A Framework for Unsupervised Multi-Objective Optimization of Transport Networks through the Use of Genetic Algorithms: Demonstrated Through the Optimization of Indian Inter-City Transport

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Abstract

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Intercity travel optimization has always been a goal of every government. Such an optimization would allow efficient and economical development. Optimizing travel would facilitate transfer of ideas and resources and promote economic growth. As such, travel can be optimized on two fronts: cost of arranging for that travel and the ease of travel i.e., the length/time of travel for a person to go from any one city to any other. This study proposes a methodology of optimizing travel pathways in any circumstance through genetic algorithm metaheuristics and network theory representations. This study demonstrates the same to optimize inter-city travel between the 25 cities with the highest population in India.
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Introduction

Inter-city travel has always been an integral part of a government’s economic and political policy. For example, the Roman Empire made highly effective use of roads to facilitate governance. An extension of travel/transport network optimization can be seen in the Silk Road and the Belt and Road initiative, which aims to not only promote global economic growth but also to improve diplomatic relations. As such, throughout history, albeit for varying reasons (from war-preparational to developmental), nations have sought to optimize transport networks.


Today, with modern computer science and mathematical techniques, this multi-objective optimization can be achieved with a much greater degree of simplicity. To do this, the study finds the need to define two functions: $C(x)$ that returns the cost of infrastructure for any given transport network and $L(x)$ that returns the per-node (per-city) average shortest weighted Dijkstra path length that provides information on the average travel time to move from one city to another, where $x$ is the network. Thus, the goal of this optimization can be condensed to the following:

$$\arg\min_x [C(x) + L(x)]$$ (1)

**Network Representation**

This study uses network graphs to represent the transport network, where each node represents a city and each edge represents some form of transport that connects the cities. Each edge is weighted with respect to the distance (in terms of latitude and longitude) between the nodes (or cities). This permits for the deterministic derivation of $C(x)$ and $L(x)$. This study assumes $C(x)$ to be linearly correlated with the distance of the same travel, allowing $C(x)$ to be described by the total length of the weighted network, normalized by the number of nodes. $L(x)$ is chosen to refer to the average length of all the shortest paths from any city to any other city. This is computed through the iterative use (and consequently averaging) of Dijkstra’s Algorithm\(^3\) which computes the shortest path between any two nodes in a weighted network.

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The Objective Function

The first issue arises when one realizes that a minimization of the cost, or $C(x)$, would reasonably imply that fewer resources were utilized and thus, $L(x)$ would increase. A minimization of $L(x)$ would mean that more resources were utilized and thus $C(x)$ would increase. This trade-off is at the core of multi-objective optimization. As such, utilizing a heuristic described in the study conducted by Frank Schweitzer et al.\textsuperscript{4} is used in this study to resolve this dilemma. This heuristic involves using a parameter (here, $\gamma$) that can be selected by the user based on their preferences. This would amend the optimization goal as seen below.

$$\arg \min_x [\gamma L(x) + (1 - \gamma)C(x)]$$ \hspace{1cm} (2)

Intuitively, a $\gamma$ of 0 would return a null (or isolated vertex) graph as there would be no connections between nodes and thus no cost to build the network. However, on discretizing the domain of optimization to only those networks wherein each node is connected to at least one other node and there is always a path to reach any arbitrary node from any other arbitrary node, a $\gamma$ of 0 would return a minimal link system.\textsuperscript{5} Conversely, a $\gamma$ of 1 would intuitively return a fully connected network where each node is connected to every other node. Thus, observably, the selection of $\gamma$ will impact the resulting output.


\textsuperscript{5} Schweitzer et al.
Genetic Algorithm

Genetic algorithms, proposed by John H. Holland,⁶ are computational metaheuristics that draw inspiration from Darwinian theories of evolution commonly used to solve discrete optimization problems. They involve sampling and simulating a particular ‘population’ of possibilities from the sample space that are then evaluated against a fitness function (here, the objective function described above). The fittest individuals or ‘chromosomes’ in the population are selected to reproduce through a process known as crossover where elements of the fittest chromosomes or ‘parents’ are mixed. However, this could lead to the model getting stuck in local maxima/minima. Thus, mutation is introduced in this study that involves the flipping of a bit of a member of the produced population randomly at a defined mutation rate. This study represents each edge as a chromosome (or single bit) that is evaluated by the negative product fitness function defined by the minimization goal above, thus resulting in a maximisation goal shown below.

\[
\arg \max_x \left[ -\gamma L(x) - (1 - \gamma)C(x) \right] \quad (3)
\]

In the simulations performed, this study defines the hyperparameters 'Population_size' as 10, 'Mutation_rate' as 0.05, and 'Percentage of Population Selected for Mating' as 0.5.

This study also uses a roulette wheel strategy\(^7\) to select the members for mating that involves a weighted selection of members of the population based on their fitness. This is done instead of objectively selecting the members of the population with the highest fitness to avoid getting stuck in local maxima.

**Environment**

This study used an implementation of the same code in Python 3,\(^8\) using the Pandas\(^9\) and NumPy\(^10\) libraries for data handling, the Networkx\(^11\) library for network representations and calculations, and the GeneAl\(^12\) library to solve the optimization problem using genetic algorithms with the defined constraints and hyperparameters. This was modified by implementing penalties whenever the algorithm produced unacceptable results (or those beyond the discretized domain defined by this study) that included subtracting a large number from the fitness score of that network. This study also used a dataset of longitudes and latitudes of the 25 most populated Indian cities to generate nodes and distance-based weights that the

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model had to optimize. These 25 cities include Delhi, Mumbai, Kolkata, Bangalore, Chennai, Hyderabad, Pune, Ahmedabad, Surat, Lucknow, Jaipur, Cawnpore, Mirzapur, Nagpur, Ghaziabad, Indore, Vadodara, Visakhapatnam, Bhopal, Chinchvad, Patna, Ludhiana, Agra, Kalyan, and Madurai.

Results

The simulations were conducted for various values of $\gamma$ (from 0 to 1 with a step of 0.1). The networks were visualized using the algorithm proposed by Tomihisa Kamada and Satoru Kawai in their paper ‘An Algorithm for Drawing General Undirected Graphs’.13

Shown below are the graphs produced on simulating the network evolution (and thus optimization) using $\gamma$ values of 0.2, 0.4, 0.6, and 0.8 respectively.

Figure 1. The Resultant Output Network Returned by the Model for a $\gamma$ value of 0.2
Figure 2. The Resultant Output Network Returned by the Model for a $\gamma$ value of 0.4
Figure 3. The Resultant Output Network Returned by the Model for a $\gamma$ value of 0.6
Figure 4. The Resultant Output Network Returned by the Model for a $\gamma$ value of 0.8
Visibly, as intuitively expected, an increase in $\gamma$ corresponds to an increase in node degrees.

Interestingly though, when simulated for $\gamma$ values from 0 to 1 with a step length of 0.1, a trendline for fitness against $\gamma$ can be plotted. In this particular situation, as seen in Figure 6, it can be seen that a $\gamma$ of 1, or a fully connected network, has the highest fitness. This could indicate that governments and decision makers might find it to their benefit to maximize their utility-cost ratio by connecting all these major cities to each other. However, obviously, there may be more specific constraints in various situations.
Figure 6. The Fitness Trendline as a Fitness Against $\gamma$ Graph (Orange Trendline and Blue Zero Reference)
Conclusions

This study aims not to show that any particular transport network design is perfect for Indian cities, but to provide a method for designing any transport networks in the world. This study shows that such a parametric approach to multi-objective optimization is feasible in network optimization, especially that of transport. This can also be extended to a larger number of objectives (for n objectives, there will be n-1 parameters to adjust according to personal preferences). Moreover, the assumption of $C(x)$ linearity with respect to distance can be circumvented through the incorporation of a more-apt model, whatever it may be, in the objective function and thus the fitness function. This framework also allows for the scaling of datasets beyond just the most populated 25 cities to how many ever cities, limited only by the hardware accessible. On increasing the number of entries in the dataset, the edge-weights can further be adjusted with respect to the population of each city. Thus, this study provides a framework for users’ multi-objective optimization, which, when used aptly, can save time, money, and resources.
Bibliography


