Adaptation Metrics for Low-Order Mobile Line-of-Sight-MIMO Links

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Abstract
Adaptation metrics for low-order mobile line-of-sight-MIMO links. These are linked to the mean-square error (MSE) and channel MIMO matrix conditioning.
Adaptation Metrics for Low-Order Mobile Line-of-Sight-MIMO Links

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Abstract—We examine adaptation metrics for mobile line-of-site MIMO (LOS-MIMO) links. An example of such a deployment is a very high-speed wireless link in the terahertz (THz) band between an indoor base-station and an untethered mixed-reality headset worn by a user whose distance from the base-station is time-variant. Due to narrow antenna beams and directional propagation at these frequencies, realization of high spectral efficiency must rely on LOS-MIMO transmission, realization of which depends critically on the distance between transmitter and receiver, as well as on the antenna-array element spacing at the transmitter and receiver. Therefore, metrics that facilitate adaptation of the element spacings in response to changes in the link length are required. We derive these metrics for MIMO orders two, three and four and show how the transmission rate of the LOS-MIMO link depends on them. We also point out the efficacy of frequency adaptation in accommodating a mobile link.

Keywords—LOS-MIMO; adaptation; metric; line-of-sight MIMO; THz, terahertz

I. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) signaling is a widely-used technique for increasing the spectral efficiency of both wired and wireless data communication links, and is enabled by the multi path propagation characteristics of the wireless channel. At carrier frequencies up to the high tens of gigahertz, the statistical nature of multi path signal propagation makes MIMO communication possible by providing uncorrelated signal transmission paths between elements of a group of transmit antennas and those of a group of receive antennas. This is primarily due to random scattering of radio signals by objects in and around the propagation environment, and uncorrelated signal-transmission paths provided by multi path propagation are realized when antenna spacings are greater than half the carrier wavelength. The resultant multi path channel MIMO matrix then contains entries that are complex Gaussian variates and, with high probability, uncorrelated.

At carrier frequencies above several hundreds of GHz, signal propagation in wireless channels becomes directional due to the short wavelength, specular reflections, and higher antenna gains required to operate wireless links. As a result, the efficacy of multi path-based MIMO communication is lost. But Line-of-Sight MIMO (LOS-MIMO) transmission can be realized by taking advantage of propagation path-length differences between pairs of transmit and receive antennas. The LOS-MIMO channel is therefore independent of the signal propagation environment and is deterministic, depending only on those propagation path-length differences. If we consider parallel transmit and receive antenna arrays with uniform element spacing in a purely LOS scenario (i.e., no scattering), the MIMO channel matrix $H = [h_{nm}]$ has entries

$$h_{nm} \approx \frac{\lambda}{4\pi D} \exp \left[ -j \frac{2\pi}{\lambda} \left( D + \frac{(md_T - nd_R)^2}{2D} \right) \right]$$

with $D$ the link length, $d_T$ and $d_R$ the transmit and receive antenna element spacing, respectively, $\lambda$ the carrier wavelength, and assuming $D \gg d_T, d_R$ [1]. This is illustrated in Fig. 1. The capacity of the LOS-MIMO link with channel matrix entries given by (1) is maximized when all singular values of $H$ are equal, i.e., when $H$ is a scaled unitary matrix (all pairs of distinct columns are orthogonal)—and we assume that $H$ is square of size $N$. When that is the case, all $N$ individual component streams comprising the MIMO transmission have equal transmission rate. It is well known [1]-[4] that $H$ is a scaled unitary matrix when $d_T d_R = \lambda D/N$.

$$d_T d_R = \lambda D/N. \quad (2)$$

A. Antenna Deployment for LOS-MIMO

Due to its relative simplicity and small size, the user equipment (UE) will typically have a few small sub-arrays with a fixed number of elements (sub-array size) and inter-sub-array spacing [throughout this paper, the base station—(gNB) in 3G Generation Partnership Project (3GPP) language—is the transmitter and the user equipment (UE) is the receiver.] These will support MIMO transmission of modest order $N$ likely two or three. The extent of UE antenna array flexibility then is limited to recruiting up to several fixed-size sub-arrays and to beam steering, i.e., the UE is able to electronically steer its antenna sub-array beams to a fixed number of different directions. The gNB, on the other hand, will have a much larger and very flexible antenna array to enable support of LOS-MIMO links with a large pool of sub-arrays that can be dynamically configured into variable sub-array sizes, and

![Fig 1. Parallel uniform linear transmit and receive arrays for $N = 3$ with $2 \times 2$ sub array size.](image-url)
groups of those sub-arrays in turn will flexibly support a wider range of MIMO communication order. We observe from (2) that, for fixed MIMO order and carrier frequency, the product of the gNB and UE sub-array spacings is proportional to the radio link length. Therefore, if the UE sub-array spacing is fixed, variable gNB sub-array spacing is required to accommodate variable link length (i.e., UE mobility.) After a LOS-MIMO link has been optimally setup according to (2), a change in the distance $D$ between gNB and UE movement will result in loss of optimality of the LOS-MIMO link and, unless restored by reconfiguration of the gNB or UE antenna array, will result in a reduction in the data transmission rate due to the reduction of the link channel capacity [8]. In practice, metrics that can be readily computed and which provide a quick indication of the deviation of link properties from (2) are indispensable.

B. Channel Matrix Condition Number vs. MIMO Rate

Here we describe how the conditioning of the LOS-MIMO channel matrix relates to the mean-square error (MSE) of the received signal vector resulting from LOS-MIMO transmission. The MSE, resulting from stream interference due to imperfect spatial equalization of the LOS-MIMO channel, sets an upper bound for the signal-to-interference-plus-noise ratio and thus also for the MIMO transmission rate. Consider MIMO transmission

$$y = Hx + n$$

of zero-mean complex signal vector $x \in \mathbb{C}^{N \times 1}$ over LOS-MIMO channel matrix $H$ of size $N \times N$ resulting in received signal $y \in \mathbb{C}^{N \times 1}$ corrupted with complex noise vector $n$ with uncorrelated normally-distributed components. Let $H = USV^H$ be a singular value decomposition of $H$, with $U$ and $V$ unitary $\in \mathbb{C}^{N \times N}$, and $S \in \mathbb{R}^{N \times N}$ diagonal comprised of the ordered singular values $\{\sigma_1 > \cdots > \sigma_N\}$ of $H$.\(^1\) With perfect channel knowledge at both ends, the transmitter transmits precoded $z = Vx$ over the channel and the receiver recovers

$$\hat{x} = S^{-1}U^H y = x + S^{-1}U^H n$$

from channel output $y = H\hat{x} + n$. Now assume that the LOS-MIMO link at a fixed carrier frequency is optimally set up for a known distance $D$ according to (2) and then, due to UE movement, a different channel $H_\Delta$ results. If antenna-element spacings $d_T$, $d_R$, the precoding $V$, spatial equalization $U$ and scaling $S$ matrices all remain unchanged, an error vector $e$ will result from use of these matrices to recover $x$ from $H_\Delta$, with $e = (I - S^{-1}U^HH_\Delta V)x$, and $I$ the size-$N$ identity matrix. If $x$ has covariance matrix $E[xx^H] = \sigma^2 I$, then the normalized MSE at the channel output is

$$\text{MSE} \triangleq \frac{E[\|e\|^2]}{E[\|x\|^2]} = \frac{1}{N} \|I - S^{-1}U^HH_\Delta V\|^2_F,$$  

with $\|M\|_F$ the Frobenius norm of $M$. Alternatively, we may contemplate the signal-to-interference ratio $1/\text{MSE}$. The matrix condition number $\kappa(H)$ is defined as $\kappa(H) \triangleq \sigma_1/\sigma_N$ and as mentioned earlier, a scaled unitary $H$ is optimal for MIMO transmission and results in $\kappa(H) = 1$ so that $S$ in (5) takes the form $sI$ with $s = \lambda \sqrt{N/(4\pi D)}$ for the channel matrix $H$ with entries given by (1). We can examine the dependence of the MSE as given by (5)–on $\kappa(H)$ for the case where antenna spacings $d_T$ and $d_R$ optimally set for a given distance $D_{\text{ref}}$ are subsequently used at distance $D_\Delta \neq D_{\text{ref}}$. Thus, for each value of $D_\Delta$ in a range centered at $D_{\text{ref}}$ we compute both the MSE and $\kappa(H)$ with fixed $d_T$ and $d_R$. We are interested in such a relationship because we later derive error metrics that estimate $\kappa(H)$ and therefore the MSE. Fig. 2 shows the empirically-determined characteristic for the dependence of the MSE on $\kappa(H)$ for $d_T = d_R$, and we observe that the MSE degrades very rapidly as $H$ deviates moderately from a scaled unitary matrix. For a practical receiver, the MSE will be much higher than thermal noise [represented by the second term on the right hand side (RHS) of (4)] over most of the range of the abscissa shown in Fig. 2, and will therefore be the major determinant of the MIMO transmission rate.

II. LINK FINE-TUNING AND ADAPTATION USING PHASE

A. Preliminaries

Here we consider a general method for guiding fine-tuning of an order-$N$ LOS-MIMO link using channel phase. A particular case for $N = 2$ of the general method we describe here was presented in [5], and our development does reduce to the specific phase metric used there although the derivation was not explicitly shown. In what follows, we derive metrics which are dependent on the phase shifts of the component MIMO paths. These metrics provide an indication of $\kappa(H)$ without explicit estimation of $H$ and subsequent computation of its singular values. They can then be monitored during fine-tuning of the LOS-MIMO link at setup or for reconfiguration of the gNB antenna as the UE moves around the service area of the gNB, we write a normalized $H$ as

$$H = \begin{bmatrix} e^{-j\phi_1} & \cdots & e^{-j\phi_N} \\ \vdots & \ddots & \vdots \\ e^{-j\phi_{N-1}} & \cdots & e^{-j\phi_{N-1+N}} \end{bmatrix}$$

\(^1\mathbb{R}$ and $\mathbb{C}$ are the real and complex field, respectively, and $M^H$ is the Hermitian transpose of $M$. 

![Fig. 2. Variation of the MIMO channel-output normalized mean-squared error with channel-matrix condition number.](image-url)
with path phase \( \phi_{mn} = 2\pi d_{mn}/\lambda \), and \( d_{mn} \) the distance between transmit antenna \( m \) and receive antenna \( n \). When \( \mathbf{H} \) is a scaled unitary matrix, \( \mathbf{HH}^H = \mathbf{H}^H \mathbf{H} = \mathbf{N I} \). The matrix product therefore takes the general form

\[
\mathbf{HH}^H = \begin{bmatrix}
1 & s_{12} & \cdots & s_{1N} \\
s_{21} & 1 & \cdots & s_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
s_{N1} & s_{N2} & \cdots & 1
\end{bmatrix}
\]  

(7)

with \( s_{mn} = (1/N)\sum_{k=1}^{N} \exp[-j(\phi_{mk} - \phi_{nk})] \). Maximizing the MIMO transmission rate requires driving all the off-diagonal entries \( s_{nm} \) to zero, i.e., for \( m \neq n \). Because \( \mathbf{HH}^H \) is Hermitian, \( s_{nm} = s_{mn}^* \), and therefore if \( s_{nn} = 0 \) then \( s_{mn} = 0 \) also. It is not evident that a general solution for the zeros of \( s_{nn} \) (with \( m \neq n \)) for arbitrary \( N \) is known, but we show solutions for \( N = 2, 3 \), and 4; these solutions lead directly to error metrics.

**B. Error Metric Derivation—\( N = 2 \)**

For \( N = 2 \), setting the single upper off-diagonal term \( s_{12} = 0 \) gives the single constraint for \( \mathbf{H} \) to be a scaled unitary matrix,

\[
e^{-j(\phi_{12} - \phi_{11})} + e^{-j(\phi_{21} - \phi_{12})} = 0,
\]  

(8)

and by setting real and imaginary parts to zero we obtain

\[
\cos A + \cos B = 0 \\
\sin A + \sin B = 0
\]  

(9)

with \( A = (\phi_{21} - \phi_{11}), B = (\phi_{22} - \phi_{12}), \) and general solution

\[
\phi_{21} - \phi_{11} = \phi_{22} - \phi_{12} + (2k\pm1)\pi, \quad k \in \mathbb{Z}.
\]  

(10)

The one-dimensional error metric \( \epsilon \) is then

\[
\epsilon = (\phi_{21} - \phi_{11}) - (\phi_{22} - \phi_{12}) - (2k\pm1)\pi,
\]  

(11)

which should be driven towards zero during link fine-tuning or adaptation. In principle, the LHS of (9) could be viewed as a pair of error metrics, but due to the oscillatory nature of the circular functions over relatively small perturbations of the path-phase differences, those expressions are ill-suited for such a purpose. The same will be observed for \( N = 3 \) and 4.

**C. Error-Metric Derivation—\( N = 3 \)**

For \( N = 3 \), the \( \binom{3}{2} \) constraints corresponding to the upper off-diagonal terms \( s_{12} = s_{13} = s_{23} = 0 \) result in

\[
\cos A + \cos B + \cos C = 0 \\
\sin A + \sin B + \sin C = 0
\]  

(12)

and \( A = (\phi_{x1,1} - \phi_{x2,1}), B = (\phi_{x1,2} - \phi_{x2,2}), C = (\phi_{x1,3} - \phi_{x2,3}) \) with pairs \((x_1, x_2)\) drawn from the set \( \{(1, 2), (1, 3), (2, 3)\} \) resulting in six equations. Equation (12) has as a general solution\(^2\)

\[
A = C + (2k\pm2/3)\pi, \quad k \in \mathbb{Z}
\]  

(13)

which is independent of \( B \), and subsequently implies an error metric corresponding to each sine and cosine pair from (12) as

\[
\epsilon = A - [C + (2k\pm2/3)\pi], \quad k \in \mathbb{Z}.
\]  

(14)

Equation (14) applies to each of the three sine-cosine pairs (12). This gives the components of a three-dimensional error-metric \( \epsilon \)

\[
\epsilon_1 = (\phi_{11} - \phi_{21}) - (\phi_{13} - \phi_{23}) - (2k_1\pm2/3)\pi \\
\epsilon_2 = (\phi_{11} - \phi_{31}) - (\phi_{13} - \phi_{33}) - (2k_2\pm2/3)\pi, \\
\epsilon_3 = (\phi_{21} - \phi_{31}) - (\phi_{23} - \phi_{33}) - (2k_3\pm2/3)\pi
\]  

with \( \{k_1, k_2, k_3\} \in \mathbb{Z} \).

**D. Error-Metric Derivation—\( N = 4 \)**

For \( N = 4 \), constraining to zero the upper off-diagonal terms \( s_{nn} \) in (7) reduces to the pair of equations

\[
\cos A + \cos B + \cos C + \cos D = 0 \\
\sin A + \sin B + \sin C + \sin D = 0
\]  

(15)

with \( A = (\phi_{x1,1} - \phi_{x2,1}), B = (\phi_{x1,2} - \phi_{x2,2}), C = (\phi_{x1,3} - \phi_{x2,3}), D = (\phi_{x1,4} - \phi_{x2,4}) \), and pairs \((x_1, x_2)\) drawn from the set of all \( \binom{4}{2} \) pairs of the integers 1-4. This results in six sine-cosine pairs (15). A solution to (15) can be approached by writing

\[
\cos A + \cos B = \cos[C + l\pi] + \cos[D + m\pi] \\
\sin A + \sin B = \sin[C + l\pi] + \sin[D + m\pi]
\]  

(16)

with \( l \) and \( m \) odd integers. This has as a solution

\[
A + B = C + D + 2k\pi, \quad k \in \mathbb{Z},
\]  

(17)

and the corresponding error metric implied by (17) is therefore

\[
\epsilon = A + B - (C + D) + 2k\pi, \quad k \in \mathbb{Z}.
\]  

(18)

which applies to each of the six sine-cosine pairs (15) giving a six-dimensional error metric \( \epsilon = [\epsilon_n], n = 1, \ldots, 6 \). For example, with \( n = 1, (x_1, x_2) = (1, 2) \) and

\[
\epsilon_1 = (\phi_{11} - \phi_{21}) + (\phi_{12} - \phi_{22}) - [(\phi_{13} - \phi_{23}) + (\phi_{14} - \phi_{24})] + 2k_1\pi
\]

In practice, minimization of each expression in (11), (14), and (18)\(^2\)We have used identities \( \sin A + \sin B = 2\sin\frac{1}{2}(A + B)\cos\frac{1}{2}(A - B) \) and \( \cos A + \cos B = 2\cos\frac{1}{2}(A + B)\cos\frac{1}{2}(A - B) \) to obtain (13).
will be achieved by a search over a few small values of the integer \( k_i \in \{0, \pm1, \pm2\} \).

III. MEASUREMENT OF PATH PHASE DIFFERENCES

The derived error metrics are simple functions of differences in path phase, and there are several methods of making these measurements \((\phi_{ik} - \phi_{jk})\) in the range \(-\pi \leq \phi_{ik}, \phi_{jk} \leq \pi\). We follow and generalize to some degree the method described in [5] for \( N = 2 \) that makes use of a pair of phase-locked sinusoidal tones with slightly different frequencies at the transmitter. A procedure for implementation of this measurement method for \( N > 2 \) is as follows: the transmitter sequentially transmits a pair of phase-locked tones using a pair of transmit antennas, and the receiver estimates the phase difference \((\phi_{ik} - \phi_{jk})\) in path phase between pair \((i, j)\) of transmit antennas and receive antenna \( k \). Thus, all of the error signals \( \epsilon \) in (14) [for \( N = 3 \)] or in (18) [for \( N = 4 \)] can be measured by sequentially stepping through all transmitter antenna pairs. Fig. 3 shows how these measurements can be made for \( N = 3 \) using three envelope detector/phase detector (ED/PD) sets and a pair of coordinated switches at the transmitter. Three states of the switch-pair are required to provide all the measurements required to compute all the error metrics. For general \( N \), the functionality depicted in Fig. 3 can also be realized with fewer than \( N \) (but a minimum of two) ED/PD sets but at the expense of a longer measurement duration. For a LOS-MIMO link, the optimal antenna configuration or adaptation setting is that for which a longer measurement duration. For a LOS-MIMO link, the optimal antenna configuration or adaptation setting is that for which a longer measurement duration. For a LOS-MIMO link, the optimal antenna configuration or adaptation setting is that for which a longer measurement duration.

IV. EMPIRICAL RESULTS AND DISCUSSION

Here we study the behavior of these adaptation metrics over perturbations of the link length \( D \). We first show how the condition number \( \kappa(\mathbf{H}) \) of the channel matrix \( \mathbf{H} \) varies with a change in \( D \).

[Fig. 3. Measurement of path-phase difference for \( N = 3 \) using three sets of envelope/phase detector pairs. Switches are set to measure \((\phi_{31} - \phi_{21})\), \((\phi_{32} - \phi_{22})\) and \((\phi_{33} - \phi_{23})\).]

We do this by assuming that an optimal LOS-MIMO link of nominal length \( D \) has been setup for a given MIMO order \( N \), i.e., \( d_T \) and \( d_R \) have been configured according to the optimality condition (1). Then, leaving both fixed, the link length is varied resulting in varying \( \mathbf{H} \) that deviate from a scaled unitary. Plots of the MSE and condition number of \( \mathbf{H} \) are shown in Fig. 4 from which we observe that the deviation of \( \mathbf{H} \) from a scaled unitary matrix is modestly greater as the UE approaches the gNB than when receding from it. Observe also that in order for the MSE to remain below about \(-20 \text{ dB} \), \( \kappa(\mathbf{H}) \) must not increase beyond about 1.1 for \( N = 2 \). This translates into a requirement for link adaptation when \( \Delta D/D \) extends beyond an operating window. From Fig. 4, the operating windows for an MSE of \(-20 \text{ dB} \) are \( \Delta D/D = 0.14, 0.09, \) and \( 0.06 \), for \( N = 2, 3, \) and \( 4 \), respectively. If \( d_T = ad_T \) when \( \mathbf{H} \) is optimal, empirical examination reveals that this operating window is maximized for \( \alpha = 1 \), i.e., \(|d_T(\mathbf{H})/dD|\) is minimized for small \( \Delta D \)—the plots in Fig. 4 are for \( \alpha = 1 \). This underscores the importance of having to reconfigure either \( d_T \) or \( d_R \) as variation of the link length exceeds the boundaries of a narrow range, particularly as the UE approaches the gNB. An important consequence of the narrow operating window is that, to ease the practical difficulty of making such small adjustments to \( d_T \) and \( d_R \) to accommodate UE movement, the operating frequency (i.e., \( \lambda \)) will likely also have to be changed as part of the adaptation. Examination of (2) makes it clear that \( \lambda \) can also be modified to re-establish optimal \( \mathbf{H} \). For a high time rate of change of the UE-gNB distance \( D \), this reconfiguration will need to be done rapidly, and such rapid adaptation will require error metrics that can be rapidly computed—like those derived in the previous section. See also the related work in [8] for fixed links.

We next examine the response of the metrics to changes in link length \( D \). In general, each component of the error metrics is a different function of \( \Delta D/D \).3 Recall from (11), (14), and (18) that \( \epsilon \) has dimension \( (\frac{N}{2}) \); we plot in Fig. 5 the first component \( \epsilon_1 \) of \( \epsilon \) against \( \Delta D/D \). A serendipitous benefit of such a monotonic and near-linear characteristic over the operating range is its suitability for also estimating the distance change between UE and gNB if the nominal link length (when \( \epsilon = 0 \)) is known. Shown in Fig. 6 is a plot of the condition number \( \kappa(\mathbf{H}) \) against the magnitude of \( \epsilon_1 \), and the monotonic characteristic makes evident the facility of

![Fig. 4. MSE (top) and channel matrix condition number (bottom) vs. normalized distance change for \( d_T = d_R \).](image-url)

3For \( N = 3 \), \( \kappa(\mathbf{H}) \) varies with \( \Delta D/D \) identically for \( \{\epsilon_1, \epsilon_3\} \), while for \( N = 4 \), variation for \( \{\epsilon_1, \epsilon_4, \epsilon_6\} \) is identical, as it is for \( \{\epsilon_2, \epsilon_6\} \).
estimation of $\kappa(H)$ from $|\epsilon_1|$. The ordinate in Fig. 6 can be readily transformed into the MSE by use of the $\kappa(H)$-to-MSE function in Fig. 2; this is shown in Fig. 7.

Lastly, a change in the relative elevation angle between the transmitter and receiver antenna arrays will also result in a degradation of the MSE. The gNB and UE arrays in Fig. 1 are both shown at 90° elevation—i.e., at relative elevation $\Delta \theta = 0^\circ$. We have examined, empirically, the MSE response to a change in the relative elevation by holding fixed the gNB and varying the optimal link has been set up at $\Delta \theta = 0^\circ$. The MSE degrades sharply with increasing $\Delta \theta$, in most cases resulting in a narrow operating window (defined by MSE < −20 dB.) For example, at 300 GHz, $D = 5$ m, and $N = 2$, the operating window is only about fifteen degrees (i.e., ±7.5° from 90° elevation.) The MSE-$\Delta \theta$ characteristic is dependent on operating frequency and link length, but the operating window is again maximized for $d_\parallel = d_R$. Since the error metrics we have derived here only give an estimate of the channel matrix condition number, a single component of the multi-dimensional error metric is unable to distinguish between MSE degradation from UE translational motion and that due to changes in UE antenna pitch. It is likely, however, that using more than one component of the metric may enable discrimination between these two modes of UE motion.

V. CONCLUDING REMARKS

We have shown the derivation of metrics which facilitate adaptation of antenna-element spacing required for optimal operation of mobile LOS-MIMO links. These metrics, derived for LOS-MIMO orders two, three, and four provide a direct measure of the signal-to-distortion ratio of the LOS-MIMO link as the distance between transmitter (gNB) and receiver (UE) changes due to UE mobility. The metrics facilitate rapid estimation of both the MIMO channel-matrix condition number and the link length (if receiver motion is purely translational.) Such rapid estimation is required to enable mobile LOS-MIMO links.

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