DC Inductor Design using the Unique Air Gap Equation

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Index Terms—Ampere’s Law, unique air gap equation, area product, core geometry coefficient

I. INTRODUCTION

The inductor is the principal component in DC-DC conversion circuits. It is the backbone to convert, to store and to transfer the energy from the source to the load. Furthermore, it is the component responsible for generating the Volt-second balance. It is also the component which contributes on the design of the capacitor filter and it plays an important role in achieving the soft switching in several converter topologies. The design and optimization of the DC inductor was and still yet a central research subject since the early days of the power electronics field. Several papers and books were published to explain the principle of operation of the inductor and its design. The area product method (AP) and the core geometry coefficient method (Kg) are two fundamental approaches applied to design the DC inductor [1-4]. In general, the principle of these two methods is to firstly find the suitable core based on the input specifications. Then, the designer needs to determine the required number of turns that fits to the designed core and finally calculate the air gap that gives the required inductance. In most of design cases, the convergence to the final solution is achieved through an iterative approach. In the literature, numerous advanced design and optimization techniques, which are based on these two fundamental methods, have been developed. Despite the significant contributions made by techniques and their remarkable impact on the efficiency and the power density of the DC inductor, we think it is not worthy to review these advanced methods because the purpose of this work is to discuss a fundamental design issue. In addition, most of the advanced methods are based on one of the two fundamental methods (AP or Kg) and eventually they suffer the same issue.

The objectives of this paper are the following:
1. Highlight the issue existing in the literature related to the dependency of the air gap on the cross section. Then, the paper introduces a new design method based on the unique air gap equation. The interaction between the inductance variables is investigated based on the proposed method.
2. Demonstrate the unique air gap equation and the independence of the air gap and on the cross section.
3. Verification of the proposed method with the AP approach.
4. Discuss the advantages of the proposed method over the existing methods.

The paper is structured as follows: section II presents the existing issue and the misconception of the air gap and the cross section dependency. The principle of the proposed method and the analysis of the inductance variables interaction is given in section III. The proofs of the air gap independence on the cross section is shown in section IV. The design steps of the proposed method and the discussion of the results are given in section V. The verification with the AP approach and the discussion is presented in section VI. Finally, the paper’s major contributions are presented in the conclusion.

II. EXISTING ISSUE

The existing issue in the literature is not in relation to the equations, used for the calculation of the air gap, but it is related to what can be understood from these equations about the interdependence between the inductance variables. The main equations used in the literature that make the confusion in the design of the DC inductor and specifically the misconception of the air gap function are the following:

\[
I_g = \frac{\mu_0 L_{max}^2}{A B_{max}^2} - \frac{I_c}{\mu_r} \tag{1}
\]

\[
E = \frac{1}{2} B_{max}^2 A I_g \quad \tag{2}
\]

The first equation gives the impression that the air gap is a function of the cross section, assuming that the remaining variables are constant. The misconception comes from considering \( L \) as a constant and not a function that depends on the variables given in equation (3). Replacing \( L \) by its expression clearly will cancel out the cross section variable.
In fact $A$ is a dummy variable in equation (1). Nevertheless, equation (1) can be used for the calculation of $I_g$ but to be understood that the air gap is dependent on the cross section is a well-known misconception as we will be demonstrated in the following sections.

The second equation shows also that for a constant energy, a change in $A$ definitely leads to a change in $I_g$. However, equation (2) cannot be used in the case we want to change the cross section while keeping the energy constant for the following reason. Equation (2) is derived from Faraday’s law and Ampere’s law and in the later law, the magnetic field depends on the air gap itself. Therefore a change in the gap length, due to the change in $A$, will force an opposite change in the magnetic field and the magnetic flux density and hence the energy density will in turn changes.

III. PRINCIPLE OF THE DESIGN METHOD AND ANALYSIS OF VARIABLES INTERDEPENDENCE

The proposed design method is based in solving for $N$ and $I_g$ the following system of equations.

\[
\begin{align*}
I_g &= \frac{2A_{\text{max}} N I_{\text{max}}}{B_{\text{max}}} - \frac{I_c}{\mu_r} \\
L &= \frac{\mu_0 \mu_r N^2 A}{I_c + \mu_r I_g}
\end{align*}
\]

(3, 4)

The first equation is defined as the unique air gap equation and the second equation is called the reluctance model of the inductance. The solution of this system of equations are given as follows:

\[
(N, I_g) = \left( \frac{L I_{\text{max}}}{A B_{\text{max}}}, \frac{\mu_0 N I_{\text{max}}}{B_{\text{max}}} - \frac{I_c}{\mu_r} \right)
\]

(5)

The first solution for $N$ is well-known, however, the solution for $I_g$ is called, here, the unique and exact equation of the air gap. It is independent on the cross section and relatively dependent on the core length as well as the magnetic material. For magnetic materials with high permeability such as ferrite and Nano-crystalline, the second term in the air gap equation is negligible which makes the air gap completely independent on the core length and the relative permeability of the magnetic material. The solution clearly shows that $N$ is inversely proportional to the cross section and $I_g$ is proportional to $N$. The solution $(N, I_g)$ of our system gives the impression that $I_g$ is also dependent on $A$, however, the reality is that $I_g$ is independent on $A$. In order to understand this independence, we need to look to the solution’s expression and to the equation of $L$ together. As it can be seen in equation (4), $L$ is proportional to $N$ squared and proportional to $A$ while inversely proportional to the air gap $I_g$. For same $L$, a change in the cross section $A$, leads to an opposite change in the number of turns. However, since $L$ is proportional to $N^2$, the change in $L$ due to $N$, is also squared. In fact it is a product of two equal changes. The first change in $N$ is to fully compensate the change due to $A$. Hence $A$ becomes with no effect as it was not changed. The second same change in $N$ needs to be compensated by the air gap in order to keep $L$ unchanged. In the following, we explore the effect of the cross section, the relative permeability and the inductance on the solution $(N, I_g)$.

A. Effect of the cross section

The effect of the cross section can be understood through the solution of the following design example.

Let’s design a DC inductor with the following specifications, using the proposed method: $L=10$ uH, $I_{\text{max}} = 20$ A, $B_{\text{max}}=0.2$ T, $\mu_r=2000$. We choose to sweep the cross section in order to explore the variation of $N$ and $I_g$ with respect to $A$, the core length can be set zero ($l_c=0$) which is the advantage of this method compared to AP and Kg methods. As it can be seen in Fig.1-a, the evolution of $N$ follows the function $(1/A)$. The air gap is implicitly decreases as $A$ increases (Fig.1-b). However, the real cause of the air gap evolution is the term $(1/N)$, which is the remaining effect in $L$ as it can be noticed in Fig.1-d and Fig.1-c.

B. Effect of the relative permeability

Since the independence of $I_g$ on the cross is justified, the effect of the relative permeability on the air gap can be understood by fixing the cross section to a constant value and changing the relative permeability. Then, we examine the results on the air gap and the number of turns.

From solution (5), it can be seen that the number of turns will keep unchanged as the cross section is fixed, however, the air gap decreases as the relative permeability increases and vice-versa (see Fig.2).

For a given $L$, $I_{\text{max}}$ and $B_{\text{max}}$, it is possible to know the solution of the relative permeability that gives a zero air gap length.

\[
\mu_r |_{I_g=0} = \frac{B_{\text{max}} I_c}{\mu_0 N I_{\text{max}}}
\]

(6)

If we suppose that the length of core is largely bigger than the cross section diameter, the core length becomes only dependent on the number of turns and the conductor diameter. The core length $l_c$ can be derived from the winding area as follows:
where \( \eta \) is the filling factor. The general expression of the core length that takes into account the diameter of the cross section is given by equation (21). In this analysis part, we suppose that diameter is negligible compared to the core length, which is a fundamental assumption of Ampere’s Law applied in the design of the magnetic components. The conductor diameter can be estimated by the following expression:

\[
d_p = \sqrt[4]{\frac{4}{\mu_0 N I_{\text{max}}}} \times 10^{-3}
\]

Substituting (8) into (7) and for \( \eta=0.78 \), we get the following expression for the core length:

\[
l_c = \frac{\pi}{7.2} \times 10^{-3} \sqrt{N I_{\text{max}}} = 1.49 \times 10^{-3} \sqrt{N I_{\text{max}}}
\]

Substituting (9) into (6), we get:

\[
\mu_r l_g = \frac{1.49 \times 10^{-3} B_{\text{max}}}{\mu_0 N I_{\text{max}}}
\]

One important metric result that can be derived is the following. For a 1 turn inductor, \( B_{\text{max}}=0.2 \) T and \( I_{\text{max}}=1 \) A, the minimum relative permeability to achieve a zero gap inductor is 237. Fig.2-a shows the evolution of the air gap with respect to the relative permeability of a typical design.

Fig.2-b shows how the number of turns is independent on the relative permeability. This means that changing the magnetic material for same inductance will only have an effect on the air gap and not on the number of turns.

Another important result is related to the maximum current, which the non-gapped inductor can handle for a given permeability. As an example, for a 1 turn inductor, \( \mu_r=2000 \) and \( B=0.2 \) T, the maximum current is equal to 0.118 A.

C. Effect of changing the inductance

In this section, we explore the evolution of \( A, l_c \), and \( N \) due a change in the inductance value.

1) Case 1: \( A, l_c \), and \( N \) fixed and \( L \) varies

If \( L \) changes by a factor of \( x \), the number of turns will change oppositely by the same factor \( x \) and consequently the air gap will change proportionally to \( N \). The resulting change of the inductance variables \( N, l_g \) and \( l_c \) is same as the original change of \( L \). It can be seen from the solution (5) that the magnetic flux density will keep unchanged. Equation (2) shows that the energy density will keep constant but the volumetric energy will change by \( x \). The change of \( l_c \) and \( N \) with respect to \( L \) is shown in Fig.3-a. As it can be seen in Fig.3-b, the air gap has always a linear dependency on the number of turns.

For customized cores, the change in \( N \) will force the core length to change by the same factor, however, its effect is insignificant on the inducance value. Therefore, it can be concluded that the size of the inductor is proportional to \( N \) and hence the inducance is also proportional to the size. This fact is evident by nature as the energy storage is proportional to the size.

This result corrects a well-known misconception that states the inducance is not size depending.

IV. PROOFS

A. Proof by Principle of conservation of energy

It can be deduced from the principle of energy conservation that the energy stored in the inductor is equal to the energy released to the load and is also equal to the energy absorbed from the source during the charging time. The energy quantities can be written mathematically as follows:

\[
\frac{1}{2} P = \frac{1}{2} B_{\text{max}}^2 l_g A + \frac{1}{2} \frac{B_{\text{max}}^2}{\mu_0 \mu_r} l_c A
\]

By substituting the average power \( P \) with its expression function of the voltage and the current, we get the following:

\[
\frac{1}{2} \int_0^T V_L(t) i_L(t) dt = \frac{1}{2} \frac{B_{\text{max}}^2 A}{\mu_0 \mu_r} \left(l_g + \frac{l_c}{\mu_r} \right)
\]

The integral term can be calculated for the case where the inductor is operating in the boundary condition mode (BCM) and then it can be generalized for any mode. This mode is chosen because of the simplicity of the integral calculation. The average power of the inductor in the BCM is as follow:

\[
P = \frac{1}{T} \int_0^T V_L(t) i_L(t) dt = V_L I_{\text{max}} D
\]
Where D is the duty cycle. The voltage source $V_s$ can be expressed using Faraday’s Law.

$$V_L = \frac{N A f B_{max}}{D}$$  \hspace{1cm} (14)

Substituting (14) and (13) in (12) and solving for $l_g$, We get the unique air gap equation (3).

### B. Proof by derivation

The common equation of the air gap is given by expression (1). By substituting $L$ by its expression (4), we get the following quadratic equation:

$$l_g^2 + \frac{2 l_g}{\mu_r} + \frac{l_c^2}{\mu_r^2} - \frac{\mu_0 N^2 l_{max}^2}{B_{max}^2} = 0$$  \hspace{1cm} (15)

One solution of this quadratic equation is the unique air gap equation (3).

### C. Proof by Identification

The unique equation of the air gap is exactly Ampèr’s Law as demonstrated by the following.

$$H_{g_{max}} l_g + H_{c_{max}} l_c = N I_{max}$$  \hspace{1cm} (16)

$$B_{max} \left( \frac{l_g}{\mu_0} + \frac{l_c}{\mu_0 \mu_r} \right) = N I_{max}$$  \hspace{1cm} (18)

$$l_g = \frac{\mu_0 N l_{max}}{B_{max}} - \frac{l_c}{\mu_r}$$  \hspace{1cm} (19)

### D. Proof by Absurd

As it can be understood from the equation of $L$ (4), the existence of $L$ necessarily requires the existence of $N$ and $A$ together and simultaneously. In other terms, the non-existence of one of these two variables leads naturally to the non-existence of the inductor. On the other hand, the existence of the inductor needs only the existence of one variable of the two variables in the denominator. The non-existence of $l_g$ gives the ideal non-gapped inductor and the non-existence of the $l_c$ gives the air core inductor. In order to understand the dependency between the air gap and the renaming variables $A$ and $N$, we can impose a change in one variable and see its effect on the air gap and on the existence of $L$ for the following hypotheses.

1) **Hypothesis 1: $l_g$ and $A$ are independent**

First, we fix the remaining variables ($N$, $l_c$) and we verify the existence and the non-existence of $L$. In this hypothesis, the existence and the non-existence of $L$ is constantly depending on $A$, by intuition, we can conclude that when $A$ exists, $L$ exist and when $A$ does not exist, $L$ does not exist. Thus, the independence between $A$ and $l_g$ is true. Since a fact cannot be true and false at the same time, the dependence between $A$ and $l_g$ is false.

2) **Hypothesis 2: $l_g$ and $A$ are dependent**

Let’s consider an inductance $L_0$ for a given cross section and air gap noted $A_0$ and $l_{g0}$ respectively. It is given by the following equation:

$$L_0 = \frac{\mu_0 \mu_r N^2 A_0}{l_c + \mu_r l_{g0}}$$  \hspace{1cm} (20)

Let’s suppose that $A_0$ is increasingly reduced by $x$ to an infinitesimal value. In this case the air gap needs to be increased by $x$. the inductance can be expressed as function of $x$ as follows:

$$L(x) = \frac{\mu_0 \mu_r N^2 A(x)}{l_c + \mu_r l_{g0} \frac{A_0}{A(x)}}$$  \hspace{1cm} (21)

Given that $A(x)$ is equal to $\frac{A_0}{x}$, $L(x)$ can be written by the following expression:

$$L(x) = \frac{\mu_0 \mu_r N^2 A_0}{x^2 A_0} = \frac{A(x)^2 l_{g0}}{A_0^2}$$  \hspace{1cm} (22)

By examining the evolution of $A(x)$ and $L(x)$, we can understand that the inexistence of $L(x)$ will occur before the occurrence of the inexistence of $A(x)$, which is impossible by nature.

3) **Hypothesis 3: $l_g$ depends on $N$**

Let’s suppose that $N$ is reduced by $x$ to an infinitesimal value but not zero. According to equation (3), the air gap will also decreases by the same factor $x$. the total effect leads to decrease the inductance by the same factor $x$. if $N$ disappears, $L$ will disappear at the same time.

In conclusion, the existence and the non-existence is only verified using only the unique relationship between the air gap and the number of turns (3).

### V. DESIGN METHOD STEP AND DISCUSSION

#### A. Steps of the proposed design method

In this part, the design steps of the proposed method, at a fixed frequency, are detailed in the following:

**Step 0:** The input data are: the target inductance $L$ (constant), $I_{max}$ (constant), $B_{max}$ (constant), conductor cross section $S_c$ (constant), cross section diameter $D_c$ (variable).

**Example:** $l_{max}=10$ A, $B_{max}=0.2$ T, $L_s=1$ uH, $l_c=0$, $D_c=[3:0.1:5]$

**Step 1:** solve (3, 4) and find the solution $(N, l_g)$.

Fig. 4 shows the evolution of the number of turns with respect to $A$ and the evolution of the air gap with respect to $N$. an increase in in the cross section results in a decrease in the number of turns. The air gap is proportional to the number of turns as given in equation (3).

![Fig. 4. Variation of $N$ with respect to $A$ for the given design example.](image)

![Fig. 5. Variation of $l_g$ with respect to $N$ for the given design example.](image)
\[ I_c = \pi \left( \frac{4A_w}{\pi} + \frac{4A}{\pi} \right) \]  \hfill (24)

**Step 3: design verification**

The results can be verified by comparing the real design results to the target results. In this step, we calculate the real value of the following quantities: \( L, B_{\text{nac}} \) and we compare \( E_t \) to \( E_m \).

\[
E_t = \frac{1}{2} L I_{\text{max}}^2 \hfill (25)
\]

\[
E_m \approx \frac{1}{2} B_{\text{max}}^2 A l_g \hfill (26)
\]

The small difference between the obtained results and the target ones is because the real results are plotted using the target value of the maximum current and not the real one. In principle, the maximum current depends on the excitation voltage, the frequency and the real inductance. In general, the obtained results and the target ones are very similar as shown in fig.6. In particular, fig.6-d shows the evolution of the core length with respect to \( A \) and with respect to the number of turns. As it can be seen, the cross section has significant effect on increasing the core length than the number of turns. The intersection between the evolution curves of \( I_c \) as function of \( A \) and \( N \) defines the optimal solution for the core volume and loss minimization in case the core loss is negligible. This is because the core loss is linearly proportional to the cross section while the winding loss is linearly proportional to the number of turns.

![Graphs](image)

**Fig. 6. Comparison of the obtained results with the target results (a, b, c) and analysis of \( I_c \) with respect to \( A \) and \( N \) (d).**

**Step 4: Determine the optimal solution**

The total loss (\( P \)) and the core volume (\( V_c \)) can be calculated using the following equations.

\[
V_c = A l_c \hfill (27)
\]

\[
P = P_c + P_w \hfill (28)
\]

The core and winding loss are given by (29) and (30) respectively:

\[
P_c = k f^\alpha B_{\text{nac}}^\beta V_c \hfill (29)
\]

\[
P_w = R_{\text{dc}} I_{\text{rms}}^2 \approx R_{\text{dc}} I_{\text{dc}}^2 \hfill (30)
\]

**Step 5: choose or customize the suitable core**

**VI. VERIFICATION WITH AP APPROACH**

In this part, the inductor with the following specifications is designed using the method of the UAGE and the AP method. The inductor to be designed is the following: \( L=1 \text{ uH}, I_{\text{dc}}=30 \text{ A}, B_{\text{dc}}=0.2 \text{ T}, u_r=2000 \). The optimization will be performed by sweeping the diameter (\( D \)) of the core cross section. The design constraints to be accounted are the following. The first constraint, given by (31), is the minimum number of turns set to one turn due to difficulty in the fabrication process to achieve a fractional turn especially for power inductor. The second constraint (32) is the minimum diameter (\( D_{\text{min}} \)) of the core cross section is set to 4mm to increase the capability of the core to damp mechanic forces such as vibration.

\[
N_{\text{min}} \geq 1 \hfill (31)
\]

\[
D_{\text{min}} \geq 4 \text{ mm} \hfill (32)
\]

**A. UAGE method**

**Fig. 8-a shows the variation of the number of turns due the change in the cross section. As it can be seen, the variation is a typical \( (l/A) \) function where an increase in the cross section, naturally leads to a decrease in the number of turns. The change rate in both variables must be equal to satisfy Faraday’s law imposed by the voltage of the sources across the inductor.** Fig.8-b shows the dependence of the air gap on the number of turns, the relationship between both is linear where an increase in \( N \) leads naturally to a same increase in the air gap. We note that the air gap has a complete independence on the cross section. This can be concluded intuitively from equation (4) as any change in the cross section.
will be fully compensated by the reverse change in the number of turns.

Fig. 8. Inductance-variables interdependence using the unique air gap equation.

Fig. 9 shows the variation of the core volume with respect to \(N\) and \(A\). As it can be seen, the core volume is inversely proportional to \(N\), however it is linearly proportional to the cross section. The latter conclusion can easily understood from equation (27). An increase in \(A\) will increase the volume by the same way. However, we know that an increase in the cross section leads to increase the mean core length. In principle, if we consider the last condition, the core volume will increase to the square of \(A\). But, we know also that the core length depends on the number of turns. Since \(N\) is inversely proportional to \(A\), therefore, the core length effect on the core volume due to \(A\) is compensated by the effect of \(N\) on the core length. The compensation rate depends on several factors such as the filling factor and the core geometry. In general, the effect of \(A\) and \(N\) are approximately equal which make the core volume depends on \(A\) and negligibly on the effect of \(A\) on the core length. The dependence of the core volume on \(N\) follows the typical \((1/N)\) function and give consistency to the dependence of the core volume with \(A\).

Fig. 10 shows the variation of the inductor loss with respect to \(N\) and \(A\). As it can be seen, an increase in \(N\) leads to increase the loss in an slight linear way within the variables solutions intervals. On the other hand, the inductor loss goes inversely proportional to \(A\). Exploring Fig.9 and Fig.10, we can deduct the relationship between the core volume and the loss. As it can be understood, an increase in the core volume allows to reduce the loss and vice-versa. The interdependence between the volume and the loss can also be seen in Fig.11. In this latter, the volume and the loss are plotted against each other.

As explained in the previous section, the verification of the design can be achieved by calculating \(B_{dc}\), \(E_{m}\) and \(L\). Fig.12-14 show that the designed quantities are in good consistency with the target values of these quantities.

Fig. 9. Core volume dependence on \(A\) and \(N\).  
Fig. 10. Loss dependence on \(A\) and \(N\).

B. AP method

In this section, the results of the proposed method is compared with the results using the area product method. The principle of the area product method is to select a magnetic core based on the applications specifications. Some of these specifications such as \(B_{m}\) is and the temperature rise is fixed by the designer, while others are imposed by the application. Once the magnetic core is determined, the designer need to determine the number of turns and the air gap.

The design steps of the AP applied to our case are detailed in the following:

1) Core selection:

The core is selected using the following expression of the area product:

\[
AP = \frac{2W_m}{K_fA}\delta_m
\]  

(33)

\(W_m\) is the energy which will be stored in the inductor. It is given by the well-known formula given as below:
\[ W_m = \frac{1}{2} L_I m^2 \]  

Assuming that the current ripple is negligible in comparison to the DC component, we can write the following:

\[ W_m \approx \frac{1}{2} L_I d_c^2 \]  

(35)

\( K_f \) is the filling factor of the winding, which depends on the number of turns, the conductor shape and the core window geometry. It is approximated to 0.8 as in this example, we expect the number of turns to be low. \( J_m \) is the current density equals to 5A/mm², which is a conservative and a reasonable choice for such application. \( B_m \) is the operating magnetic flux density, usually chosen by the designer. Its selection depends on several considerations such as the saturation flux of magnetic material and the magnitude of the swing flux. In our example it is equal to 0.2 T, which is about 50% of the common saturation flux in ferrite materials and we expect the swing flux to be negligible. Numerical application of equation (33) allows to choose a magnetic core having an AP higher or equal to 0.1125 cm². A suitable core could be EE19/8/5. The geometry specifications are summarized in Tab. II.

<table>
<thead>
<tr>
<th>Tab. II EE19/8/5 Geometry Details</th>
<th>A(mm²)</th>
<th>l_c (mm)</th>
<th>V_c (cm³)</th>
<th>AP(cm²)</th>
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<tr>
<td>45.5</td>
<td>40.1</td>
<td>1.82</td>
<td>0.14</td>
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</tr>
</tbody>
</table>

2) Air gap and number of turns calculation

The equations derived from the principal laws in magnetism allows a bit of freedom to choose which variable to be calculated first. That’s why, there are several versions of the design steps of the AP method. In this paper, we choose to calculate the air gap using the following expression:

\[ l_g = \frac{\mu_0 L d_c^2}{A B_{dc}^2} - \frac{l_c}{\mu_r} \]  

(36)

\( A \) and \( l_c \) are, respectively, the cross section and the length of the selected core. Usually, these quantities are expressed in terms of their average or mean values and applied for \( l_g \) calculation in the design of the inductive components. The second term in equation (36) can be neglected if \( \mu_r \) is reasonably high. \( l_c \) can also be expressed using the peak values of the current and the magnetic flux density, however, it doesn’t matter as the ratio between both is the real effect that determine the air gap and one variable of them is an image to the other and vice-versa. Numerical application of (36) gives an air gap of 0.61 mm required to our design. This result is quite similar to the result obtained for the optimal solution given in Tab. I. This shows that the calculation of the air gap using the UAGE is accurate.

The number of turns \( N \) is equal to 3.3 computed using equation (37) and is equal to 4 using equation (38). In principle, both equations should give the same result, however, the small difference between both numerical results is expected as the real AP of the core is slightly higher than the calculated one. The ratio between the calculated and real AP and the ratio between the number of turns of (37 and 38) are equal. The question is which result to be considered by the designer. The correct solution is a number in between these two values. From a practical point of view, the right number of turns depends on the air gap. Since a small change in the air gap (few micrometer) can lead to a significant change in the inductance, it is much straightforward to determine the number of turns after gapping the core.

\[ N = \frac{L d_c}{K_f W_a} \]  

(37)

\[ N = \frac{K_f}{W} \]  

(38)

The value of the obtained inductance can be calculated from the reluctance model given in equation (2). It is equal to 0.97 uH using the solution of \( N \) obtained from equation (21). The second solution will give an inductance with considerable error as a small error in \( N \) is converted into an error in \( L \) proportional to to the square of the error in \( N \). For that reason, a trade-off between the air gap and the number of turns is usually required to achieve the right inductance.

Comparing to the results obtained from the proposed method, we can see that number of turns of the optimal solution is in good agreement with the result given by the area product.

In General, the proposed and the area product solution are converging to approximately the same solution. The advantages of the proposed method over the area product method are the following:

- The designer do not need to determine the suitable core in preliminary. The design can be done without known the core length. The design need only to start by a random selection of the cross section variable.
- The designer can design the winding without knowing the core.
- The proposed method gives much better understanding of the inter-connection between the design variables.
- The convergence time of the proposed method is much smaller than the area product method when applied for multi-objective optimization problems.

VII. DISCUSSION

This study has presented a deep and clear explanation of the relationships between the inductance variables, from which some useful conclusions can be extracted. The first conclusion is that the air gap is fully independent on the core cross section. As shown, in equation (3), the air gap is proportional to the number of turn, and inversely proportional to the relative permeability. Another important conclusion of this study is that the number of turns is independent on the magnetic material (relative permeability). This fundamental point is of great importance as it allows to design the inductor without knowing the core length, even for low permeability materials where the core length has a significant contribution on the air gap calculation. In fact, the designer can design the inductor with high permeability materials so that the second term in equation (3) becomes negligible and thereafter, the solution with low permeability could be determined. The study shows also that it is preferable to change the number of turns, rather than any other variable, when scaling up or down the inductance. Furthermore and unlike to what is found in the literature, this study shows that the inductance is size dependent, which means that an increase or decrease in the inductance value, at same DC current, leads to a similar change in the size. We should mention that this conclusion becomes invalid when the DC current changes. This point will be investigated in a future work.
From a point of view optimization, the proposed method has several advantages over the existing methods. The first advantage is the significant improved design flow with reduced iteration time to reach an optimal solution. In the existing optimization methods, it is common to sweep all the inductance variables to cover all the design space, which increases significantly the optimization time. Using the proposed method, only one optimization variable is required. For example, in the evolutionary algorithms like the Genetic algorithms, the computational complexity is function of the population size and the number of the objective functions. For instance, the computation complexity of the non-dominated sorting algorithms is $O(MN^3)$, while for dominated sorting algorithms is $O(MN^2)$ (where $M$ is the number of objectives and $N$ is the population size) [5]. As it can be clearly understood, the proposed method can reduce the population size by five (5) times, which results in a significant reduction in the simulation time. In fact, it is possible to consider the number of turns as the solely optimization variable, while the four remaining variables ($A$, $l_a$, $\mu_r$, $l_c$) could be derived as explained in section II. Another advantage of this method consists in improving the accuracy and the reduction of the sources of error. The latter is achieved by reducing the number of the design constraints and the number of the optimization variables, required in the existing methods. As an example, in the existing method, an inferiority and superiority constraint-equations are needed to meet with the target inductance value because the equal constraint is not feasible. Using the proposed method, there is no need for such constraints.

VIII. CONCLUSION

This study has revised a fundamental ambiguity that often lead to misconceptions about the dependency between the cross section and the air gap. In addition to that, this paper has proposed, a new design method based on the unique air gap equation. The proposed method enables to design the winding without the need to know the core length. Furthermore, the proposed method has several advantages over the existing method such as better accuracy and improved design flow with reduced convergence time when applied in the optimization of the DC inductor.

IX. REFERENCES