Waveguide-Floquet Mapping Based on Surface Susceptibilities for Metasurface Unit Cell Characterization

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Abstract

A simple method is proposed to characterize metasurface unit cells using rectangular waveguide. The method maps the S-parameters obtained using a dominant-mode waveguide to the S-parameters of fundamental Floquet mode for a plane wave incident at any desired angle. The mapping between the dominant waveguide mode and the fundamental Floquet mode is executed by extracting the equivalent surface susceptibilities of the metasurface unit cell. The results obtained from the proposed method show good agreement with the full-wave simulation results of the metasurface unit cell.
1 Introduction

Metasurfaces are finding many new applications nowadays. By controlling the amplitude and/or phase of reflection and/or transmission coefficients from a metasurface, properties of the emanating beams can be flexibly manipulated. Thus, such metasurfaces are being used as perfect absorbers, polarization converters, reflector antennas, lenses, filters, sensors, and so on, throughout the frequency spectrum [1]. To experimentally characterize these metasurfaces, the most popular method is to irradiate an array with sufficiently large number of its unit elements using plane wave in free-space and measure the reflection and/or transmission characteristics with a receiving antenna [2]. However, this method becomes expensive most of the times, especially for reconfigurable metasurfaces where active elements, such as PIN diodes, varactor diodes, and so on, are loaded onto the printed PCBs. Therefore, it is highly desired to obtain the measured characteristics of the unit cell in a less expensive way before going to full array implementations and measurements.

Waveguide simulator [3, 4], which is known for a long time, can be one such low-cost method to obtain experimental characteristics of the metasurface unit cells. The method uses only few elements inside a waveguide. However, this method has limitations. The frequency response obtained from a waveguide simulator corresponds to certain plane wave component with specific frequency dependent incident angles only following the dispersion characteristics of the waveguide [3]. Thus, this method is found not so effective and popular for characterizing metasurface for a plane wave incidence with a normal or a fixed incident angle.

In this paper, an approach is proposed to map the waveguide response, which is function of incident angle of plane wave, to the Floquet mode response of unit cell at any fixed angle. In this method, the metasurface is represented by effective surface susceptibilities, which are the effective constitutive parameters of the unit cell independent of the incident angle [5, 6], extracted from the waveguide response. After that, the reflection and/or transmission coefficient for fundamental Floquet mode can be calculated for any desired incident angle. To validate the proposed method, results from full-wave simulations inside a waveguide are transformed to the Floquet mode response with the help of surface susceptibilities. The transformed responses are compared with the full-wave simulations of the metasurface unit cell with periodic boundaries.

2 Proposed Waveguide-Floquet Mapping

The wave with dominant mode (TE_{10}) inside a rectangular waveguide can be reconstructed using a plane wave with an incident angle \( \theta \) using the relation [3],

\[
\sin(\theta) = \frac{\lambda_0}{2b}
\]

where \( \lambda_0 \) is free space wavelength, and \( b \) is the longer dimension of the waveguide cross-section. The plane wave component with its plane of incidence and polarization related to TE_{10} mode inside the waveguide is illustrated in Figure 1(a). Thus, if a X-band (8-12 GHz) waveguide is used, the incident angle of equivalent plane wave will vary from 55.11° to 33.15° with changing frequency.

Therefore, for a waveguide terminated with a metasurface (e.g. a reflector cell) that needs to be characterized, a TE_{10} mode excitation at a specific frequency \( f_0 \) is equivalent to an infinitely periodic array of unit cells excited with a uniform plane wave at an angle \( \theta(f_0) \) given by (1). The question is if the reflectance \( R(f_0) \) measured inside a waveguide can predict the reflection response of the cell when excited by plane wave with any other arbitrary angle of incidence. Here we propose a method based on surface susceptibilities of the unit cell, that achieves this goal.
2.1 Surface Susceptibilities-Based Mapping

As explained in [5], [6], metasurface resonators can be homogenized using spatially varying electric and magnetic surface susceptibility models, which are tensors in general. For example, under plane-wave incidence, when parallel electric currents are induced on the metasurface, an electric resonance is formed. This can be modeled using tangential electric surface susceptibilities ($\chi_{EE}^{\theta}$). Whereas, when a time-varying magnetic flux generates a circulating current normal to it, a magnetic dipolar moment will be created. This can be modeled using a normal magnetic surface susceptibility ($\chi_{MM}^{\theta}$). By symmetry and reciprocity considerations of a unit element, the general tensor form of the surface susceptibilities maybe reduced to fewer terms [7]. The surface susceptibilities represent the constitutive parameters of the unit cell and are independent of the angle of incidence for deeply sub-wavelength structures. They are typically extracted from discrete set of plane-wave excitations of the unit cells with normal and oblique angles of incidences. Once obtained, the scattering response of the reflective or transmissive cell can be reconstructed at any other arbitrary angle. For instance, the reflection ($R$) and/or transmission coefficients ($T$) for any incident angle ($\theta$) can be calculated from these susceptibility components using, (2a) and/or (2b)

$$R(\theta, \alpha) = \frac{-jk_0 (\chi_{EE}^{\theta} + \chi_{MM}^{\theta} \sin^2 \theta)}{2 \cos \theta - jk_0 (\chi_{EE}^{\theta} + \chi_{MM}^{\theta} \sin^2 \theta)}$$

(2a)

$$T(\theta, \alpha) = \frac{2 \cos \theta}{2 \cos \theta + jk_0 (\chi_{EE}^{\theta} + \chi_{MM}^{\theta} \sin^2 \theta)}$$

(2b)

where $k_0$ is the free-space wavenumber. If these surface susceptibilities can somehow be extracted from the waveguide measurements, the response of the unit cell in an infinite periodic configurations (i.e. the Floquet response) at an arbitrary angle can be readily reconstructed. This is the basic principle of the proposed method. The steps involving this process are provided in Figure 1(b).

3 Numerical Demonstration

3.1 Dog-bone Unit Cell Structure

The above method is validated here using full-wave simulation results obtained from CST MWS v2022. For the sake of simplicity and demonstration of the proposed method, consider a reflective metasurface with dog-bone unit cell, as shown in Figure 2. The dog-bone unit cell is designed on a ground-backed Rogers RO4003C substrate ($\varepsilon_r = 3.55, \tan \delta = 0.0027$) with thickness ($h$) of 32 mil. The unit cell dimensions are chosen in such a way that an X-band waveguide can be used for the demonstration. The geometry parameters are given in Figure 2. It can be noted that the unit cell is subwavelength to satisfy the homogeneity criteria. The waveguide can accommodate $1 \times 4$ unit cells of the dog-bone metasurface inside it, as shown in Figure 2(b).

This purely reflective dog-bone metasurface can be characterized by a single susceptibility term ($\chi_{yy}^{\theta}$) based on physical considerations. Specifically, it can be shown that for a TE (or y−) polarized incident wave, this structure can be represented by an equivalent tangential electric susceptibility ($\chi_{yy}^{\theta}$), which can be extracted from the reflectance at a specific angle $\theta$ as [8]

$$\chi_{yy}^{\theta} = \frac{4 j \cos(\theta)}{k_0} \left[ \frac{R(\theta) - 1}{R(\theta) + 1} \right].$$

(3)
Once $\chi_{ee}^{yy}$ is known, the reflectance at all other angles can be obtained from

$$R(\theta = \theta_0) = \frac{4\cos(\theta_0) - jk_0\chi_{ee}^{yy}}{4\cos(\theta_0) + jk_0\chi_{ee}^{yy}}.$$  \hspace{1cm} (4)$$

Thus, from the waveguide reflection response, $R(f)$, $\chi_{ee}^{yy}(f)$ can be easily extracted using (3) and (1). Thereafter, the reflection coefficient corresponding to a fixed incident angle ($\theta_0$) of a TE-polarized plane wave can be calculated using (4).

![Figure 4. Extracted electric susceptibilities of the dog-bone metasurface from the waveguide S-parameters.](image-url)

### 3.2 Numerical Results

The magnitude and phase of the reflection coefficient obtained from the waveguide simulation using CST MWS are shown in Figure 3. The waveguide simulation result shows a maximum reflection loss of 2.7 dB at 9.48 GHz. The extracted real and imaginary tangential electric surface susceptibilities ($\chi_{ee}^{yy}$) are shown in Figure 4. From the extracted $\chi_{ee}^{yy}$, the reflection coefficient is computed for $\theta = \theta_0(f)$, where $\theta_0 = 0^\circ$, and plotted in Figure 3. The magnitude and phase of the reflection coefficient of a unit cell with 2-D Floquet boundaries for TE polarized normal incidence obtained from full-wave simulation are also given in Figure 3. It can be observed that both the magnitude and phase response obtained from the proposed method via the waveguide simulator shows an excellent agreement with the Floquet full-wave simulation results of the unit cell.

### 4 Conclusion

A simple method to map the dominant mode waveguide results to fundamental Floquet mode responses for metasurface unit cells has been presented. The method has been demonstrated on a reflective dog-bone metasurface using numerically simulated results. The results obtained using the proposed method shows an excellent agreement with the full-wave Floquet simulation results of unit cell. Since the waveguide measurement at a fixed frequency corresponds to a single angle of incidence, the current methods works only for structures which are characterized by a single susceptibility term. The mapping will be explored further for variety of passive and dynamic metasurfaces in a future work and possible extension to unit cells with increased susceptibility tensor components. More detailed analysis of this method will further be performed establishing the limitations of this method. It will help to obtain measured responses of the metasurfaces in a low-cost process before going for full array fabrication and measurement.

### References


