Nonuniform Transmission Line Model for Electromagnetic Radiation in Free Space

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Abstract

An equivalent nonuniform transmission line model for electromagnetic radiation in free space is developed. By properly defining a voltage and a current associated with the transverse component of the mode fields, a kind of Telegraphers’ equation is derived for each spherical harmonic mode. Based on the equivalent distribution inductance and capacitance, the local characteristic impedance and phase velocity are derived. For each spherical mode, a cutoff spherical surface and an associated cutoff radius are introduced to separate the space into an evanescent region and a propagating region. A spherical mode field will decay approximately exponentially in the evanescent region and experience local reflection in the propagating region. The proposed model may provide an intuitive illustration for the radiation process in free space.

This work is also part of my efforts in revisiting the issue of electromagnetic radiation and mutual coupling. Although the concepts I proposed, like the macroscopic Schott energy and this NTL model, may look strange and different from the conventional formulations, I believe they can provide intuitive and insightful interpretation to the issue of EM radiation.
Abstract—An equivalent nonuniform transmission line model for electromagnetic radiation in free space is developed. By properly defining a voltage and a current associated with the transverse component of the mode fields, a kind of Telegraphers’ equation is derived for each spherical harmonic mode. Based on the equivalent distribution inductance and capacitance, the local characteristic impedance and phase velocity are derived. For each spherical mode, a cutoff spherical surface and an associated cutoff radius are introduced to separate the space into an evanescent region and a propagating region. A spherical mode field will decay approximately exponentially in the evanescent region and experience local reflection in the propagating region. The proposed model may provide an intuitive illustration for the radiation process in free space.

Index Terms—Electromagnetic radiation, spherical harmonic mode, nonuniform transmission line, characteristic impedance

I. INTRODUCTION

The electromagnetic radiation problem has been extensively investigated over the past century. Except several issues concerning with the electromagnetic powers and energies [1]-[3], the theoretic aspects of the electromagnetic radiation of antennas are well understood by researchers and engineers. In free space, the radiation fields of a current source distribution in a bounded region can be directly calculated with integrals involving the dyadic Green’s function. These integrals generally have to be performed over the source region with some kind of numerical methods. However, in many practical applications, such as designing antennas with better directivity, higher efficiency or wider bandwidth, it may require more insightful information and interpretation of the radiation process than those the numerical solutions can provide.

Consider a source distribution \( \mathbf{J}(\mathbf{r}) \) in a bounded region \( V_s \).

In a spherical coordinate system shown in Fig.1(a), the vector spherical basis functions form a complete bases for the functional space of the radiation fields of the source [4]-[8]. Let \( S_0 \) with a radius of \( r_0 \) denote the smallest sphere containing the source region. The electric and magnetic fields outside \( S_0 \) can be expanded with \( TM_{nm} \) mode and \( TE_{nm} \) mode,

\[
\begin{align*}
\mathbf{E}(\mathbf{r}) &= \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \left[ E_{n}^{TE}(\mathbf{r}) + E_{n}^{TM}(\mathbf{r}) \right] \\
\mathbf{H}(\mathbf{r}) &= \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \left[ H_{n}^{TE}(\mathbf{r}) + H_{n}^{TM}(\mathbf{r}) \right]
\end{align*}
\]

where \( n \) is the degree and \( m \) the order of the mode, respectively.

The electromagnetic fields of the spherical mode can be explicitly expressed with a set of vector eigenfunctions. As can be found in many literatures [4]-[8], the fields of the spherical modes \( TM_{nm} \) can be expressed as

\[
E_{n}^{TM}(\mathbf{r}) = b_{n}^{n} k n (n+1) \frac{h_{n}(u)}{u} Y_{nm}(\theta, \phi) \hat{r}
\]

\[
+ b_{n}^{n} \sqrt{n(n+1)} \left[ u h_{n}(u) \right] \pi_{nm}(\theta, \phi)
\]

\[
H_{n}^{TM}(\mathbf{r}) = b_{n}^{n} \frac{j k n(n+1)}{u} h_{n}(u) \psi_{nm}(\theta, \phi)
\]

and for \( TE_{nm} \) spherical modes, we have

\[
E_{n}^{TE}(\mathbf{r}) = a_{n}^{n} k \sqrt{n(n+1)} h_{n}(u) \psi_{nm}(\theta, \phi)
\]

\[
H_{n}^{TE}(\mathbf{r}) = a_{n}^{n} \frac{j k n(n+1)}{u} \frac{n h_{n}(u)}{u} \pi_{nm}(\theta, \phi)
\]

The coefficients \( a_{n}^{n} \) and \( b_{n}^{n} \) in (2) - (5) are determined by the sources in \( V_s \in S_0 \). \( k \) is the wavenumber and \( u \) is the intrinsic impedance in free space. \( u \mathbf{r} \) represents the spatial phase shift. \( h_{n}(u) \) is the spherical Hankel function of the second kind. We use the notation \( f'(u) \) for \( df(u)/du \) throughout this paper. The spherical coordinate \( r, \theta \), and \( \phi \) are defined in their usual way, and their unit vectors are respectively denoted by \( \hat{r}, \hat{\theta}, \) and \( \hat{\phi} \). The normalized vector harmonics are expressed as

Fig.1 The electromagnetic radiation. (a) a source in a bounded region. (b) antenna with symmetrical structure.
\[
\Psi_n^m(\theta, \phi) = \frac{1}{\sqrt{n(n+1)}} \left[ j^m \frac{Y_n^m(\theta, \phi)}{\sin \theta} - \frac{dY_n^m(\theta, \phi)}{d\theta} \phi \right] \\
\pi_n^m(\theta, \phi) = \frac{1}{\sqrt{n(n+1)}} \left[ dY_n^m(\theta, \phi) \cdot \sin \theta + j^{m} \frac{Y_n^m(\theta, \phi)}{\sin \theta} \phi \right] 
\]
where \( Y_n^m(\theta, \phi) \) is the harmonic function on the unit sphere, and \( P_n^m(\cos \theta) \) is the associated Legendre function. The normalized spherical harmonics form a complete set of orthogonal basis functions,

\[
\int_0^\pi \int_0^{2\pi} \Psi_n^m(\theta, \phi) \cdot \Psi_{n'}^{m'}(\theta, \phi) d\Omega = \delta_{nm} \delta_{m'm'} \\
\int_0^\pi \int_0^{2\pi} \pi_n^m(\theta, \phi) \cdot \pi_{n'}^{m'}(\theta, \phi) d\Omega = \delta_{nm} \delta_{m'm'}.
\] (7)

where the subscript \( d \Omega \) denotes \( \int_0^\pi \int_0^{2\pi} \sin \theta d\theta d\phi \).

At region far away from the source, \( kr \gg 1 \). Making use of the asymptotic expansions of the spherical Hankel functions for large argument, the far fields of the spherical modes can be approximately considered as spherical TEM waves with radiation patterns described by the corresponding spherical harmonic functions. However, the behavior of the near zone fields has not been revealed so intuitively. By numerically evaluating the amplitudes or some other quantities related to spherical Hankel functions [9]-[12], the fields of the mode with large argument, the far fields of the spherical modes can be the asymptotic expansions of the spherical Hankel functions for mode-dependent constant \( \delta_n \).

According to (2) and (3), we can write

\[
V_n = j\sqrt{\eta a_n} \left[ u_0 h_u(u_0) \right] \\
I_n = \sqrt{\eta a_n} u_0 h_u(u_0) 
\] (8)
where \( a_n \) is a mode-dependent constant and \( u_0 = kr_0 \). The normalized input impedance is then expressed using the ratio of the voltage and current,

\[
z_n = \frac{V_n}{I_n} = \frac{\int u_0 h_u(u) \, d\tau}{u_0 h_u(u_0)} 
\]
(9)

The TE\(_{\theta_0}\) modes were addressed in a similar way in [13]. The radiation power, energies and Q factors of the mode are evaluated accordingly. Furthermore, the normalized impedance of TM\(_{\theta_0}\) or the normalized admittance of TE\(_{\theta_0}\) is expanded with a continuous fraction that can help us to establish an equivalent circuit consisting of cascaded inductances, capacitances and a termination of a unitary resistance. All the elements in the equivalent circuit are normalized with the intrinsic impedance of free space. This circuit model is very popular and has been widely used to predict the minimum radiation Q-factor of an antenna bounded in a small sphere. However, it mainly describes the characteristics on the input port \( S_0 \) and does not provide intuitive information about the radiation process from \( S_0 \) to \( S_\infty \).

In this paper, a nonuniform transmission line (NTL) model for the radiation process from \( S_0 \) to \( S_\infty \) is developed, in which \( S_0 \) serves as the input port and \( S_\infty \) the matched terminal port. The equivalent voltage and current at any concentric sphere with a radius \( r \) \( (r_0 \leq r < \infty) \) are properly defined. The governing differential equation is obtained and found to be similar to the conventional Telegraphers’ equation [14] except that the equivalent distribution inductance and capacitance are dependent on \( kr \). It can be interpreted as a kind of nonuniform transmission line or a nonuniform waveguide. The generalized input impedance at any position on the NTL can be obtained, and the normalized impedance defined by Chu is simply the special case at the input port \( S_0 \), where \( r = r_0 \). It will be shown that the NTL model can provide more insightful interpretation for the radiation process from \( S_0 \) to \( S_\infty \) than conventional theories.

II. NONUNIFORM TRANSMISSION LINE MODEL

In order to develop the NTL model for the radiation problem, we will multiply the mode fields with \( r \) and find the relationship between the transverse electric field and the rotated transverse magnetic field. Let’s consider TM\(_{\theta_\infty}\) mode at first. According to (2) and (3), we can write

\[
\mathbf{r} E_{\theta_\infty}^{TM}(\mathbf{r}) = b_n^{TM} \sqrt{n(n+1)} [u_0 h_u(u)] \pi_n^m(\theta, \phi) \\
\mathbf{r} \times \mathbf{H}_{\theta_\infty}^{TM}(\mathbf{r}) = -b_n^{TM} \sqrt{n(n+1)} \frac{u_0}{\sqrt{\eta}} h_u(u) \pi_n^m(\theta, \phi) 
\]
(10)(11)
in which \( \mathbf{r} \times \Psi_n^m(\theta, \phi) = \pi_n^m(\theta, \phi) \) has been applied. The subscript \( \tau \) represents transverse component. Note that both \( r \) and \( u = kr \) are used in the derivation for the sake of brevity. Taking the derivative of both side of (10) with respect to \( u \), and making use of the relationship [9] of

\[
d^2 [u_0 h_u(u)] = \left[ u - \frac{n(n+1)}{u} \right] h_u(u) 
\]
(12)
we get
\[ \frac{\partial}{\partial u} \left[ r_{{E}_{TM}}(r) \right] = -\frac{ju}{u} \left[ 1 - \frac{n(n+1)}{u^2} \right] \left[ -r \times r_{{H}_{TM}}(r) \right] \] (13)

Taking the derivative of both side of (11) with respect to \( u \), we obtain
\[ \frac{\partial}{\partial u} \left[ -r \times r_{{H}_{TM}}(r) \right] = -\frac{j}{\eta} \left[ r_{{E}_{TM}}(r) \right] \] (14)

Define the equivalent voltage and current on a concentric sphere with radius \( r \) as
\[ V_{TM}^n(u) = b_n^u \sqrt{n(n+1)} \left[ u h_n(u) \right] \] (15)
\[ I_{TM}^n(u) = -b_n^u \sqrt{n(n+1)} u h_n(u) \] (16)

Then the electromagnetic fields can be expressed by
\[ r_{E_{TM}}(r) = V_{TM}^n(u) \pi_u^n(\theta, \phi) \] (17)
\[ -r \times r_{H_{TM}}(r) = I_{TM}^n(u) \pi_u^n(\theta, \phi) \] (18)

It can be verified that the radiation power crossing the sphere can be correctly calculated with the voltage and current,\[ P_{TM}^n(u) = \text{Re} \int \frac{1}{2} \left[ \frac{1}{2} r_{E_{TM}}^2 + r_{H_{TM}}^2 \right] \cdot \mathbf{r} dS \]
\[ = \text{Re} \int \left[ \frac{1}{2} V_{TM}^2 + I_{TM}^2 \right] \frac{n(n+1)}{2\eta} \] (19)

Substituting (17) and (18) into (13) and (14) gives the governing differential equation for the \( TM_{mn} \) mode,
\[ \frac{\partial V_{TM}^n(u)}{\partial u} = \eta \left[ 1 - \frac{n(n+1)}{u^2} \right] I_{TM}^n(u) \] (20)
\[ \frac{\partial I_{TM}^n(u)}{\partial u} = \frac{1}{\eta} V_{TM}^n(u) \]

Note that equation (20) are with respect to the phase shift \( u \) instead of the radial distance \( r \). It is valid for all \( u_0 \leq u < \infty \).

Obviously, (20) is similar to the Telegraphers’ equation that the voltage \( V(u) \) and current \( I(u) \) satisfy on a transmission line with distribution inductance \( L(u) \) and capacitance \( C(u) \) [14]-[16],
\[ \frac{\partial V(u)}{\partial u} = -j\omega L(u) I(u) \]
\[ \frac{\partial I(u)}{\partial u} = -j\omega C(u) V(u) \] (21)

Comparing (20) with (21), we can introduce an equivalent distribution inductance and capacitance for \( TM_{mn} \) mode as
\[ L_{TM}^n(u) = \eta \left[ 1 - \frac{n(n+1)}{u^2} \right] \]
\[ C_{TM}^n(u) = \frac{1}{\eta} \] (22)

At a given frequency, the capacitance for \( TM_{mn} \) is constant, while the inductance varies with \( u = kr \). Therefore, (21) describes the voltage-current relationship on an NTL. The field propagation from \( S_i \) to \( S_o \) is then transformed to the voltage and current signal transmission on an NTL from \( u_i \) to \( u_o \).

Note that the inductance and capacitance are only dependent on the degree \( n \) of the mode. All \( TM_{mn} \) modes of the same degree \( n \) share the same NTL model.

We borrow the concept in transmission line theory and define a local characteristic impedance for the NTL as
\[ Z_c(u) = \frac{\eta}{\sqrt{LC}} \left[ 1 - \frac{n(n+1)}{u^2} \right] \] (23)

Similarly, a local phase velocity is defined as,
\[ v(u) = \frac{du}{dt} = \frac{1}{\sqrt{LC}} \frac{\omega}{\sqrt{1 - n(n+1)/u^2}} \] (24)

We have to take care that the phase velocity defined in (24) is with respect to the phase shift \( u \). Since \( du/dt = k dr/dt \), the conventional phase velocity is obtained to be
\[ v_p(u = kr) = \frac{1}{k\sqrt{LC}} \frac{c}{\sqrt{1 - n(n+1)/u^2}} \] (25)

where \( c \) is the light velocity in free space.

A typical plot of the local characteristic impedance and the phase velocity of an NTL for \( TM_{mn} \) mode are shown in Fig.2.

Fig. 2 Typical characteristic impedance and phase velocity of \( TM_{mn} \) mode.

We can summarize from Fig.2 that:

i) for \( u = kr \to \infty \), the characteristic impedance approaches \( \eta \) and the phase velocity approaches the light velocity \( c \).

ii) for \( u^2 > n(n+1) \), the characteristic impedance and the phase velocity are all real. It is an ordinary transmission line. The voltage and current signals will propagate on the line. However, the line is not a uniform transmission line (UTL) but an NTL. Generally, an NTL can be approximately considered as a section-wise-uniform transmission line, i.e., cascaded by short UTL sections with different characteristic impedances. Therefore, local reflections will inevitably occur [15][16].

iii) for \( u^2 < n(n+1) \), the characteristic impedance and the phase velocity of all are imaginary, as can be seen from (22). We can treat it as a nonuniform transmission line with negative inductance and positive capacitance. For a transmission line with a constant negative inductance and a constant positive capacitance, the general solution of (21) is \( \exp(\pm ju) \), is real. The voltage and current on the line are evanescent. They will not propagate like waves but decay exponentially. This is similar to the case that has been widely analyzed in handling left-handed materials or transmission lines [17][18] when there is a negative permittivity or a negative permeability. The transmission line has similar behavior when the negative inductance and positive capacitance are position-dependent.
It is obvious that \( u^2 = n(n+1) \) represents the critical state between the propagating and the evanescent state. We can consider the space between \( S_0 \) and \( S_{\infty} \) as a two-port radial waveguide. Therefore, it is natural to introduce a cutoff spatial phase shift \( u_c \) as
\[
u_c = \sqrt{n(n+1)} \quad (26)
\]
It can be observed that \( TM_{nm} \) is a propagating mode for \( u > u_c \) and an evanescent mode for \( u < u_c \).

Since \( u = kr \), at a given frequency, we can accordingly define a cutoff radius \( r_c = u_c/k \) associated with a cutoff sphere. The mode \( TM_{nm} \) is an evanescent one inside the cutoff sphere and becomes a propagating one outside the cutoff sphere. We can then write the propagation condition as,
\[
u > u_c \text{ or } r > r_c \quad (27)
\]

On the other hand, for an arbitrarily specified concentric sphere with radius \( r \), \( TM_{nm} \) is a propagating mode outside the sphere if its degree \( n \) satisfies \( n(n+1) < (kr)^2 \). Otherwise, it remains to be an evanescent mode till it reaches a larger sphere that meets the propagation condition. Therefore, for a sphere with radius \( r \), we can define a cutoff mode degree \( n_c \) for \( TM_{nm} \) mode, which is the largest integer satisfies,
\[
n_c (n_c + 1) = (kr)^2 \quad (28)
\]
Solving (28) gives
\[
n_c = 0.5\left(\sqrt{1+4(kr)^2} - 1\right) \quad (29)
\]
We may take the largest integer smaller than the right-hand side of (29). The \( TM_{nm} \) mode becomes a propagating mode outside the sphere if \( n \leq n_c \). Note that for large \( kr \), \( n_c \approx [kr-0.5] \).

Based on these observations, we can develop the equivalent NTL model for \( TM_{nm} \) as shown in Fig.3. In general situations, the NTL consists of two parts divided by the cutoff phase shift \( u_c \). The end port is perfectly matched with \( \eta \). Intuitively, the mode fields enter the input port \( S_0 \), travel through an evanescent zone from \( u_0 \) to \( u_c \), pass the cutoff line, then enter the propagating zone, and propagate from \( u_c \) to \( u_{\infty} \). In the evanescent zone, the fields will decay approximately exponentially. In the propagating zone, the fields may also decrease much faster than \( \mu/r \) at the region near to \( u_c \) because of strong local reflections. Only when \( u \) is much larger than \( u_c \), the fields begin to propagate like a spherical wave with amplitude decreasing approximately in the order of \( 1/r \).

The parameters of \( TM_{nm} \) mode with \( n = 5, 15, \) and \( 25 \) are plotted in Fig.4, where the cutoff phase shift \( u_c \) are respectively 5.48, 15.49, and 25.495. It can be seen that, at a fixed input port \( S_0 \), the \( TM_{nm} \) mode with larger \( n \) may experience longer evanescent zone and suffer larger attenuation. It is required to excite larger fields at \( S_0 \) to send the same amount of radiation power to \( S_{\infty} \), in other words, it is generally less efficient to excite mode with larger degree.

In order to efficiently excite a \( TM_{nm} \) mode, we may choose \( u_c > u_n \) so that the evanescent zone and the high local reflection zone are excluded in the propagating path of that mode, intuitively speaking, we may put \( S_0 \) to the right side of the red line in Fig.3.

![Fig. 3 NTL model for \( TM_{nm} \) mode.](image)

![Fig. 4 Parameters for \( TM_{nm} \) mode. \( n = 5, 15, 25. \) (a) characteristic impedance; (b) phase velocity.](image)

The NTL model for \( TE_{nm} \) modes can be developed similarly. Starting from (4) and (5), we can write
\[
,r E_r^{TE} (r) = a_n^r j\sqrt{n(n+1)}n h_n (u) \psi_n (\theta, \phi) \quad (30)
\]
\[
-\hat{r} \times r H_z^{TE} (r) = a_n^z j\sqrt{n(n+1)}n h_n (u) \psi_n (\theta, \phi) \quad (31)
\]
in which \( \hat{r} \times \pi_n^z (\theta, \phi) = -\psi_n (\theta, \phi) \) has been applied. Define the equivalent voltage and current for \( TE_{nm} \) as
\[
V_{E_{\mu}}^{TE}(u) = a_n^r j\sqrt{n(n+1)}n h_n (u) \quad (32)
\]
\[
I_{E_{\nu}}^{TE}(u) = a_n^z j\sqrt{n(n+1)}n h_n (u) \quad (33)
\]
It can be verified that
Making use of (12), the Telegrapher’s equation for $TE_{nm}$ mode is derived to be

\[
\frac{\partial V_{TE}^n(u)}{\partial u} = -j\eta I_{TE}^n(u) \\
\frac{\partial I_{TE}^n(u)}{\partial u} = -j \left[ u^2 - n(n+1) \right] V_{TE}^n(u)
\]

(35)

Accordingly, the equivalent inductance and capacitance are

\[
\begin{align*}
L_{TE}^n(u) &= \frac{\eta}{\omega} \\
C_{TE}^n(u) &= \frac{u^2 - n(n+1)}{\omega \eta u}
\end{align*}
\]

(36)

Note that the capacitance varies with $u$ while the inductance is constant at a given frequency, which is different from that for $TM_{nm}$ mode.

The local characteristic impedance for $TE_{nm}$ mode is

\[
Z_c^e(u) = \frac{\eta}{\sqrt{1 - n(n+1)/u^2}}
\]

(37)

Fig. 5 Characteristic impedance of $TE_{nm}$ mode for $n=5$ and 15.

For a $TM_{nm}$ mode with fixed amplitude of $[I_{TM}^n(u)]$ at $S_o$, it will transfer larger radiation power with larger $R_{nm}^T(u_b)$. $R_{nm}^T(u_b)$ is actually the radiation resistance of $TM_{nm}$ mode. It can be used to predict the radiation capability of the mode. Large $R_{nm}^T(u_b)$ means stronger radiation capability.

The generalized input impedance of $TM_{nm}$ mode for $n=5, 15$, and 25 are plotted in Fig.6, where the dash-line indicate the cutoff phase shift for the corresponding mode. The input resistances in Fig.6(a) clearly reveal that the radiation of a mode is weak when $u_b < u_c$.

For $TE_{nm}$ mode, the generalized input admittance at point $u$ is defined as

\[
Y_{TE}^n(u) = \frac{I_{TE}^n(u)}{V_{TE}^n(u)} = \frac{j \eta h_b(u)}{u h_b(u)} = R_{nm}^T + jX_{nm}^T
\]

(38)

and Chu’s normalized admittance for $TE_{n0}$ is just the special case at the input port ($u = u_b$). Furthermore, (38) and (40) show that mathematically the normalized input impedance of the $TM_{nm}$ mode equals to the normalized input admittance of the $TE_{nm}$ mode, i.e.,

\[
Z_{nm}^T(u) = \frac{1}{\eta Y_{TE}^n(u)} = \frac{1}{\eta h_b(u)} = \frac{j}{u h_b(u)} = \frac{1}{2} R_{nm}^T |V_{TM}^n|^2
\]

(39)

III. DISCUSSIONS

For $TM_{nm}$ mode, the generalized input impedance at point $u$, $u_b \leq u < \infty$, is found to be

\[
Z_{nm}^T(u) = Y_{TM}^n(u) = \frac{1}{\eta h_b(u)} = R_{nm}^T + jX_{nm}^T
\]

(40)

For $TE_{nm}$ mode, the generalized input admittance at point $u$ is defined as

\[
y_{TE}^T(u) = \frac{I_{TE}^n(u)}{V_{TE}^n(u)} = \frac{j \eta h_b(u)}{u h_b(u)}
\]

(41)

which is exactly the same expression used by Chu to develop the equivalent circuit model. Therefore, if necessary, the type of Chu’s equivalent circuit model can be developed for all $TM_{nm}$ and $TE_{nm}$ mode at any spheres between $S_o$ and $S_e$. 

Fig. 6 Generalized input impedance of $TM_{nm}$ mode for $n=5, 15$ and 25. (a) real part. (b) imaginary part.
It is reasonable to take the intrinsic impedance $\eta$ as the reference impedance to define a generalized reflection coefficient at $u$, 

$$\Gamma(u) = \frac{Z_{in}(u) - \eta}{Z_{in}(u) + \eta}$$  (41)

where $Z_{in}(u)$ is the generalized input impedance of a spherical wave mode. In the evanescent regions, the reflection coefficient is very large. Most of the electromagnetic power carried by the mode will be reflected, so the radiation efficiency tends to be very low.

A Hertzian dipole generates $TM_{10}$ mode field in the free space. We can check that the equivalent voltage and current on the NTL are respectively,

$$V_{TM}^{TM}(u) = b_0 \sqrt{2} \left[ j + \frac{1}{u} - j \frac{1}{u^2} \right] e^{-ju}$$  (42)

$$I_{TM}^{TM}(u) = b_0 \sqrt{2} \frac{1}{\eta} \left( j + \frac{1}{u} \right) e^{-ju}$$  (43)

The cutoff radius of $TM_{10}$ mode is $r_c = \sqrt{2/k} = \lambda/\sqrt{2}\pi < \lambda/4$. The shadow area in Fig.8 shows the evanescent region of the $TM_{10}$ mode. The fields of the Hertzian dipole in this area will decay exponentially. As is well known, the Hertzian dipole is not an efficient radiator.

Intuitively, we have two basic methods to improve the radiation efficiency. The first one is to efficiently guide the electromagnetic power outside the cutoff region using some kind of metal structures. A typical structure is the half wavelength dipole shown in Fig.8. Its end reaches $S_0$ with $r_0 = \lambda/4$, which is apparently locating in the propagating region of the $TM_{10}$ mode. Because of the two metal arms of the dipole, spherical TEM mode can exist within $S_0$. The transmission of spherical TEM mode can be modelled using a UTL with cutoff radius of $r_c = 0$ [14]. It has the potential to efficiently transfer electromagnetic power from the feeding point to $S_0$.

The second method is to fill the region surrounding the feeding source with some kind of dielectrics. For a dielectric with relative permittivity of $\varepsilon > 1$, the cutoff radius of all spherical modes will decrease by a factor of $\sqrt{\varepsilon}$. The radiation efficiency may be improved due to the shrink of the evanescent region.

IV. CONCLUSIONS

The proposed NTL model for electromagnetic radiation of sources in a bounded region can provide an intuitive way for illustrating the propagation process of mode fields. As the NTL model is independent on the order $m$ of the spherical modes, $TM_{n0}$ and $TE_{n0}$ share the same NTL with $TM_{00}$ and $TE_{00}$, respectively.

By introducing the cutoff spatial phase shift $u_c = kr_c$, the traveling path of a spherical mode can be divided into an evanescent zone and a propagating zone. Although the amplitudes of the mode fields will decay in both regions, the mechanism and behaviors in the two regions are quite different. The NTL model does not provide information about how the feeding source distributes its electromagnetic power to each spherical mode. However, it can help us to predict which modes can effectively penetrate the barrier in the near region and propagate to far region.

REFERENCES


