powers $\cos^{2k}(\theta)$ and odd powers $\cos^{2k-1}(\theta)$ are known [15] and given below:

$$\cos^{2k-1}(\theta) = \frac{1}{2k} \left( \frac{2k-1}{k} \right) \cos ((2k-2r-1) \theta)$$

$$\cos^{2k}(\theta) = \frac{1}{2k} \left( \frac{2k}{k} \right) + \frac{1}{2k} \sum_{r=0}^{k-1} \left( \frac{2k}{r} \right) \cos (2(k-r) \theta).$$

Substituting (10) into (9) yields:

$$H_{GS}^m = 1 + \sum_{k=1}^{m} (-1)^k \binom{m}{k} \left( \frac{2k}{k} \right) + \sum_{r=0}^{k-1} \frac{1}{2k} \left( \frac{2k}{r} \right) \cos (2(k-r) \theta).$$

Since $\theta = \frac{1}{2} \beta_2 L \omega^2$, the analytical expression for the inverse Fourier transform of $C_{r,k}(\theta)$ is:

$$c_{r,k}(t) = \mathcal{F}^{-1} \left\{ C_{r,k}(\theta) \right\} = \sqrt{\frac{2}{L |\beta_2|}} a_{r,k}(t)$$

$$a_{r,k}(t) = \frac{-2 \left( \frac{2k-1}{r} \right)}{\sqrt{|r-k+\frac{1}{2}|}} \sin \left( \frac{t^2}{4L |\beta_2|} \right) + \frac{2k-1}{r} \sin \left( \frac{t^2}{4L |\beta_2|} (k-r) + \frac{\pi}{4} \right).$$

Finally, the inverse Fourier Transform of $H_{GS}^m(t)$ is:

$$h_{GS}^m(t) = \sqrt{\frac{4\pi}{L |\beta_2|}} \left[ 1 + \sum_{k=1}^{m} \frac{(-1)^k}{2k} \binom{m}{k} \left( \frac{2k}{k} \right) \right] \delta(t) + \frac{\sqrt{2}}{L |\beta_2|} \left[ \sum_{k=1}^{m} \frac{(-1)^k}{2k} \binom{m}{k} \sum_{r=0}^{k-1} a_{r,k}(t) \right].$$

The coefficient of the first term depends only on the number of iterations $m$ and can be simplified:

$$1 + \sum_{k=1}^{m} \frac{(-1)^k}{2k} \binom{m}{k} \left( \frac{2k}{k} \right) = \frac{\Gamma(m + 1/2)}{\sqrt{\pi} \Gamma(m + 1)}.$$

Substituting (14) into (13), yields (6).

REFERENCES


