Online Estimation of Inertia Distribution for both Ambient and Transient Power System Conditions

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Abstract

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Debargha Brahma, Member, IEEE, Abhinav Kumar Singh, Member, IEEE, Nilanjan Senroy, Senior Member, IEEE, Abdul Saleem Mir, Member, IEEE, and Bikash C. Pal, Fellow, IEEE

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Index Terms—Nodal inertia, inertia distribution, power system dynamics, rate of change of frequency, uncertainties.

I. INTRODUCTION

The adverse impacts of depleting power system inertia due to the transition from synchronous to non-synchronous power generation are well documented [1]. This has primarily led to extensive research in the estimation of power system inertia in various spatial and topological levels, like system-wide inertia (or simply system inertia) [2]-[5], and area-wide inertia (or regional inertia) [6]-[8]. System and regional inertia refer to the total aggregated inertia in a power system and its areas, respectively. However, the distribution of these aggregated inertia for a practical power system is non-uniform, which can be mainly attributed to (a) diversified power generation technologies of varying capacities distributed non-uniformly in the grid [9], [10], and (b) zero, low or varying inertial contribution from inverter-based resources (IBRs), depending on their control methodologies [11]-[14]. This locational unevenness in inertia distribution affect both local and global dynamic behavior of the system, like rate of change of frequency (ROCOF), and inter-area power oscillations. Hence, rather than an aggregated inertial estimation, accurate estimation of inertial distribution at node or bus level, also known as nodal inertia [15], [16], becomes more critical and necessary, especially as the generation portfolio changes due to increasing penetration of IBRs.

In existing literature, most of the inertia distribution estimation techniques are based on either ambient [17] or ring-down system condition [18], [19] which utilize linear techniques or swing equation, respectively. Alternatively, there are some perturbation injection-based methods which involves active injection of perturbed signals [15]. Therefore, the applicability of these methods are limited to specific data-sets (ambient or event-based data), or system response to perturbation injection. Hence, there is a definitive lack of a perturbation-free unified method, with capability to effectively estimate and track inertia during both ambient and transient conditions. Such capability would enhance credibility for practical online applications. Also, most of these existing methods only provide the estimation of effective generator inertia [18] rather than a nodal estimate. Very few methods in existing literature which estimate the nodal inertia are limited to specific data-sets (ambient or event-based data), or system response to perturbation injection. Although these methods are useful in providing a general overview of the inertia distribution and in identifying the inertia deficient nodes or areas, an absolute-valued index (instead of relative-valued index) is still desirable for more efficient and tractable estimation which can be applied for developing under-frequency load shedding (UFLS) schemes [24], and actuator placements [25]. Moreover, some of the inertia estimation methods [20], [21] are mode-dependent and require information of the electromechanical modes of the system. On the other hand, the accuracy of several ring-down methods which estimate inertia using the generator swing equations are limited by the accuracy of the techniques used for the estimation of ROCOF, the rate of change of ROCOF, and the rate of change of power injection [18]. These methods also require defining the near-zero ROCOF thresholds, which can be system-specific or node-specific and may thus lead to numerical error and noise in the inertial estimate, if chosen inaccurately.

In this paper, a robust two-level estimation method for inertia distribution is proposed, which is capable of providing absolute-valued nodal inertia for (i) any given system operating condition (planning level), and (ii) both ambient and transient power system conditions (monitoring and tracking level), depending on the transparency of available data. The novelty of the proposed method and its advantages over current inertia distribution techniques are manifold: (a) capable of estimating and tracking the nodal inertia for both ambient and transient condition, and hence more tractable for online monitoring, (b) provides absolute-valued nodal inertia rather than normalized indices, and thus more suitable for planning UFLS measures and actuator placement studies, (c) mode-agnostic approach, and therefore more generalized than dominant mode-based methods, (d) does not require ROCOF measurements and
calculating rate of change of power injections, thus immune to
the estimation methods of these quantities and their associated
accuracy levels and noise, and (e) does not require defining
thresholds for near-zero ROCOFs, which is a non-trivial
system-specific problem.

The rest of this paper is organized as follows. Section II
introduces the rationale behind the concept of nodal inertia, and
then follows up with the analytical derivation of the proposed
two-level nodal inertia estimation method. In Section III, the
applicability of the proposed nodal inertia estimation method is
tested for the IEEE 39-bus system at any given operating
condition, and during ambient and transient system conditions.
Section IV discusses the robustness of the proposed method in
the presence of model uncertainties and measurement noise.
Conclusions are drawn in Section V.

II. NODAL INERTIA ESTIMATION METHOD

The well-known concepts of system-wide inertia, or area-
wide inertia quantifies the opposition to the rate of change
of center-of-inertia (COI) frequency, or the rate of change of area-
aggregated frequency due to the aggregated active power
imbalance, respectively. This interpretation can be further
extended to the nodes (network buses) of the system, where the
kth bus inertia would indicate the opposition to the ROCOF at
bus k due to any imbalance in the aggregated incoming or
outgoing active power flows at the kth bus (ΔP_k). This is
hypothetically expressed as

$$\text{ROCOF}_k = \frac{1}{2} H_k^{-1} \Delta P_k$$  \hspace{1cm} (1)

Here, the term $H_k$ denotes the effective inertia at the kth bus,
which is to be estimated. The objective for the rest of this
section, is to establish the relationship given in (1) for all
network buses, and thereby extract the information of effective
inertia, without actually requiring the measurement of ROCOF
and ΔP.

The proposed nodal inertia estimation method is discussed
for a power system with $N_g$ generators and $N_B$ number of buses.
The electromechanical dynamics of such a system can be
defined using the swing equation and is given in vector notation

$$2 H_g \omega_g = \Delta P - D_g \Delta \omega_g$$  \hspace{1cm} (2)

where $H_g$ is a $N_g \times N_g$ diagonal matrix with generator inertia
constants as diagonal entries, $\omega_g$ is a $N_g \times 1$ vector of generator
rotor speeds and $\Delta \omega_g$ represents the change in generator rotor
speeds, $\Delta P$ is a $N_g \times 1$ vector of active power imbalance as the
difference between the mechanical power input ($P_{\text{mech}}$) and
electrical power output ($P_g$), and $D_g$ is a $N_g \times N_g$ diagonal
matrix with generator damping coefficients as diagonal entries.
Considering the slower dynamics of the mechanical power with
respect to electrical power for synchronous generators (SGs)
[27], the general swing equation given in (2), excluding the damping
can be modified to

$$\omega_g = \frac{1}{2} H_g^{-1} \Delta P_g$$  \hspace{1cm} (3)

Here, $\Delta P_g$ represents the generator output power deviation.
From (3), the importance of inertia in restraining the rate of
change of rotor speed (ROCOF) during any imbalance (which
causes $\Delta P_g$) can be understood. It should be noted that the
effect of damping is not neglected but excluded from (3) for the
ease of analytical derivation, as the focus is on estimating inertia, not damping. Eqn. (3) forms the basis of several
measurement-based techniques for inertia estimation of
synchronous and non-synchronous generators during large
disturbances (transients). As ROCOF measurements are not
readily available, they are replaced by the generator terminal
ROCOF measurements [28].

Proceeding with (3), the next steps are to develop an
equivalent expression to achieve (1). The first step towards that is
by linking the network bus frequencies to the generator rotor
speeds, which is provided by the frequency divider (FD) formula [29]. The FD formula is model-dependent which
requires the information of the network admittances, generator
internal reactance ($x_a'$ for transient generator model, or $x_a''$ for
sub-transient generator model). By taking the time derivative of
the FD formula, a relationship can be established between the
ROCOF in the network buses ($\tilde{\omega}_B$) with the ROCOF ($\omega_g$)
which is given as

$$\tilde{\omega}_B = D \omega_g$$  \hspace{1cm} (4)

where $\omega_g$ is a $N_B \times 1$ vector of network bus frequencies, and $D$
is an $N_B \times N_g$ matrix obtained from the imaginary part of the
augmented network admittance matrix which is given as

$$D = -(\text{imag}(Y) + B_{BG})^{-1} B_{BG}$$  \hspace{1cm} (5)

Here, $Y$ is the $N_B \times N_B$ standard network admittance matrix.
$B_{BG}$ is an $N_B \times N_B$ diagonal matrix, and $B_{BG}$ is an $N_B \times N_B$
matrix which are obtained from the generator internal
reactances [29]. Since $D$ is obtained from the structure-
-preserving model-based [30] augmented admittance matrix, it is
time-invariant but event-dependent. Therefore, it needs to be
updated only after contingencies which results in network
topological changes.

By multiplying from the left of both sides of (3) with $D$, and
plugging in (4), a relationship between the ROCOF in the
network buses with the generator power variations can be
established as

$$\omega_g = \frac{1}{2} D H_g^{-1} \Delta P_g$$  \hspace{1cm} (6)

Eqn. (6) can be useful in estimating the network bus ROCOFs,
provided the synchronized power measurements are available
only at the generator terminals. However, it still does not satisfy
the objective to achieve (1) analytically.

The remaining step in order to do so, is to link the nodal
active power of the network buses to the generator terminal
power. This mapping of nodal powers from the generator
terminal to all network buses can be performed by using the
Two-level method:
(a) Planning level: Nodal inertia at any given operating condition.
(b) Monitoring level: Online nodal inertia

upstream distribution matrix, which acts as the mapping matrix. Using the upstream method of power flow tracing [31], the contribution of every generator to each nodal power can be calculated, and is given as

\[
H \cdot \omega = A_B \cdot P_G
\]

Here, \(P_G\) is a \(N_B \times 1\) vector of total nodal inflow power, or simply nodal power, \(A\) is the \(N_B \times N_B\) upstream distribution matrix, where the sub-matrices \(A_g\) \((N_g \times N_B)\) and \(A_i\) \((N_i \times N_B)\) provide information of the upstream distribution of generator buses, and the upstream distribution of the rest of the buses \((N_i = N_B - N_g)\) in the network, respectively. The \((i, j)\)th element of \(A\) is calculated as

\[
[A]_{i,j} = \begin{cases} 
1 & \text{for } i = j \\
\frac{P_{i,j}}{P_i} & \text{for } P_{i,j} < 0 \\
0 & \text{otherwise}
\end{cases}
\]

Hence, the \(p\)th non-zero off-diagonal entries in the \(p\)th row of the matrix \(A\), represents the incoming power from bus \(j\) to bus \(i\). Considering only the generator buses in (7) i.e., the generator upstream distribution, results in

\[
P_g = A_g P_B
\]

Thus, the mapping matrix \(A_g\) links the nodal power of the network buses to the generator terminal buses. Next, the incremental form of (9) is required which can then be plugged in (6). Rewriting (9) in the incremental form results in

\[
\begin{bmatrix}
\Delta P_{g1} \\
\Delta P_{g2} \\
\vdots \\
\Delta P_{gm}
\end{bmatrix} =
\begin{bmatrix}
\Delta P_{1-2} & \cdots & \Delta P_{1-n} \\
\Delta P_{2-1} & 1 & \cdots & \Delta P_{2-n} \\
\vdots & \vdots & \ddots & \vdots \\
\Delta P_{m-1} & \Delta P_{m-2} & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
\Delta P_1 \\
\Delta P_2 \\
\vdots \\
\Delta P_m
\end{bmatrix}
\]

Hence for the \(p\)th generator, the power deviation at its terminal can be expressed as

\[
\Delta P_{p,p} = \Delta P_p + \sum_{i=p}^{n} \left( \frac{\Delta P_{p-i}}{\Delta P_i} \right) \Delta P_i
\]

Here the terms \(\Delta P_{p-i}/\Delta P_i\) represent the ratio of the power deviations in the incoming lines \((P_{p-i} < 0)\) to the nodal power deviations. These power deviations will be dictated by the line admittances and will distribute as per the network admittance matrix. Hence \(\Delta P_{p-i}/\Delta P_i\) can be approximated as \(P_{p-i}/P_i\), using which the incremental form of (9) can be written as \(\Delta P_{g} \approx A_g \Delta P_B\). Plugging \(\Delta P_g\) in (6) results in

\[
\dot{\omega}_B = \frac{1}{2} W \Delta P_B
\]

where \(W = DH_g^{-1} A_g\) is an \(N_B \times N_B\) weight matrix, which provides the spatial dependence of ROCOF on nodal power variations. By comparing the derived analytical expression in (12) with the desired hypothetical expression provided in (1), it can be concluded that the decoupled relation between the ROCOF of a bus and its nodal power variation as given in (1) cannot be exclusively obtained. However, the ROCOF of the \(k\)th bus can be calculated as a weighted sum of the nodal power variations and is given as

\[
\dot{\omega}_k = \frac{1}{2} \sum_{i=1}^{n} W_{k,i} \Delta P_i
\]

Therefore, the inverse of the weighted terms i.e., \(1/W_{k,i}\) denote the opposition to the \(k\)th bus ROCOF due to power imbalance at the \(j\)th bus. Hence, the total opposition can be calculated as the \(k\)th bus inertia, and is given as

\[
\hat{H}_k = \frac{1}{\sum_{i=1}^{n} W_{k,i}}
\]

which defines the estimated nodal inertia at the \(k\)th bus.

The first objective of the proposed method is to provide absolute-valued nodal inertia for any given operating condition (snapshot) of the system, and therefore primarily represents a
model-based planning problem. It can be summarized from this section that the required data for effective estimation of the nodal inertia values at any given snapshot are the network admittance matrix \( Y \), generator model information like internal reactance \( x_g \) and inertia constant \( H_g \), and the power flow solution at that snapshot (to obtain \( A_g \)). For any model-based planning problem, these data are readily available. The proposed method also provides an additional level of transparency. With the availability of synchronized power measurements at generator terminals through phasor measurement units (PMUs), \( A_g \) can be updated in real-time using (8). Given the network topological information, \( D \) may remain unchanged (ambient conditions), or may be modified after an event (contingencies). Assuming \( D \) to be updated according to the occurrence of any event, would enable online calculation of \( W \), and thereby the nodal inertia of the network buses. Such additional level of transparency can be utilized for monitoring the nodal inertia values. This two-level proposed method architecture is highlighted in Fig. 1.

Hence, it can be seen through the analytical derivation in this section that the proposed nodal inertia formula given in (14) does not require the ROCOF or the rate of change of generator power \( dP_g/dt \) information, which makes it immune to the method used to estimate ROCOF or \( dP_g/dt \). Another advantage of the proposed method over traditional swing equation-based inertia estimation methods is that this method does not require defining thresholds for near-zero ROCOF values, as determining these thresholds can be system-specific and therefore a non-trivial problem.

### III. CASE STUDY

In this section, the applicability of the proposed method is tested on the IEEE-39 bus test system [32] for several system scenarios. Firstly, the planning level nodal inertia estimation is performed for a given operating condition. The additional transparency of monitoring the nodal inertia online during ambient and transient system conditions is discussed later.

For performing the simulation studies, the test system model was created separately in MATLAB/Simulink and PSS/E environments, thus enabling cross-validation and securing modeling accuracy. The 6th order (sub-transient) generator models, with static excitors and power system stabilizers were used.

#### A. Given Operating Condition

First, the first-level of the proposed method was performed as part of a planning problem. The required data was extracted from the test model, i.e., (a) \( Y \) and \( x_g \) to calculate \( D \) using (5), (b) \( P_{l-f} \) \((l \in \text{PV buses})\) from branch power flows obtained from the power flow solution to calculate \( A_g \), and (c) generator \( H_g \).

Next, the weight matrix \( W \) was calculated using \( W = DH_g^{-1}A_g \), from which the absolute-valued nodal inertias were estimated using (14).

This was performed for the initial equilibrium condition and the results are shown in Fig. 2. In Fig. 2(a) the estimated machine inertia constants are shown. Numerically these estimated values are seen to be equivalent to the actual values to within 4% of prediction error. For the non-generator buses of the network, the estimated nodal inertia is provided in descending order in Fig. 2(b). It is observed that buses 1, and 9 have the highest nodal inertia, and buses 19, 20, 23, and 22 have the lowest nodal inertia. The validation of these numerical values of nodal inertia will be made in the following sub-section while estimating the ROCOF following a disturbance, where the accuracy of the estimated ROCOF would depend on the numerical values of these nodal inertia. However, the ranking of the buses as seen in Fig. 2(b), is similar to the bus rankings obtained using relative-valued inertial index methods [20]-[23] for identical operating condition.

#### B. Online Estimation

To test the second-level, i.e., the applicability of the proposed method in nodal inertia tracking for both ambient and transient condition, synthetic measurements were obtained through dynamic simulations of the test system. The additional data required were obtained through generator terminal measurements which are used to update \( A_g \).

Ambient system conditions attributed to continual changes in system loads are considered first. To mimic the ambient condition, the loads were excited with additive white noise [33] at every simulation time step \( \Delta t \). In Fig. 3, the ambient system condition is demonstrated through bus 6 voltage angle and active power flow in line 19-16. For a 60-seconds ambient data set, the nodal inertias are estimated. The estimated nodal inertias for generators at bus 30 and 38, and non-generator buses 8, 19, 27 and 3 are shown in Fig. 4. Here, for each bus, the nodal estimates for every time step are provided (in grey) through a scatter plot, along with the moving average over a 100 samples window (in black). To evaluate the performance of the proposed method in ambient condition, the mean absolute percentage error (MAPE) [34] was calculated using
Nodal Inertia estimated for bus 30, 38, 8, 19, 27, and 3 in ambient conditions.

\[
\text{MAPE}_{k,\text{amb}}(\%) = \left( \frac{1}{n} \sum_{n} \left| \frac{\tilde{H}_{k,0} - \tilde{H}_{k,\text{amb},[n]}}{\tilde{H}_{k,0}} \right| \right) \times 100
\]

where, \( \tilde{H}_{k,\text{amb},[n]} \) denotes the \( n \)th sample ambient nodal inertia for the \( k \)th bus, and \( \tilde{H}_{k,0} \) denotes the estimated nodal inertia obtained previously for the given initial system condition. For the 60-seconds ambient data set, the MAPE was calculated for all the buses of the 39-bus system, as shown in Table I. It can be observed that the maximum MAPE was less than 0.5% for the test system, which validates the accuracy and tractability of the proposed method in estimating nodal inertia for ambient system conditions.

Next, the capability of the proposed method in effectively estimating and tracking the nodal inertias was tested for a transient condition. No additional data is required (with respect to the data requirement for ambient condition) for performing the nodal inertia estimation during transient conditions. This is a key advantage of the proposed method over other inertia tracking methods for transient conditions, as for the later, additional information like ROCOF, and defining near-zero ROCOF and \( dP/dt \) thresholds are minimum requirements.

The system transient condition was simulated through outage of generator 9 connected at bus 38 at 1 s. The fault was sustained for the rest of the simulation period (till 5 s). The nodal inertias for buses 30, 33 and 35 (generator-connected buses), and 2, 6, 16, 20, 22 and 28 (network buses) are shown in Fig. 5. It can be seen that the nodal inertias hold their initial pre-fault value till the fault is triggered. The inertial values are reduced after the fault at 1 s. Reductions in nodal inertia is observed in all the buses, which is justified as the overall system inertia is reduced due to the generator outage. However, the numerical values of the redistributed nodal inertias need to be validated.

To validate the authenticity of the post-disturbance nodal inertia redistribution, the ROCOF of the network buses were estimated using (13), and compared with the ROCOF obtained from the bus measurements. The nodal powers required in (13) are calculated using (7) as \( P_B = \text{inv}(A)P_G \). The estimated ROCOFs were compared against the measured ROCOFs for the same set of buses, and is shown in Fig. 6. It is observed that barring the first transient swing, which can be attributed to the sample window for calculating \( df/dt \) (and hence beyond the scope of this work), the estimated ROCOF matches with the measured ROCOF, which suggests that the nodal inertial redistributions after the disturbance are accurate, which verifies the ability of the proposed method in accurately estimating and tracking the nodal inertias. From (13), it can be also concluded that these nodal inertias along with the nodal power variations are able to capture the frequency dynamics with considerable accuracy.

### Table I

<table>
<thead>
<tr>
<th>Bus</th>
<th>MAPE_{amb} (in %)</th>
<th>Bus</th>
<th>MAPE_{amb} (in %)</th>
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<tbody>
<tr>
<td>1</td>
<td>0.109</td>
<td>21</td>
<td>0.006</td>
</tr>
<tr>
<td>2</td>
<td>0.015</td>
<td>22</td>
<td>0.004</td>
</tr>
<tr>
<td>3</td>
<td>0.013</td>
<td>23</td>
<td>0.004</td>
</tr>
<tr>
<td>4</td>
<td>0.015</td>
<td>24</td>
<td>0.007</td>
</tr>
<tr>
<td>5</td>
<td>0.019</td>
<td>25</td>
<td>0.01</td>
</tr>
<tr>
<td>6</td>
<td>0.018</td>
<td>26</td>
<td>0.008</td>
</tr>
<tr>
<td>7</td>
<td>0.023</td>
<td>27</td>
<td>0.008</td>
</tr>
<tr>
<td>8</td>
<td>0.025</td>
<td>28</td>
<td>0.005</td>
</tr>
<tr>
<td>9</td>
<td>0.016</td>
<td>29</td>
<td>0.005</td>
</tr>
<tr>
<td>10</td>
<td>0.014</td>
<td>30</td>
<td>0.008</td>
</tr>
<tr>
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<td>0.011</td>
</tr>
<tr>
<td>12</td>
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<td>0.009</td>
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<tr>
<td>13</td>
<td>0.014</td>
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</tr>
<tr>
<td>14</td>
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</tr>
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<td>0.009</td>
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<td>0.003</td>
</tr>
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<td>0.003</td>
</tr>
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<td>0.432</td>
</tr>
<tr>
<td>20</td>
<td>0.003</td>
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<td></td>
</tr>
</tbody>
</table>

The previous section emphasizes on the tractability of the proposed method in effectively estimating the nodal inertias in varying system conditions. To improve the credibility of the proposed method further, its robustness is tested for uncertainties in system model data (model data required for the method), and also in the presence of measurement noise.
First, uncertainty in line reactances for the transmission network is considered to study the impact of model data uncertainty. To emulate uncertain line reactances, zero mean Gaussian noise, with differing variances was added to the basecase line reactances \cite{35}, \cite{36}. While choosing the maximum variance, it is ensured that the steady-state load flow converges. For the IEEE 39-bus test system, this was observed to be 20%. The nodal inertias were estimated for the same transient condition that was previously performed i.e., outage of generator 9 at 1 s, considering different variance (from 5% to 20%) for the additive noise in the line reactances. For every noise variance level, 5 sets of nodal inertia estimates were obtained by running the simulation 5 times. The estimated nodal inertias for bus 2, 6, 16, and 20 were compared with respect to the basecase (without uncertainty), and is provided in Fig. 7. It can be seen that for increasing noise variance in the model data – 15% (shown in red) and 20% (shown in purple), the degree of variation in the estimated nodal inertia values increases. Whereas for low variance – 5% (shown in green) and 10% (shown in blue), the nodal estimates are comparatively near the basecase estimate (shown in black). To numerically evaluate the degree of estimation error, the MAPE for every set of test scenarios (with model uncertainty) were calculated with respect to the base case using the following expression

$$\text{MAPE}_{k,\text{case}}(\%) = \left( \frac{1}{n} \sum_{n} \left| \frac{\tilde{H}_{k,\text{base}}[n] - \tilde{H}_{k,\text{case}}[n]}{\tilde{H}_{k,\text{base}}[n]} \right| \right) \times 100$$ \hspace{1cm} (16)

This is provided in Table II, where for different levels of additive noise variance, the MAPE is calculated for the buses 2, 6, 16, and 20. It can be seen that even for the largest variance of 20% in all the line reactances, the highest estimation error is less than 1.9%.

Next, the accuracy of the proposed method is tested in the presence of measurement noise. For this, the test system was...
subjected to three different types of disturbances, and the nodal inertia as well as the estimated ROCOFs were calculated. This is highlighted in Fig. 8. First, the system was subjected to a load increase at bus 5 by 500 MW at 1 s. Then at 15 s, a solid bus fault is applied at bus 24 which is cleared after 100 ms. Finally, generator 9 connected at bus 38 was taken out at 40 s. The estimated inertia without and with measurement noise were calculated separately and are plotted for bus 22 as shown in Fig 8(a) and 8(b), respectively. Also, to validate the inertial estimate, the estimated ROCOF were compared with the actual ROCOF obtained through terminal measurement. This is shown separately for both the cases, i.e., without and with measurement noise, in Fig. 8(c) and 8(d). It can thus be concluded from these results that the proposed method is robust against uncertainties in both data and measurement and is able to accurately monitor and track the nodal inertias.

Fig. 7. Nodal inertias at bus 2, 6, 16, and 20 for the basecase (without model uncertainty) and test cases with different variance level in the additive gaussian noise added in all line reactances.

Table II

<table>
<thead>
<tr>
<th>Noise Variance</th>
<th>Bus 2</th>
<th>Bus 6</th>
<th>Bus 16</th>
<th>Bus 20</th>
</tr>
</thead>
<tbody>
<tr>
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Fig. 8. Estimated nodal inertia for bus 22 (a) without measurement noise, (b) with measurement noise. Comparison of the estimated ROCOF (using (13)) with the actual ROCOF (c) without measurement noise, (d) with measurement noise.

V. CONCLUSION

A two-level method for estimating inertia distribution is proposed. Depending on the availability of system and measurement data, the proposed levels of the method can be applied. The first level is part of a planning problem which provides inertial distribution for any given operating condition. The second level is for online monitoring and tracking of the nodal inertias for both ambient and transient system conditions. The advantages of the proposed method over other inertia distribution methods are discussed in terms of its (i) absolute-valued results, (ii) mode-agnostic approach, (iii) non-requirement of ROCOF, or rate of change of power injection, and (iv) non-requirement of ROCOF threshold. The validation of the method is tested in the IEEE 39-bus system for all system condition. Also, the robustness of the method is in its
effectiveness and accuracy in tracking and monitoring the nodal inertias is discussed in the presence of modeling uncertainties and measurement noise.

REFERENCES


