Self-Identification of Reluctance Synchronous Machines with Analytical Flux Linkage Prototype Functions

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Abstract

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Abstract—This paper presents a self-identification approach for reluctance synchronous machines (RSMs) in combination with analytical flux linkage prototype functions. Due to the severe magnetic saturation, good machine knowledge is required for high-performance electrical drive systems. The proposed self-identification method injects bipolar voltage pulses to an inverter-fed machine at standstill, without additional test equipment and requirements. Within an extremely short time, nonlinear flux linkages, including the magnetic saturation and cross-coupling effects, can be obtained. Instead of saving the samples as lookup tables (LUTs), they are approximated by analytical flux linkage prototype functions, which are parametrized by few parameters, and continuously differentiable throughout the whole operation range. The effectiveness of the developed self-identification method is experimentally validated for a nonlinear 4.0 kW RSM. The results prove (i) the simple and time-saving procedure of the self-identification method, (ii) the high approximation accuracy and (iii) the benefits of the flux linkage prototype functions for real-time applications.

Index Terms—analytical flux linkage prototype functions, reluctance synchronous machine, self-identification, saturation effects

I. Introduction

Reluctance synchronous machines (RSMs) [1], [2] are considered as a promising technology in industrial applications, owing to the good cost-effectiveness and high energy efficiency. However, due to the magnetic saturation and cross-coupling effects, their flux linkages and differential inductances exhibit highly nonlinear characteristics and thus vary enormously during operation. In order to accomplish the best possible drive performance, these effects must be taken into account during controller design. Unfortunately, it is common that only key nameplate specifications are available for commercial machines. Therefore, parameter identification [3] must be carried out to extract the nonlinear flux linkages and/or differential inductances experimentally.

To measure the nonlinear machine maps, traditional methods require specialized setups with power supply and data acquisition equipment, which might not be available in every laboratory. The difficulty of considering the cross-coupling limits the applicability for highly nonlinear machines, such as RSMs. In contrast, approaches which use inverters are more preferable, as they can cover the entire nonlinearity by a scheduled identification procedure in real-time control systems. Besides, inverters are already commonly equipped in modern electrical drive systems due to the increasing demand of variable speed drives. According to the rotor conditions during test, these inverter-fed approaches can be mainly categorized into two groups: constant speed and free-wheeling shaft methods. For the former, the tested machine must be coupled to a prime mover rotating at a constant speed [4], [5]. A large number of current pairs over the complete operation range must be tested thoroughly, which directly yields a sufficiently amount of data to generate LUTs for flux linkages or differential inductances. Hence, the obtained data by this method often serves as a reference to be compared with other identification methods. Nevertheless, for both, the traditional and constant-speed approaches, the time-consuming procedure and the mechanical and set-up requirements might not be valid for all applications and users.

On the other hand, for free-wheeling shafts, the “self-identification” [6]–[10] applies a sequence of test voltages or currents to acquire nonlinear machine parameters within a short time period while usually no further hardware modifications are needed. This is perfectly suited for self-commissioning, which is a desired functionality in modern electrical drive systems to start up the controlled machine successfully without knowing any system information beforehand. It can be performed either with varying speeds or at standstill. Dynamically [6], [7], constant current references are set to switch between motor and generator operation modes, such that the nonlinear machine maps can be extracted during machine acceleration and deceleration. However, some current pairs with higher values cause a rapid acceleration without being able to collect sufficient samples; therefore, the admissible measurement range might be narrow. Alternatively, at standstill [8]–[10], injecting voltage and/or current test pulses to the machine under test is also feasible. This identification procedure is extremely fast and requires (nearly) no additional mechanical or set-up requirements, and is, therefore, the preferred approach in this paper. Both [9] and [10] are based on the method proposed in [8] by means of applying similar voltage test sequences but handle the identified data differently. In [9], specially designed current functions are employed to represent the nonlinear flux-to-current saturation characteristics of RSMs. The inverse modeling facilitates the inclusion of the cross-coupling, which yields smooth current surfaces. In [10], the measured data is trained by a simple neural network invoking artificial intelligence methods, which
also results in smooth flux linkage surfaces. Nevertheless, the flux or current functions in [8], [9] contain absolute value functions and are thus not continuously differentiable, and the solution in [10] can hardly consider the energy conservation rule in the neural network training. Due to these difficulties, their applicability is limited.

To compensate for the magnetic nonlinearity of RSMs, prototype functions [8], [9], [11]–[13] are used to approximate the nonlinear flux linkages and differential inductances instead of lookup tables (LUTs). Their features, such as a low parameter number and an extrapolation capability, are desirable in real-time systems with limited storage. Among all existing solutions, the analytical flux linkage prototype functions for synchronous machines (SMs) [13] are: (i) physically motivated and designed to present the respective saturation effects including cross-coupling; (ii) continuous and even continuously differentiable; and (iii) obey the energy conservation rule. Furthermore, the applicability has been approved by utilizing them in a nonlinear current control system for RSMs [14].

The significant potential of flux linkage prototype functions, as proposed in [13], motivates the work in this paper to approximate the identified flux linkages during the simple and fast self-identification using flux linkage prototype functions. The comprehensive properties of the prototype functions are expected to be capable of overcoming the above mentioned drawbacks [8]–[10]. After providing the state-of-the-art on this topic in this section, the self-identification method and the analytical flux linkage prototype functions are discussed. At the end, in order to confirm the effectiveness of the proposed identification method, measurement results of the proposed self-identification methods for a RSM and the approximation results using the flux linkage prototype functions are presented.

II. Modeling and Parameter Identification

First, the adopted machine model of RSMs is briefly revisited, where the dependence of the flux linkages on temperature, speed and position is neglected. Only the current dependency is considered for simplicity. Afterwards, the self-identification method, including the self-axis and cross-coupling tests, is introduced to identify the nonlinear flux linkages. Besides, additional key considerations for the implementation are provided.

A. Nonlinear RSM Model

The electrical machine model in the rotating (d,q)-reference frame is given by [15, Chapter 14]

\[
\psi_d^q = \frac{\partial}{\partial t} \psi_d^q(t_s) + \frac{d}{dt}\psi_d^q(t_s),
\]

where \( \psi_d^q := (\psi_d^q, \psi_q^q) \) are the stator flux linkages, \( \psi_d^{q\sigma} := (\psi_d^{q\sigma}, \psi_q^{q\sigma}) \) are the flux linkages (which are functions of \( \text{e}^d \)) and \( R_s \) is the stator resistance. The electrical angular frequency \( \omega_p = \Omega_{rpm} \) rotates the (d,q)-reference frame synchronously, where \( \Omega_{rpm} \) and \( \omega_m \) denote the number of pole pairs and the mechanical angular frequency, respectively.

B. Self-Identification Method

When the machine is at standstill (\( \omega_p = 0 \)), the back electromotive force (EMF) term \( \omega_p J \psi_d^q \) in (1) is usually neglected. Then, the flux linkage derivative (the last term in (1)) serves as a key component in the self-identification. Integration of the voltage signal, subtracting the resistance voltage drop from the stator voltages, can lead to the flux linkages as follows

\[
\psi_d^q(t_s) = \int u_d^q - R_s i_d^q dt.
\]

For the discrete-time implementation, (2) can be discretized by means of the explicit Euler method, i.e.

\[
\psi_d^q[n] = \psi_d^q[n-1] + \frac{T_{sam}}{2}\left[u_d^q[n] - R_s i_d^q[n]\right]
\]

with the sampling time \( T_{sam} \), the known stator resistance \( R_s \), the measured currents \( i_d^q \) and the voltages \( u_d^q \) that can be approximated by their references, i.e. \( u_d^q[n] \approx u_d^q[n-1] \). Based on this concept, the following tests must be implemented to obtain the entire flux linkage maps including the self-axis and cross-coupling saturation effects.

1) Self-axis test

To extract the self-axis saturated flux linkages, voltage test sequences are applied separately to the \( d \) and \( q \) components. The voltage references \( u_d^q[n] := (u_d^q, u_q^q) \) can be generated by a step-like control function

\[
u_d^q, u_q^q \approx (u_d^q, u_q^q) \]

\[
\begin{cases}
 u_d^q, u_q^q \approx (u_d^q, u_q^q) \\
 u_d^q, u_q^q \approx (u_d^q, u_q^q) \\
 u_d^q, u_q^q \approx (u_d^q, u_q^q) \\
 u_d^q, u_q^q \approx (u_d^q, u_q^q)
\end{cases}
\]

where \( U_d^d \) and \( U_q^d \) are the testing voltage amplitudes and current limits, respectively. To avoid invoking the cross-axis current, the cross-axis voltage component must be set to zero, i.e. \( u_d^q[n] = (\pm U_d^d)^T \) for the \( d \)-axis test and \( u_d^q[n] = (0, \pm U_d^q)^T \) for the \( q \)-axis test. Similar to a hysteresis controller, the stator current increases and decreases with the positive and negative testing voltages, respectively. When the current exceeds the corresponding limit, the polarity of the testing voltage changes. Consequently, a high-frequency square-wave voltage signal is injected to the machine under identification. With the induced stator current and the flux linkage from (2), the desired current-to-flux relationship can be derived.

2) Cross-coupling test

To extract the cross-coupling saturation effects on the flux linkages, voltage test sequences are applied simultaneously to the \( d \) and \( q \) components. Reusing (3) and selecting proper voltage amplitudes for both components, the resulting voltage references \( u_d^q[n] = (\pm U_d^q)^T \) lead to the \( d \)-axis current \( i_d^d \) and the \( q \)-axis current \( i_q^d \) over the whole operation range. Even though the generated torque is nonzero, as \( i_d^d \) and \( i_q^d \) are nonzero during this test, the average torque is approximately zero, which prevents a dramatic acceleration of the tested machine. In addition, the pulsating torque can be filtered out to a certain extent, depending on the mechan-
cal loading condition. However, the inevitably slight rotor movement induces the back EMF, i.e. \( \omega_p J \psi_{dq} \neq 0 \). The neglect of it causes wrong voltage values in (2), which leads to a drift of the estimated flux linkages. Therefore, during the cross-coupling test in order to derive accurate flux linkage values, the back EMF must be taken into account as follows

\[
\psi_{dq}(\epsilon_{dq}) = \int u_{dq} - R_{dq} \epsilon_{dq} - \omega_p J \psi_{dq}(\epsilon_{dq}) \, dt. \tag{4}
\]

3) Key considerations

Even though the constant-speed method (using the back EMF term in (1) as main source) is commonly utilized to identify the flux linkages of SMs, the machine under test must be mechanically coupled to a speed-controlled load machine. Due to its long-lasting identification procedure, the machine temperature will increase significantly if it is not carefully monitored and regulated to a constant level. Consequently, the identified accuracy may be deteriorated. In contrast to that, these difficulties can be overcome by the introduced self-identification method at standstill, since it requires (nearly) no mechanical requirements and can be performed within a very short testing period. Furthermore, no current controllers are needed; therefore, this approach is well-suited for applications without being aware of any machine parameters.

Along with the above-mentioned advantages, few crucial points must be considered while implementing the self-identification method:

- Testing variables: The testing voltage amplitudes \( U_{test}^{dq} \) should be chosen as high as possible (close to the inverter voltage limit), such that the inaccuracies caused by incorrect stator resistance, inverter nonlinearities and measurement errors can be mitigated. Due to different differential inductances for different machines, not only \( U_{test}^{dq} \) but also the current limits \( I_{max}^{dq} \) must be set adaptively online to increase the measurement range.
- Acquisition window: Sufficient measurement samples should be acquired during tests. However, a huge drift due to the integration in (2) and (4) must be avoided. Therefore, selecting an adequate length of the acquisition window is of uttermost importance. Otherwise, an effective method to counteract the integration problem must be considered as well (e.g. by cascaded low-pass filters [10]).
- Rotor movement: For the \( q \)-axis and cross-coupling tests, the rotor may rotate easily due to rotor misalignment and nonzero torques, which result in an inaccurate flux linkages and limited measurement range. Therefore, the rotor position, required for the Park transformation, should be updated by e.g. an encoder in standard applications or encoderless position detection algorithms (future work).

III. FLUX LINKAGE PROTOTYPE FUNCTION

In this section, the analytical flux linkage prototype functions of RSMs are presented. The prototype functions are physically motivated and designed to mimic the magnetic characteristics of real nonlinear RSMs, including the self-axis and cross-coupling saturation effects. Afterwards, the fitting procedure is briefly discussed in order to obtain an optimal parametrization of the prototype functions.

A. Analytical Prototype Functions

The RSM flux linkages can be approximated by the following form of analytical prototype functions [13]

\[
\begin{align*}
\hat{\psi}_{s,d}^{d}(i_s^d, i_q^d) &= \hat{\psi}_{s,d}^{self}(i_s^d) - \hat{\psi}_{s,d}^{cross}(i_s^d, i_q^d), \\
\hat{\psi}_{s,q}^{q}(i_s^q, i_q^q) &= \hat{\psi}_{s,q}^{self}(i_s^q) - \hat{\psi}_{s,q}^{cross}(i_s^q, i_q^q),
\end{align*}
\]

where \( \hat{\psi}_{s,d}^{self}(i_s^d) \) and \( \hat{\psi}_{s,q}^{self}(i_s^q) \) and \( \hat{\psi}_{s,d}^{cross}(i_s^d, i_q^q) \) & \( \hat{\psi}_{s,q}^{cross}(i_s^q, i_q^q) \) represent self-axis and cross-coupling saturation terms, respectively. The overall approximated flux linkages \( \hat{\psi}_{s}^{dq} := (\hat{\psi}_{s,d}^{dq})^{T} \) are obtained by subtracting the cross-coupling saturation terms from the self-axis saturation terms. The cross-coupling saturation terms can be expressed in a general form as follows

\[
\begin{align*}
\hat{\psi}_{s,d}^{cross} & (i_s^d, i_q^q) = \sum_{i=1}^{n} k_i F_i(i_s^q) G_i(i_q^q), \\
\hat{\psi}_{s,q}^{cross} & (i_s^d, i_q^q) = \sum_{i=1}^{n} k_i F_i(i_s^d) G_i(i_q^q),
\end{align*}
\]

where \( k_i \) is a cross-coupling constant, \( F_i(i_s^q) \) & \( G_i(i_q^q) \) describe the cross-coupling effects and \( F_i(i_s^d) \) & \( G_i(i_q^q) \) control the impact of the cross-coupling effect for different current levels of the prototype function; \( F_i(i_s^q) := \frac{d}{di_s^q} F_i(i_s^q) \) and \( G_i(i_q^q) := \frac{d}{di_q^q} G_i(i_q^q) \) denote the respective derivatives of the functions \( F_i(i_s^q) \) and \( G_i(i_q^q) \). The number \( n \) of cross-coupling terms in (6) can be chosen arbitrarily according to given accuracy requirements. Normally, \( n = 2 \) or \( n = 3 \) can achieve satisfactory fitting accuracies [13].

To describe the self-axis saturation effects of the flux linkages, a combination of a hyperbolic function and a straight line can represent well the smooth transition between the non-saturated and saturated regions. Therefore, the \( d \) and \( q \)-axis self-axis saturation terms in (5) for zero cross-coupling current are defined as follows [13]

\[
\begin{align*}
\hat{\psi}_{s,d}^{self} & (i_s^d) = a_{d1} \tanh (a_{d2} i_s^d) + a_{d3} i_s^d, \\
\hat{\psi}_{s,q}^{self} & (i_s^q) = a_{q1} \tanh (a_{q2} i_s^q) + a_{q3} i_s^q,
\end{align*}
\]

which will be denoted as \( \hat{\psi}_{s,d}^{self}(i_s^d, i_q^q) = 0 \) and \( \hat{\psi}_{s,q}^{self}(i_s^d, i_q^q) = 0 \), respectively. Both \( \hat{\psi}_{s,d}^{self}(i_s^d) \) and \( \hat{\psi}_{s,q}^{self}(i_s^q) \) are dependent only on their self-axis currents \( i_s^d \) and \( i_s^q \) with separate function parameters \( a_{d1}, a_{d2}, a_{d3}, a_{q1}, a_{q2}, a_{q3}, \) respectively.

For the cross-coupling saturation effects, the flux linkages decreases gradually due to the increase of the cross-coupling currents. To describe this saturation trend, Gaussian functions with bell-shaped and symmetric curves are good candidates. Hence, a modified Gaussian function, which is negative and shifted, is designed. For \( n = 3 \) (to achieve a better fitting accuracy), the resulting cross-coupling saturation terms \( F_i(i_s^d), F_i(i_s^q), F_i(i_s^d) \) and \( G_i(i_s^q) \), \( G_i(i_s^d) \), which describe how \( i_s^d \) affects \( \hat{\psi}_{s}^{dq} \) and how \( i_q^q \) affects \( \hat{\psi}_{s}^{dq} \), respectively, are composed of
three modified Gaussian functions as [13]

\[
F_1(i_d^q) = 1 - e^{-a_{d4}i_d^q}, \quad F_2(i_q^d) = 1 - e^{-a_{d5}i_q^d}, \quad F_3(i_q^q) = 1 - e^{-a_{d6}i_q^q}, \\
G_1(i_d^q) = 1 - e^{-a_{q4}i_d^q}, \quad G_2(i_q^d) = 1 - e^{-a_{q5}i_q^d}, \quad G_3(i_q^q) = 1 - e^{-a_{q6}i_q^q},
\]

where \(a_{d4}, a_{d5}, a_{d6}\) and \(a_{q4}, a_{q5}, a_{q6}\) are shared parameters which affect the widths of the corresponding Gaussian functions. The derivatives of (8) can easily be computed and derived as follows

\[
F_1'(i_d^q) = 2a_{d4}^2i_d^q e^{-a_{d4}i_d^q}, \quad F_2'(i_q^d) = 2a_{d5}^2i_q^d e^{-a_{d5}i_q^d}, \quad F_3'(i_q^q) = 2a_{d6}^2i_q^q e^{-a_{d6}i_q^q}, \\
G_1'(i_d^q) = 2a_{q4}^2i_d^q e^{-a_{q4}i_d^q}, \quad G_2'(i_q^d) = 2a_{q5}^2i_q^d e^{-a_{q5}i_q^d}, \quad G_3'(i_q^q) = 2a_{q6}^2i_q^q e^{-a_{q6}i_q^q},
\]

which modulate the extent of the cross-saturation effects for different self-axis current levels. Finally, all the required functions for the entire RSM flux linkage prototype functions, as in (5), are found.

B. Fitting

With the presented flux linkage prototype functions (5), an effective fitting is needed to find the optimal set of function parameters. As fitting references, the real flux linkages of RSMs must be obtained by finite element analysis (FEA) or experimental measurements. With respect to the previously introduced self-identification method, the identified samples from the self-axis and cross-coupling tests correspond to the designed self-axis and cross-saturation terms in the flux linkage prototype functions.

To find the optimal function parameters effectively and efficiently, a fitting procedure is proposed by performing the fitting in a step-by-step manner, i.e. finding of (i) the separate parameters \(a_{d4}, a_{d5}, a_{d6}\) and \(a_{q4}, a_{q5}, a_{q6}\) in the self-axis saturation terms \(\psi_{d\text{self}}^d\) and \(\psi_{q\text{self}}^q\) (7), respectively; (ii) the shared parameters \(a_{d1}, a_{d2}, a_{d3}, a_{q1}, a_{q2}, a_{q3}\) in the cross-coupling terms \(\psi_{d\text{cross}}^q\) and \(\psi_{q\text{cross}}^q\) [recall (6), (8) and (9)]; and (iii) all parameters \(\alpha_{d4q} = (a_{d1}, \ldots, a_{d6}, a_{q1}, \ldots, a_{q6})^T\) of the overall flux linkage prototype functions \(\tilde{\psi}_d^d\) and \(\tilde{\psi}_q^q\) as in (5). The individually fitted parameters from step (i) and (ii) should be used as initial guess for step (iii) in order to find the optimal parameter set \(\alpha_{d4q}\) within a few fitting iterations and with the best fitting accuracy. Here, \(\alpha_{d4q}\) contains in total 15 parameters [for chosen \(n = 3\) in (6)]. This number may decrease or increase according to the selection of \(n\) for the cross-saturation terms.

IV. EXPERIMENTAL VALIDATION

This section demonstrates the effectiveness of the proposed self-identification method using flux linkage prototype functions for RSMs. First, the self-identification method at standstill is applied to identify the nonlinear flux linkages of a commercial 4.0 kW RSM from ABB. Second, the extracted flux linkages are employed as references to fit the flux linkage prototype functions (5) by obtaining the optimal parameter set \(\alpha_{d4q}\). Eventually, the approximated flux linkages by using the prototype functions can represent the magnetic saturation effects accurately and make LUTs obsolete.

A. Measurement Results (see Fig. 1 & Fig. 2)

Key parameters of laboratory setup and implementation are collected in Table I. At the laboratory setup, the tested RSM is controlled by a dSPACE real-time system through a voltage source inverter and coupled to a permanent magnet synchronous machine (PMSM) (free-wheeling, not controlled). Although the mechanical coupling is not necessary for the applied approach, the resulting load increases the inertia in the mechanical subsystem, which can limit the extent of rotor movement during test. Based on the introduced self-identification method at standstill (see Section II-B), in total three experimental tests, including self-axis \((d\text{ and }q\text{-axes})\) and cross-coupling tests, must be implemented. For simplicity, the test variables in the voltage reference generation function (3), i.e. testing voltage amplitude \(U_{\text{test}}\) and current limit \(I_{\text{max}}\), are selected identically for all tests.

For the self-axis tests, the \(d\) and \(q\)-axis testing procedures and the voltage testing sequences are demonstrated in Fig. 1(a) and 1(b), respectively. Both figures display the respective testing signals and responses separately including: stator voltages \(u_d^d\) & \(u_q^d\), stator currents \(i_d^d\) & \(i_q^d\), flux linkages \(\psi_d^d\) & \(\psi_q^d\) [computed online using (2)] and mechanical angular velocity \(\omega_m\). As illustrated, acquisition windows last two voltage pulse cycles for both tests, which will normally give a sufficient number of samples for the fitting of the prototype functions in the following step. Besides, it is beneficial to select an integer for the number of voltage pulse, such that the offset, due to the (unknown) initial value of the integration,
can be eliminated. The \( q \)-axis test is already completed after 17.4 ms (140 samples) in contrast to the \( d \)-axis test with 66.9 ms (536 samples). This noticeable difference is due to the smaller \( q \)-axis differential inductance \( L_{qs}^q \) over the (almost) entire current range compared to the \( d \)-axis differential inductance \( L_{ds}^d \). Due to this characteristic of RSMs, an adequate length of the acquisition window with a sufficient number of samples must be ensured (e.g. two voltage pulses are sufficient here). By extracting the resulting current responses and the calculated flux linkages from Fig. 1(a) and 1(b), the desired nonlinear current-to-flux relationship can be extracted. As a consequence, the self-saturated flux linkages \( \psi_{ds}^d \) and \( \psi_{ds}^q \) with zero cross-axis currents are shown in Fig. 3(a) and 3(b) [•], respectively.

For the cross-axis test, the testing procedure is illustrated in Fig. 2. The shown signals/quantities are identical to those shown in Fig. 1. Additionally, \( \psi_{qs}^d \) and \( \psi_{qs}^q \) with and without consideration of the back EMF are depicted. A longer acquisition window with eight \( d \) voltage pulse cycles (i.e. 267.9 ms with 2144 samples) is chosen intentionally in order to acquire more samples covering a wide measurement range. During the test, voltage sequences as proposed in (3) are applied to both \( d \) and \( q \) axis at the same time. The pulsating torque is a result of the nonzero \( i_{ds}^d \) and \( i_{qs}^q \). Consequently, the mechanical speed \( \omega_m \) in Fig. 2 differs from standstill to a much greater extent in comparison to the self-axis tests in Fig. 1. This undesired rotor movement induces the speed-dependent back EMF, which is not included in the flux linkage derivation based on (2). As a result of this incorrect voltages, in particular, a drift in \( \psi_{qs}^q \) can be observed if (2) without considering the back EMF term is used; whereas, for \( \psi_{qs}^d \), the drifting problem is not so prominent.

In contrast to these rather bad estimation results, it can be seen that the compensated \( \psi_{qs}^q \) using (4) with the consideration of the back EMF term allows to suppress the integration drift very effectively and almost completely. As a result of the cross-coupling test in Fig. 2, the (compensated) cross-saturated flux linkages \( \psi_{qs}^q \) and \( \psi_{qs}^d \) are shown in Fig. 3(a) and 3(b) [see •], respectively.

Finally, the entire testing procedure of the self-identification method completes within only a fraction of a second (352.2 ms). The experimentally identified flux...
linkages from the self-axis and cross-coupling tests (see [●] and [▪] in Fig. 3) cover wide areas of the overall operation range and represent well the self-axis and cross-coupling saturation effects of the considered RSM. Hence, its effectiveness is confirmed. Nevertheless, the extracted flux linkages with hysteresis effects scatter in the given measurement range without allowing to represent the whole flux linkage maps or at specific points (e.g. not covered by [●] and [▪] points). Due to the scattered nature, saving of LUTs on a uniform grid or deriving the differential inductances is not possible and, hence, the compensation of the magnetic nonlinearity in real-time applications is not feasible yet. Therefore, post-processing tools must be utilized, e.g. by interpolation, neural network (as in [10]) or the proposed analytical prototype function (which will be demonstrated in the next subsection).

B. Fitting Results (see Fig. 3)

After the experimental tests by means of the self-identification method, the obtained data points of the flux linkages of the considered RSM (as shown in Fig. 3 [●] & [▪]) can be used as reference data points to fit the flux linkage prototype functions as in (5). The optimal function parameter set $a_{d0}$ is found through the described fitting procedure above. In Fig. 3(a) and 3(b), the fitted $d$-axis flux linkage $\psi^d_s$ and $q$-axis flux linkage $\psi^q_s$ are shown as colored surfaces, respectively. Both fitted flux linkage maps are (almost) identical to the dotted reference samples. They are continuously differentiable. Due to the generic nature of the prototype functions, an additional interpolation is not needed for the scattered samples obtained from the self-identification. A significant reduction of the memory requirement can be achieved. In contrast to several hundreds of data points necessary for the
commonly used LUTs, only 15 parameters in \(\alpha_{dq}\) allow to achieve a very accurate approximation of the magnetic saturation and cross-coupling effects of RSMs. Moreover, due to the continuous differentiability, the differential inductances \(\hat{L}_{dq}\) can easily be computed by analytical differentiation of the flux linkage prototype functions as in (5), i.e.

\[
\hat{L}_{dq} = \frac{\partial b_{\psi_{dq}}}{\partial b_{i_{ds}}}.
\]

In conclusion, the complete presentation of nonlinear flux linkages is successfully accomplished by applying the proposed flux linkage prototype functions in combination with the proposed self-identification method. Due to the physically motivated and analytical prototype functions, the natural inter-/extrapolation capability overcomes the addressed issues of scattered data and/or LUTs. Together with other advantages (e.g. memory and differentiability), this property further highlights the interesting potential and benefits of utilizing flux linkage prototype functions in real-time applications such as flux-identification (self-commissioning) and control algorithms (e.g. nonlinear current control strategies [14] and optimal feedforward torque control (OFTC) [16]).

V. Conclusion

In this paper, a self-identification method for RSMs in combination with analytical flux linkage prototype functions has been developed. The self-identification features significant time-savings, simple implementation and standard experimental setup (no torque sensor, nor prime mover required). Applying bipolar voltage pulses to the tested machine at standstill, the nonlinear flux linkages, including the self-axis and cross-coupling saturation effects, can be identified. Instead of directly saving the obtained data in LUTs, the extracted samples are used to fit the few parameters of the analytical flux linkage prototype functions. The effectiveness of the developed self-identification method was validated by identifying a real nonlinear RSM. The entire testing procedure is extremely fast and is completed within a fraction of a second. The fitted flux linkage prototype functions give not only a very accurate approximation but also provide an intrinsic inter-/extrapolation capability (due to the physically motivated structure of the prototype functions).

For the very first time, the successful combination of self-identification (self-commissioning) and flux linkage prototype functions was demonstrated. The fitting of the prototype functions can easily be performed with standard post-processing tools based on few (scattered) data points. As a consequence, the full approximation of nonlinear flux linkages, including magnetic saturation and cross-coupling effects, can be achieved and even applied to compensate for the magnetic nonlinearity in control and operation management approaches (future work).

References


