Threads and Spirals in a Noninteger Dimensional Universe

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Abstract

The paper presents analysis showing that the earliest structures in the evolution of noninteger dimensionality in a system will be threads and spirals. Before the dimensionality has reached 1, the structures are linear, and thereafter these threads or strings begin to bend and fold to be in accord with fundamental scale-invariance requirements. As the evolution proceeds, threads turn into spirals in a manner that parallels phase changes in a liquid crystal. Evidence is presented that the emergence of threads and spirals is consistent with the findings in bioscience, liquid crystals, galactic center filaments, and early spiral galaxies.
Abstract. The paper presents analysis showing that the earliest structures in the evolution of noninteger dimensionality in a system will be threads and spirals. Before the dimensionality has reached 1, the structures are linear, and thereafter these threads or strings begin to bend and fold to be in accord with fundamental scale-invariance requirements. As the evolution proceeds, threads turn into spirals in a manner that parallels phase changes in a liquid crystal. Evidence is presented that the emergence of threads and spirals is consistent with the findings in bioscience, liquid crystals, galactic center filaments, and early spiral galaxies.

Keywords: Noninteger dimensionality, evolutionary stages, cosmology, bioinformatics, liquid crystals, information theory

1. Introduction

The onset of a new phase in the evolution of a physical system may be viewed from the perspective of topological defects, which are emergent stable configurations of matter. These defects are irregularities or disruptions that can take various forms such as points, lines, disks, and spirals. They play a significant role in describing structures in physics and engineering, which include cosmology and nanomaterials [1-3].

Topological defects lead to linear or spiral structures as well as more complex forms [4]. The consequence of the destruction of symmetry in a core region is an elastic variable that changes slowly in space in a far-field region [4]. In analogy with the effect of an electric charge, its presence can be determined by measurements of an appropriate field on a surface that encloses the core.

Beyond their presence in condensed matter physics and engineering materials, cosmic strings as topological defects were investigated by Kibble [5], who considered various domain structures that can arise in a spontaneously broken gauge theory and showed their emergence depends on the homotopy groups of the manifold of degenerate vacuum states. Ruling out some structures because of their unacceptable gravitational
effects, he concluded that a cosmic network of strings may have been formed due to fundamental cosmological processes.

The recently proposed information-theoretic approach to physics and cosmology [6,7] comes with the postulation of dimensionality energy. In it, the Big Bang is a result of the “explosion” of $d = 0$ space into higher dimensionality, which provides a meta-narrative to the current understanding of the subject. Specifically, it was shown [8] that it provides estimates of change in the gravitational constant that is in accord with Dirac’s Large Numbers Hypothesis [9]. It also showed the expansion going through different stages that are quite like those of the standard cosmology theory, excepting that in the future the accelerating expansion will reverse [10,11], which from an epistemological perspective is much better than a universe that will end in the Big Freeze [12,13].

The information-theoretic view, a top-down view of physical system evolution, must be consistent with analytical bottoms-up methods based on relationships governed by local constraints. The satisfaction of these constraints provides a new perspective on the origins of local forces.

Topological defects leading to threads and strings may be seen from the standpoint of dynamic change in dimensionality. Linear structures created in the $d \sim 1$ ($d=$-dimensionality) regime bend and further transform into spirals and helices to provide a local fractal, scale-independent, representation.

The idea of dimensionality evolution also appears to operate at different scales in aggregates of matter. Linear structures are thus observed terrestrially in the dunes in the Great Sand Sea in Southwest Egypt [14].

Liquid crystals provide another example of linear structures. In the simplest phase in the nematic crystal, calamitic organic molecules lack a crystalline positional order,
but do self-align with their long axes roughly parallel. The molecules are free to flow and their center of mass positions are randomly distributed as in a liquid, but their orientation is constrained to form long-range directional order.

In this paper we begin with a review of the theory of dimensionality-based energy potential and its associated force. A simple relationship between dimensionality and the degree of order in a system is derived. Since structures repeat across scale, we use the example of biological systems to propose that threads, circles, and spirals [15] are primitives of complex systems and illustrate it with evidence from liquid crystals and cosmology.

2. Dimensionality potential

Let us consider the dimensionality potential of an object. Consider two objects in a \( d < 1 \) space. They will tend to fall towards each other and the extent of the fall will be greater if they are further apart [7].

In general, the potential \( p(r) \) will increase with distance \( r \), but it should be divided by the surface area, \( A \), of the equipotential surface around it. In other words,

\[
p(r) \propto \frac{r}{A} \tag{1}
\]

3-D world

A three-dimensional universe consists of \( 2 \leq d \leq 3 \). The surface area of the equipotential sphere is \( A = 4\pi r^2 \). Therefore, \( p(r) \propto \frac{r}{4\pi r^2} \). Or \( p(r) \propto \frac{1}{r} \).

Since force is the derivative of the potential, we find that the dimensional force between two objects is inversely proportional to square of the distance, and this could well be the explanation for the attractive gravitational force.

2-D world

A two-dimensional universe represents the case \( 1 \leq d \leq 2 \). The surface area of the circle, the equipotential surface in two dimensions, is \( A = 2\pi r \). Therefore, \( p(r) \propto \frac{r}{2\pi r} \). Or, \( p(r) \) is constant.

The derivative of a constant is zero, and thus there is no dimensional force between two objects in a two-dimensional universe. This is the case of asymptotic freedom, and it could be the explanation of a lack of attraction between near object in particle
physics, as for quarks. Objects that operate in the 2-D world will experience no attractive force.

**Force in 3-D Universe**

On symmetry grounds, it was proposed that the force in the 3-D universe will be proportional to \((2 - d)(3 - d)\), since it is zero at \(d = 2\) and \(3\), when the dimensionality has integer value [16][17]:

\[
\text{Force} \propto (d - 2)(3 - d)
\]  

(2)

This is shown in Figure 2.

![Figure 2. The dimensional force for 2< d <3](image)

The scale invariance associated with noninteger dimensionality may be viewed either from the perspective of local forces [16], or from that of scale invariant probability distribution [17].

In a nematic crystal, the structures will be linear at first (for \(d \leq 1\)) and thereafter bend and fold according to the constraints defined by the dynamic changing dimensionality and probability requirements underlying scale invariance. As \(d\) becomes larger than 1, which is associated with higher temperature, one can assume an energy function acting as a random tension \(T\) on the string that causes transverse waves to travel along the string with speed \(v = \sqrt{\frac{T}{\rho_L}}\), where \(\rho_L\) is the linear density of the string, \(L\) is the length of the string, and the natural frequency of the fundamental mode is \(\omega = \frac{\pi v}{L}\).
It is significant that the dimensionality constraint for \(d > 1\) will impose further changes in the line segment so that at the local level it curls into a structure that has a fractal dimension. Furthermore, a power law distribution will be associated with a one-variable system [14,15].

In a reductionist or local theory, one must explain the aggregate properties in terms of the interactions amongst the components. Noninteger dimensional systems do not lend to straightforward reductionist analysis and they are characterized by nonlocal properties. If the data does not conform to the appropriate scale-invariant distribution of the relevant dimensionality, there will arise local forces and processes (either from the environment or from the data itself, or both) that will tend to change the distribution towards the correct one [16].

We can see this as a consequence of a dimensionality potential energy that, in analogy with the elastic potential energy, equals

\[
PE(d) = c\Delta d^2
\]  

where \(c\) is an appropriate constant and \(\Delta d\) is the change in dimensionality.

3. Order in liquid crystals

Let us consider a liquid crystal to focus on the workings of dimensionality for a collective. The molecules in liquid crystal do not exhibit any positional order, but they do possess a degree of directional order even though they may not all point the same direction all the time. The order parameter \(S\), highly dependent on the temperature of the sample, describes the orientational order of the crystalline material while allowing for the individual deviation of the molecules from the director, which represents the average over the collection. In other words, \(S\) measures the degree of alignment of molecules’ symmetry axes:

\[
S = \frac{3\cos^2\theta - 1}{2}
\]  

where \(\theta\) is the angle between the director and the long axis of each molecule. Averaging this function of \(\theta\) over all molecules gives a value between 0 and 1 for the amount of directional order.

From the perspective of the effect of the evolution of dimension, the function of disorder will increase in a quadratic manner due to the constraints of equation (2).
The average value of $\theta$, if each molecule is oriented through the entire range from 0 to $\pi/2$, will be $\pi/4$. We are considering order on a surface where $\theta = 0$ corresponds to $d = 1$, and $\theta = \pi/4$ corresponds to $d = 1.5$.

Since order is naturally large for small $\theta$, the function will have a form that is a shifted version of that in Figure 2. A natural function to use here is the cosine one.

Thus we consider the quadratic to be $a \cos^2 \theta + b$.

Solving for the constraints that the order is maximum ($= 1$) for $\theta = 0$ and minimum ($= 0$) for $\theta = \pi/4$, we obtain that the order parameter, $S$, equals

$$S = 2 \cos^2 \theta - 1$$

(5)

The difference between the order parameter expressions for small $\theta$ is very small and, therefore, (4) and (5) effectively imply the same result.

4. Structures across different scales

We have already alluded to the evolutionary sequence that begins with linear or thread structures; this is followed by disk, and spiral structures. In cosmology, early thread structures have been estimated using computational methods inspired by the *Physarum polycephalum* slime mold [18]. In a similar spirit, we examine parallels with the folding of proteins [19,20], but the real justification in doing so is that the same information-theoretic constraints drive evolution in both cosmology and biological systems.

![Figure 3. α-helices and β sheets B [20]](image)
The ordering of the amino acids determines the nature of the folding of the one-dimensional strings into proteins as their detailed chemical structure is different. Typically, two types of structures form in the secondary structure. Some regions coil up into formations called α-helices, while other regions fold into β pleated sheets that are formed with the backbone bending over itself to form the hydrogen bonds (Figure 3). These two forms can interact to form more complex structures. The α-helices are three-dimensional spirals, whereas the β pleated sheets are threads.

In the context of DNA, the fractal dimension of chromatin in the cell nucleus where DNA is stored is about 2.7 or approximately $e$, which is the optimal value [20]. In its detailed structure, chromatin consists of helices that are wound further in different ways as shown in Figure 4 [21].

5. Evidence in support of early filaments and spirals

If cosmic strings and filaments are the analogs of the β sheets, objects in helical motion (like stars in a galaxy and planets in a star-system) are the analogs of the α-helices. Noninteger dimensionality theory postulates that the earliest structures to arise in the universe will be threads and spirals. This is in contrast to standard cosmology where threads and spirals come much later [22].

Galaxies are gathered not only into clusters, but also into vast interconnected filamentary structures with gigantic voids in between. It is believed that this cosmic web started out tenuous and became more distinct over time under the effect of gravity. One may see the evolution of the web as beginning with linear structures that arose when the dimensionality of the universe was less than 1 or just above and these linear structures anchoring more complex forms.
Using NASA's James Webb Space Telescope, a thread-like arrangement of 10 galaxies that existed just 830 million years after the Big Bang has been discovered [23]. It is believed that the filament will eventually evolve into a massive cluster of galaxies.

Figure 5. The deep galaxy field from Webb’s Near-Infrared Camera shows an arrangement of 10 distant galaxies marked by eight white circles in a diagonal, thread-like line. [23] Credits: NASA, ESA, CSA, Feige Wang (University of Arizona)

As another example, a population of nearly 1,000 one-dimensional filaments, which are vertical and much larger at up to 150 light-years long each, are near the galaxy's center [24,25]. The one-dimensional cosmic threads are hundreds of horizontal or radial filaments dating back to when outflow from Sagittarius A, the Milky Way's central supermassive black hole. While such black holes accrete matter, they also can power tremendous outflows of material and one way to see this outflow will be as emergence from a d < 1 range to a higher one.

Figure 6. (a) The most ancient spiral galaxy found so far, BRI 1335-0417 [26]; (b) An artist’s rendering of galaxy BX442 and its companion dwarf galaxy Credit: Dunlap Institute for Astronomy & Astrophysics/Joe Bergeron [27]
Another galaxy, BRI 1335-0417, formed around 1.4 billion years after the Big Bang, making it the earliest example of a spiral galaxy [26]. It measures 15,000 light-years across, making it a third as big as the Milky Way (Figure 6a).

Yet another very ancient galaxy with clear spirals is BX442 (Figure 6b) at roughly 3 billion years after the Big Bang [27].

Let us return to structures formed in liquid crystals. K Akagi et al. synthesized several crown ether type binaphthyl derivatives (CEBDs) and used them as chiral dopants to induce chiral nematic (N*) liquid crystals (LCs) [28]. They investigated the twisting powers of the CEBDs for phenylcyclohexane (PCH)-derived nematic LCs and found that the twisting powers of the CEBDs increased with decreasing ring size of the crown ether. This example shows how the structure is created by the interaction in specific local properties. Their spiral structure is shown in Figure 7.

6. Conclusions

The paper presents arguments in support of the theory that the earliest structures in the evolution of noninteger dimensionality will be threads and spirals. The logic behind this is that before dimensionality has reached 1, the structures are linear, and thereafter the threads will bend and fold to be in accord with the requirements of the dimensionality of the system.

This theory is consistent with the findings in bioscience, liquid crystals, galactic center filaments, and the discovery of very early spiral galaxies.
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