Chance-Constrained Planning for Dynamically Stable Motion of Reconfigurable Vehicles

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Abstract

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Chance-Constrained Planning for Dynamically Stable Motion of Reconfigurable Vehicles

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Index Terms—Autonomous vehicles, rollover avoidance, motion planning, rough surfaces, chance-constrained planning.

I. INTRODUCTION

RECONFIGURABLE vehicles (RVs) are prevalent in many applications such as extraterrestrial exploration, open-pit mining, excavation, and timber harvesting. All of the applications mentioned above require RVs to travel in unstructured outdoor environments, typically on uneven terrains that may present significant rollover risks for the vehicle. To reduce these risks, methods to quantify the closeness to rollover have been introduced and examples include the static stability method introduced in [1], the dynamic force-angle stability measure in [2], [3], the dynamic lateral load transfer (LLT) parameter [4], [5], and the zero moment point (ZMP) dynamic stability measure introduced in [6]. Due to the nonlinear and non-convex characteristics of the stability measures, they can drastically increase the computation time of traditional motion planning formulations when employed as constraints.

To achieve short computation time for complex formulations, hierarchical planning frameworks that reduce the problem dimension by employing multiple planning stages have been demonstrated in [7] and [8]. To solve the complete motion planning problem, the hierarchical frameworks first employ a sampling-based path planning method, such as the Rapidly-exploring Random Tree (RRT) [9], for geometric path planning. Then, Nonlinear Optimal Control Problem (NOCP)-based methods, such as those illustrated in [10]–[13], can efficiently find optimal trajectories based on the geometric paths that already satisfy nonlinear and non-convex constraints.

For online rollover-free motion planning on 3-D terrain specifically, we utilized the zero moment point (ZMP) dynamic stability measure introduced in [6] as a constraint on the planning problem in our previous work [8]. However, both in our previous work and in the literature, the ZMP stability constraint has so far been treated in a deterministic way, under the assumption of having perfect knowledge of the vehicle orientation on the terrain. In reality, it is practically impossible to perfectly estimate vehicle orientations based on a terrain model due to complex vehicle-terrain interactions caused by ground composition in-homogeneity and weather events. Additionally, for autonomous vehicles that rely on cameras or LIDARs to produce local terrain maps, terrain occlusion caused by snow, leaves, and dense vegetation can cause further mapping inaccuracies. As a result, unsafe motions can occur due to terrain uncertainties when motion planning relies on accurate terrain modelling.

Although implementing a safety margin to the ZMP constraint based on the worst-case scenario to accommodate the orientation uncertainties appears tempting, it is necessary to point out, as will be seen later through our analysis, that the uncertainties can affect the stability of a vehicle differently along different directions, especially on sloped terrain. Therefore, if a safety margin is derived based on the worst-case scenario, the stability constraint can become unnecessarily conservative and, as a result, may cause the planner to fail due to infeasibility when, in fact, a feasible motion plan may exist. To avoid the conservatism caused by worst-case safety margins, the chance-constrained formulation is a natural candidate to allow calculated risk-taking [14].

A chance-constrained trajectory planning formulation that bounds the probability of ZMP constraint satisfaction is proposed in this paper to address the terrain uncertainty issue. Contrary to a safety margin, the chance-constrained formulation can capture the effect of changing vehicle orien-
tation on ZMP distribution since the distribution of the ZMPs can be explicitly modeled. The chance constraint can also be adapted to different uncertainty levels that corresponds to different types of environments and vehicle characteristics due to uncertainty level being a parameter in the formulation. Additionally, if the uncertain event is bounded by a ‘small enough’ distribution, the chance-constrained formulation can easily accommodate such a scenario since the 100% bound is well-defined.

Several recent research works [14]–[17] addressing problems of autonomous driving and flight have demonstrated the benefits of chance-constrained formulations. However, all of the aforementioned works focus on the obstacle avoidance problem which is drastically different from the rollover avoidance problem that this paper aims to address. Fundamentally, the probabilistic obstacle avoidance problem is formulated in Euclidean space and uncertainties can be added onto the location of an object. For the probabilistic rollover avoidance problem, terrain-induced orientation uncertainties reside in the 3-D rotation group, SO(3), and require special care when formulating. Additionally, the aforementioned works dealt with vehicles such as cars and drones so their vehicle models cannot accommodate RVs.

To formulate the chance-constrained dynamically stable motion planning problem, it is assumed that prior knowledge of terrain is limited to a nominal 3-D surface model with known uncertainty levels distributed throughout the map in patches. These patches can correspond to different road surface types (e.g., asphalt vs. dirt road) or, as in the application of interest here, to different forest types. This assumption emulates a realistic scenario where the terrain surface is mapped using a combination of topographic maps, satellite imagery, and visual sensors onboard the RVs [18]. The uncertainty levels of different terrain types are achieved through testing measured vehicle orientations obtained with IMUs against the estimated ones based on the nominal terrain model. Consequently, the motion planner can use the terrain knowledge to steer the RV around areas that have high likelihood of rollover or reconfigure the RV so that it is less likely to roll over when navigating through such areas.

It is worth mentioning that compared to continuously generating stability-compensating motions by using configuration redundancy, as in [19], or formulating terrain properties into an artificial potential field to avoid slopes, as in [20], [21], employing the stability measure as a constraint allows the RV to operate without having to constantly reconfigure or unnecessarily avoid steep yet safe slopes unless stability is compromised. This allows the RV to travel through any terrain in any necessary configuration to complete tasks, as long as it does not cause rollover. This is essential for applications that focus on both efficiency and safety. Compared to previous publications that aim to plan collision and rollover-free trajectories for autonomous trucks used for commercial transportation [22] and open-pit mining [23], our work focuses on the terrain uncertainty aspect vis a vis stability and it also deals with RVs that travel without the guidance of marked roads.

A partial solution to the chance-constrained rollover-free trajectory planning problem has been presented in our previous work [24] where the problem formulation does not consider how the uncertainty patterns of the ZMP change as a result of dynamic motions. The major addition in contribution of this paper compared to the previous work is the inclusion of the dynamic component in the problem formulation and the new simulation results to showcase the added component. To justify the proposed chance constraint characterization, another important addition to this paper is the inclusion of experimental terrain data to showcase the characteristics and differences in surface uncertainties between different types of terrains. To the best of the authors’ knowledge, an algorithm that handles the above issues while being computationally efficient enough for online planning does not exist.

The structure of this paper is as follows: In Section II, a brief theoretical background will be provided for the kinematic equation of a skid-steered reconfigurable ground vehicle, the ZMP stability measure, and the chance-constrained optimal trajectory planning problem. In Section III, a geometrical transformation of the chance constraint for the ZMP measure using assumptions suitable for SO(3) operations will be introduced for improved computational efficiency. In Section IV, an overview of the planning framework to address the chance-constrained optimal trajectory planning problem utilizing the geometrical transformation is given. In Section V, the performance of the proposed geometrical transformation will be shown through demonstrations using randomly generated and experimental data. Finally, the complete formulation will be put to test by solving the trajectory generation problem for a feller-buncher machine, and its computational performance will be evaluated. Section VI will conclude the paper.

II. THEORETICAL BACKGROUND

A. Kinematic Model of Reconfigurable Vehicle

The kinematic model of a RV equipped with a five-DoF manipulator-like crane, as is typical of many reconfigurable machines, can be derived by treating it as a system of links connected by various joints, with link index \( i \in \{0, 1, \cdots, 5\} \), as exemplified schematically in Figure 1. This model can be succinctly written as:

\[
\dot{x} = g(x, u),
\]

where the state \( x = [q^T, \dot{q}^T]^T \) is constructed with \( q = [p_b^T, \psi, q^T]^T \) that contains the vehicle’s position, heading angle, and crane joint angle information. The 3-D terrain function is included in (1) so the vehicle’s movement is constrained to the terrain surface. The vehicle’s 3-D orientation can also be found using its position, heading angle, and the surface function. In deriving the kinematic model, the mobile base is described by a unicycle model and the vehicle is therefore subject to the kinematic control input.
\( u = \left[u_a, u_q, u_j^T\right]^T \), where \( u_a, u_q, \) and \( u_j \) represent the vehicle’s acceleration along heading direction, its yaw angular acceleration, and crane joint accelerations, respectively.

\[ z_{mp} = \sum_{i=0}^{n} (\mathbf{p}_i^{F_0} - \mathbf{p}_i^{z_{mp}}) \times m_i(\mathbf{p}_i^{F_0} - \mathbf{g}^{F_0}) \]  

where \( n \) stands for the highest link index, \( M_{zmp}^{F_0} = [0, 0, M_z]^T \), \( \mathbf{p}_i^{z_{mp}} - \mathbf{p}_i^{F_0} = [x_{z_{mp}}, y_{z_{mp}}, 0]^T \) is the relative ZMP location, \( \mathbf{p}_i^{F_0} = [x_i, y_i, z_i]^T \) is each link’s relative center of mass coordinates, and \( \mathbf{p}_i^{z_{mp}} = [\tilde{p}_{x,i}, \tilde{p}_{y,i}, \tilde{p}_{z,i}]^T \) is each link CoM’s absolute acceleration in 3-D space. The superscript \( F_0 \) signifies a quantity expressed in the base-fixed frame \( F_0 \).

Subsequently, the relative ZMP coordinates in \( \mathbf{p}_i^{z_{mp}} \) are written as

\[
\begin{align*}
    x_{z_{mp}} &= \sum_{i=0}^{n} m_i(\tilde{p}_{z,i} - g_z)x_i - \sum_{i=0}^{n} m_i(\tilde{p}_{x,i} - g_x)z_i \\
    y_{z_{mp}} &= \sum_{i=0}^{n} m_i(\tilde{p}_{z,i} - g_z)y_i - \sum_{i=0}^{n} m_i(\tilde{p}_{y,i} - g_y)z_i
\end{align*}
\]

where \([g_x, g_y, g_z]^T\) represents the coordinates of \( g^{F_0} \). Thus, the vehicle is dynamically stable when \( \mathbf{p}_i^{z_{mp}} \in \text{Conv}(S) \), but has the tendency to roll over otherwise. More details on (1), (2), and (3) can be found in [8] but are omitted here to allow for the main focus of this paper.

\[ \Pr \left( p_{z_{mp}}(t) \in \text{Conv}(S) \right) > \rho \quad \forall \quad t \in [t_0, t_f] \]  

where \( \rho \) is defined as vector component-wise inequality, underline \( _- \) and overline \( _\bar{\ } \) stand for the lower and upper bounds on a variable, respectively. The state and input constraints ensure that the machine’s configuration, the corresponding rates, and accelerations are feasible. Due to the uncertainty in vehicle orientation affecting ZMP coordinates as expressed in (3), \( p_{z_{mp}} \) is treated as a random variable. Therefore, the inequality constraint (4d) is the chance constraint that ensures probability of the ZMP being within \( \text{Conv}(S) \) is greater than the user-specified threshold \( \rho \in [0, 1] \) at any time instant \( t \).

It is worth noting that computing the probability of constraint satisfaction, \( \Pr \left( p_{z_{mp}} \in \text{Conv}(S) \right) \), is not simple due to nonlinearity of the ZMP constraint. Additionally, a feasible solution to the NOCP (4) is not readily obtainable since (4) contains the highly nonlinear ZMP chance constraint (4d), the nonlinearity of a general terrain surface function in (1), and the nonconvex obstacle avoidance constraint \( x \in x_\text{free} \). To employ the formulation of (4) for real-time motion guidance of RVs on 3-D terrain, we solve (4) using a hierarchical motion planning approach that shares the general idea of our previous work [8] with the major addition of chance-constrained ZMP stability component.

### III. Geometric Interpretation of Chance-Constrained ZMP

To efficiently solve a chance-constrained trajectory planning problem, the probabilistic properties of the ZMP location with respect to terrain uncertainty are explored in this section. First, a formal definition of the perturbation to the vehicle orientation in SO(3), resulting from terrain uncertainty is presented. Then, we derive a linearized model representing the ZMP distribution as a function of orientation perturbation. From this linear model, we show that the perturbation to the location of ZMP is a bivariate normal distribution and establish an elliptical boundary for its cumulative probability as a function of \( \rho \). Finally, we arrive at a practical formulation of the ZMP chance constraint suitable for an efficient solution of the planning problem.
A. Vehicle orientation as a multivariate random variable

For any feasible orientation-terrain pair, there exists an orientation mapping matrix \( R \in \text{SO}(3) \) that describes the RV’s base orientation in inertial frame \( T \). Hence, any vector \( v \) whose coordinates are expressed in the base-fixed frame \( F_0 \) can be expressed in the inertial frame using the following relationship:

\[
v^T = R^T v_{F_0}.
\]

Since an accurate representation of the terrain surface is hard to achieve due to reasons mentioned in Section I, only the nominal surface of a 3-D terrain is found and described by the following function:

\[
z_n = f_s(x_s, y_s),
\]

where \( x_s, y_s, \) and \( z_s \) represent the 3-D coordinates in Cartesian space. The vehicle’s nominal orientation mapping matrix \( R \) can be found using (6) based on its coordinates \( (x_s, y_s) \) and heading angle \( \psi \).

It is worth noting that an \( \text{SO}(3) \) perturbation cannot be applied in additive form. Consider two quantities \( M, M' \in \text{SO}(3) \) such that \( M + M' \notin \text{SO}(3) \) in general. The actual perturbed orientation mapping matrix \( \tilde{R} \) is therefore expressed as

\[
\tilde{R} = \tilde{R} = R \tilde{R}
\]

where \( \tilde{R} \in \text{SO}(3) \) represents the deviation from nominal orientation as caused by terrain modeling inaccuracy.

In this work, it is assumed that the perturbation \( \tilde{R} \) is “small” and concentrated along the identity transformation and hence can be expressed as:

\[
\tilde{R} = \exp_m(\epsilon^\wedge),
\]

where the random variable \( \epsilon \in \mathbb{R}^3 \) can be described by a zero-mean multivariate normal distribution \( \sim N(0, \Sigma) \).

In Assumption 1, \( \exp_m() \) is the matrix exponential, and the \( \wedge \) operator transforms \( \epsilon \) into a member of the Lie algebra \( \text{so}(3) \):

\[
\epsilon^\wedge = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{bmatrix} = \begin{bmatrix} 0 & -\epsilon_3 & \epsilon_2 \\ \epsilon_3 & 0 & -\epsilon_1 \\ -\epsilon_2 & \epsilon_1 & 0 \end{bmatrix}.
\]

In consequence of Assumption 1, the expected value of the random perturbation is the identity matrix, that is \( E(\tilde{R}) = I_3 \).

B. Dynamic ZMP distribution characterization under terrain uncertainty

In this work, the simultaneous reconfiguration and relocation of a vehicle is not explored since the necessity of executing both motions at the same time to negotiate for stability is low. Hence, with the added assumption that the terrain is locally geometrically flat, the RV experiences zero acceleration along the base-fixed \( z_0 \) direction. The ZMP (3) can therefore be approximated as:

\[
x_{zmp} = \sum_i m_i g_x x_i - \sum_i m_i g_z z_i + \sum_i m_i \tilde{p}_{x,i} z_i
\]

\[
y_{zmp} = \sum_i m_i g_y y_i - \sum_i m_i g_z z_i + \sum_i m_i \tilde{p}_{y,i} z_i
\]

(10)

Let us introduce \( [x_c, y_c, z_c]^T = \bar{p}_{c} - p_{c,0} = \sum_i m_i (p_i - p_{i,0}) \) as the center of mass coordinates of the RV in \( F_0 \). Since no reconfiguration is needed during relocation, by approximating the vehicle as a single point mass, the simplification \( \sum_i m_i \tilde{p}_{x,i} z_i = \frac{\tilde{p}_{x,c}}{g} z_c \) and \( \sum_i m_i \tilde{p}_{y,i} z_i = \frac{\tilde{p}_{y,c}}{g} z_c \) can be employed to modify (10) as

\[
x_{zmp} = x_c - \frac{g_x}{g} z_c + \frac{\tilde{p}_{x,c}}{g} z_c
\]

\[
y_{zmp} = y_c - \frac{g_y}{g} z_c + \frac{\tilde{p}_{y,c}}{g} z_c
\]

(11)

which can be re-arranged into the following form:

\[
\begin{bmatrix}
x_{zmp} \\
y_{zmp}
\end{bmatrix} = \begin{bmatrix}
x_c - \frac{r_{13}(\epsilon)}{r_{33}(\epsilon)} z_c \\
y_c - \frac{r_{23}(\epsilon)}{r_{33}(\epsilon)} z_c
\end{bmatrix} + \begin{bmatrix}
z_c \\
\frac{r_{33}(\epsilon) g_z}{r_{33}(\epsilon)}
\end{bmatrix} \begin{bmatrix}
\frac{\tilde{p}_{x,c}(\epsilon)}{g} \\
\frac{\tilde{p}_{y,c}(\epsilon)}{g}
\end{bmatrix},
\]

(12)

where \( [r_{13}, r_{23}, r_{33}]^T \) is the 3-rd column of the actual \( R \) defined in (7) which contains information on the modeled terrain noise \( \epsilon \), and \( g \) is the gravitational constant. The accelerations \( \tilde{p}_{x,c} \) and \( \tilde{p}_{y,c} \) are functions of the control input \( u \). The division in (12) is valid as long as \( r_{33} \neq 0 \), which holds true for ground vehicles.

The nonlinearity of (12) with respect to elements of \( R \) and hence \( \epsilon \) brings in added difficulties to finding a geometrical bound for the chance constraint. Linearization can provide an approximation of the probability distribution of the ZMP. Accordingly, the Jacobian matrix of (12) with respect to \( \epsilon \),

\[
I_f = \begin{bmatrix}
\frac{\partial f_x}{\partial \epsilon_1} & \frac{\partial f_x}{\partial \epsilon_2} & \frac{\partial f_x}{\partial \epsilon_3} \\
\frac{\partial f_y}{\partial \epsilon_1} & \frac{\partial f_y}{\partial \epsilon_2} & \frac{\partial f_y}{\partial \epsilon_3}
\end{bmatrix}
\]

(13)

is found. The linearized ZMP coordinates, denoted with a \( \hat{\epsilon} \) symbol to differentiate from the actual ZMP, as per the linearization embodied in \( I_f \) above can then be determined from:

\[
\hat{p}_{zmp} = I_f(u) \epsilon + \hat{p}_{zmp}^0,
\]

(14)

where \( \hat{p}_{zmp}^0 \) is the ZMP coordinates found based on the nominal RV orientation \( \tilde{R} \).

The Jacobian (13) consists of two components: the static component \( J_{fs} \), whose elements only vary with the nominal attitude; and the dynamic component \( J_{fd} \), whose elements vary also with the accelerations experienced by the RV’s center of mass.
Defining \( \tilde{p} = \rho \tilde{p}_{0} \), with the linearized \( \rho \tilde{p}_{0} \), as per (14) being a linear function of the multivariate normally distributed \( \epsilon \), it is easy to see that \( \tilde{p} \sim \mathcal{N}(0, \Sigma_{f}) \), where \( \Sigma_{f} = J_{f}(u)\Sigma_{f}(u)^{T} \), is a bivariate normal distribution whose density function can be written as:

\[
p_{p}(\tilde{x}_{\text{zmp}}, \tilde{y}_{\text{zmp}}) = \frac{1}{\sqrt{(2\pi)^2 |\Sigma_{f}|}} \exp\left(-\frac{1}{2} \tilde{p}^{T} \Sigma_{f}^{-1} \tilde{p}\right).
\]

The chance constraint (4d) can therefore be expressed as:

\[
\int_{S} p_{p}(\tilde{x}_{\text{zmp}}, \tilde{y}_{\text{zmp}}) dS > \rho \forall t.
\]

However, the double integral of bivariate density function (16) does not have an analytical solution. As a result, solving (16) can be computationally expensive, especially for NOCP solvers that employ nonlinear programming (NLP) methods where a large amount of control points and iterations have to be accounted for. To increase the computational efficiency, a geometrical approximation of (16) will be introduced.

Since the iso-probability quadratic function \( \tilde{p}^{T} \Sigma_{f}^{-1} \tilde{p} \) of (15) has chi-squared distribution \( \chi_{2}^{2}(\rho) \), the probability ellipse that bounds a cumulative probability of (15) at \( \rho \) can be expressed as:

\[
\Pr(\tilde{p}^{T} \Sigma_{f}^{-1} \tilde{p} \leq \chi_{2}^{2}(1 - \rho)) = \rho.
\]

To find an expression of the bounding ellipse, we employ the diagonalization of \( \Sigma_{f} \), \( \Sigma_{f} = V \Lambda V^{T} \), where \( \Lambda = \text{diag}(\lambda_{1}, \lambda_{2}) \) is a positive diagonal matrix of the eigenvalues of \( \Sigma_{f} \), and \( V \) is an orthogonal matrix whose column space consists of the eigenvectors of \( \Sigma_{f} \). By further denoting \( \omega = [w_{1}, w_{2}]^{T} = V^{T} \tilde{p} \), we can write the elliptical boundary as \( \omega^{T} \Lambda^{-1} \omega = \chi_{2}^{2}(1 - \rho) \), or explicitly in \( \omega \) coordinates as:

\[
\frac{w_{1}^{2}}{\lambda_{1} \chi_{2}^{2}(1 - \rho)} + \frac{w_{2}^{2}}{\lambda_{2} \chi_{2}^{2}(1 - \rho)} = 1.
\]

\[\text{Fig. 2: An bounding ellipse that encircles a probability } \rho = 0.9 \text{ of } \tilde{p} \text{'s distribution.}\]

By coordinate transformation \( \tilde{p} = V \omega \), the ellipse \( \mathcal{E}(\rho, u) \) can be centered at \( \rho \tilde{p}_{0} \) on the support polygon as shown in Figure 2. It is worth pointing out that \( \mathcal{E}(\rho, u) \) encircles an area of probability \( \rho \) that has the highest probability density.

We, therefore, replace (4d) with the approximation:

\[
\mathcal{E}(\rho, u, t) \subset \text{Conv}(S) \forall t,
\]

when solving (4). It is noted that \( \mathcal{E} \) is presented in (19) as a function of \( t \) to emphasize that it is time-varying due to the vehicle’s ever-changing orientation in the context of trajectory planning.

In our previous work [24], the elliptical bound \( \mathcal{E} \) was derived under the static assumption where the vehicle does not experience any accelerations. On the contrary, the bound derived in this work is arrived from the linearization of (12) where acceleration terms are kept. Hence, in the remainder of this paper, the bounding ellipse proposed in our previous work [24] will be referred to as the ‘static bound’ while the bounding ellipse of this paper will be referred to as the ‘dynamic bound’.

D. Chance-aware support polygon margining

To implement (19) as a set of inequality constraints for efficient constraint checking, the original support polygon can be reduced by a varying safety margin based on the geometry of the bounding ellipse (18). Consequently, representing the margined support polygon with \( \bar{S} \), the task of verifying the enclosure of \( \mathcal{E}(\rho, u) \) within \( \text{Conv}(S) \) can be carried out analytically by checking if the condition, \( \bar{p}_{\text{zmp}} \in \text{Conv}(\bar{S}) \), holds true. Here, \( \bar{S} \) represents the new set of support points of the margined support polygon. It is worth pointing out that most RVs have finite sets \( S \) and \( \bar{S} \) due to their legged/wheeled/tracked design.

To determine the margined support polygon, we define \( j \in \{1, 2, \cdots, \text{card}(S)\} \), where \( \text{card}(S) \) represents the cardinality of \( S \). The \( j \)-th edge that defines \( \text{Conv}(S) \) is found by translating the \( j \)-th edge of \( \text{Conv}(S) \) towards the interior of the polygon by a distance of \( d_{v,j}^{*} \) for all \( j \). The distance \( d_{v,j}^{*} \) is the solution of the following:

\[
\begin{align*}
\min_{d_{v,j}} & d_{v,j}^{*} \\
\text{s.t.} & \left[ \begin{array}{c} d_{h,j} \\ d_{v,j} \\ d_{j} \end{array} \right] W \Lambda^{-1} W^{T} \left[ \begin{array}{c} d_{h,j} \\ d_{v,j} \\ d_{j} \end{array} \right] = \lambda_{2}^{2}(1 - \rho),
\end{align*}
\]

where the equality constraint is found directly from (18) by transforming the coordinates such that the equality constraint is found directly from (18). The solution to (20) can then be expressed as:

\[
d_{j}^{*} = W \Lambda W^{T} \left[ \begin{array}{c} 0 \\ \sqrt{\frac{\chi_{2}^{2}(1 - \rho)}{\text{det}(W \Lambda W^{T})}} \end{array} \right].
\]
and therefore the second component of \( d^* \), \( d^* \), can be used to find the margined support polygon \( \text{Conv}(S) \).

IV. CHANCE-CONSTRAINED ZMP-STABLE MOTION PLANNING

The previous section introduced a computationally efficient approximation of the original chance constraint (4d) so that the margining quantity of the ZMP constraint can be found efficiently. However, nonlinearities and nonconvexities that are inherent to the ZMP constraint, 3D terrain surface map, and obstacle avoidance constraint in the NOCP (4) remain. Although well-established numerical solvers such as IPOPT [28] can solve the subsequent nonlinear programming (NLP) of the NOCP, their performance is severely affected by the initial guess due to the aforementioned undesirable mathematical properties. Long computation times and not finding a feasible solution are both commonly encountered when the NOCP is solved directly.

As a result, extra modifications to the purely optimization-based formulation are required before it can be solved efficiently and reliably. The modified solution process of the overall trajectory planning problem follows the hierarchical framework proposed in [8] where the problem (4) is decomposed into two stages: a chance-constrained ZMP-stable path planning stage followed by a chance-constrained ZMP-stable trajectory optimization stage. Although the hierarchical framework is not a novel contribution of this work, a brief description of the path-planning stage is presented next for completeness and to help the understanding of the overall planning process.

A. Chance-constrained ZMP-stable Path Planning

Although a geometric path does not take the dynamic aspect of motion into account, it can be viewed as a chance-constrained ZMP-stable motion with infinitesimally low speed and acceleration if the path is planned to satisfy a vehicle’s obstacle avoidance, state, kinematic, and static ZMP chance constraints. To plan the constrained path, Rapidly-exploring Random Tree (RRT) type sampling-based planners can be employed due to their ability to take nonlinear and non-holonomic constraints into account while being computationally efficient. Since most sampling-based algorithms require constraints to be convex sets [29] with respect to the state while the ZMP constraint is not, the ZMP constraint is modified in the previous work [8] to adapt to the sampling-based planning framework.

To obtain a chance-constrained ZMP-stable path, the simplified chance constraint (19) is implemented in the planner by margining the support polygon according to (22). To reduce the computational cost, the planner samples on the level of vehicle position and is designed such that it only randomly alters the vehicle crane’s configuration when node generation is slow. Finally, the generated path is obtained by connecting individual waypoints. More details on the implementation of the sampling-based planner can be found in [8]. Additionally, to achieve path curvature continuity, the Repetitive Control Point Addition (RCPA) path smoothing technique developed from the Bézier curve introduced in our previous work [30] is implemented. Finally, a path-parametrized curvature-continuous path that satisfies the obstacle avoidance, state, kinematic, and chance constraints is obtained.

B. Chance-constrained ZMP-stable Trajectory Optimization

As a benefit of the path planning technique illustrated previously in Section IV-A, the path can be directly used in the trajectory optimization stage as an initial guess to warm-start the NLP solver. Additionally, due to the path being curvature continuous and path-parametrized, the time optimal NOCP can be formulated as:

\[
\begin{align*}
\min_{\mathbf{u}} \int_{t_0}^{t_f} 1 \, dt, \\
\text{s.t.} \quad \mathbf{x}(t) = \mathbf{g}(s, \dot{s}), \quad \mathbf{u} = \mathbf{g}_u(s, \dot{s}, \ddot{s}) \\
\quad \mathbf{x}(t_0) = \mathbf{x}_0, \quad \mathbf{x}(t_f) = \mathbf{x}_f \\
\quad \mathcal{E}(\rho, \mathbf{u}, t) \subseteq \text{Conv}(S) \\
\quad \mathbf{x} \leq \mathbf{f}, \quad \mathbf{u} \leq \mathbf{u} \leq \mathbf{u} \\
\quad s(t_0) = 0, \quad s(t_f) = 1, \quad \dot{s} \geq 0,
\end{align*}
\]

where \( s \in [0, 1] \) is the path parameter. Since the vehicle’s kinematics in the new formulation (23) is path-parametrized, the free-space constraint is no longer necessary since the path is already obstacle-free. The original chance constraint (4d) is replaced with the proposed simplification (19) and the margin which varies with nominal terrain and dynamic control input is obtained with (22).

V. SIMULATION RESULTS

To showcase the properties and effectiveness of the proposed method, a comparison with other methods is presented in the form of Monte Carlo simulations. Some demonstrations are also presented to validate the proposed model and its computational efficiency. In this section, we consider a RV that simulates a feller-buncher machine modeled after the Tigercat 855E. The vehicle can be described with the schematic diagram in Figure 1. The vehicle’s geometric and mass parameters are summarized in Table I and its support polygon is a 3.23 m \times 5 m rectangle. All problems are solved on a Windows desktop with an AMD Ryzen 5 4600G 3.7GHz 6-core processor.

<table>
<thead>
<tr>
<th>Link Number</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, ( l_i ) (m)</td>
<td>1.60</td>
<td>0.96</td>
<td>3.27</td>
<td>3.27</td>
<td>0.458</td>
<td>0.677</td>
</tr>
<tr>
<td>Mass, ( m_i ) (kg)</td>
<td>13000</td>
<td>5000</td>
<td>2000</td>
<td>1000</td>
<td>50</td>
<td>2000</td>
</tr>
</tbody>
</table>

TABLE I: Reconfigurable vehicle parameters.

A. Monte Carlo simulations comparing dynamic SO(3) Gaussian noise model and other models

To showcase the dynamic bound’s ability to approximate the original nonlinear chance constraint and enclose the
correct probability distribution, Monte Carlo simulations are carried out for different vehicle orientations and accelerations. For each acceleration and orientation, the ZMP coordinates are found through the nonlinear representation (12) for 5,000 noise values sampled from the distribution $\epsilon \sim N(0, 0.05 \cdot I_3)$ which corresponds to a 90% confidence interval of about 30° of rotation about any axis. In this demonstration, $\rho = 0.9$ is chosen as the chance constraint for 90% of the ZMP distribution to fall within $\mathcal{E}$.

For the static simulations shown in Figure 4, the three bounds are confirmed to be identical for the static 0-degree scenario. Following the proposed method, the dynamic bound is found for each acceleration using (18). For a static scenario with 30-degree slope, the identical dynamic and static bounds are observed to increase in size compared to the 0-degree case, while the i.i.d bound’s size remains the same.

For the dynamic simulations, the vehicle’s nominal pitch angle is fixed at −30 degrees (30 degrees nose down as shown in Figure 3) and the vehicle is given accelerations along its longitudinal axis that vary from −6 m/s² to 6 m/s² with a 1 m/s² increment. In Figure 5, the three types of bounds are compared for a scenario with the minimum negative acceleration (left figure) and a scenario with the maximum positive acceleration (right figure). It can be observed from the left plot that when the acceleration is up the incline, the dynamic probability bound becomes more elongated compared to the static one, and vice versa, as seen in the right plot. The change in the dynamic bound’s shape mirrors that of the sampled ZMPs’ distribution. By contrast, the static bound’s shape does not change with respect to the acceleration.

For the same data set, the number of sampled ZMPs encircled by the three iso-probability bounds (dynamic, static, and naive) is calculated for each acceleration tested. It is shown in Figure 6 that the coverage rate of the dynamic bound derived from (18) remains nearly constant at approximately 86%. On the other hand, that of the naive i.i.d bound varies from 66% to 83% and that of the static bound varies from 79% (under bounding) to 91% (over bounding) as the acceleration increases from −6 m/s² to 6 m/s². Combining the observations of Figures 5 and 6, it can be concluded that the dynamic bound correctly captures the dependence of ZMP distribution on the vehicle’s acceleration and provides a very good and consistent estimate of the distribution.
Fig. 6: Comparison between coverage rates of the iso-probability bounds generated using the dynamic bound, static bound, and the naive i.i.d. assumption.

The benefit of this property is that the bounding ellipse can approximate the chance constraint for various dynamic scenarios without being conservative which can result in the reduction of problem feasibility.

Fig. 7: Comparison between iso-probability bounds generated using the dynamic model for two vehicles with different CoM heights on a 30-degree nominal slope with accelerations of \(-6\, \text{m/s}^2\) and \(6\, \text{m/s}^2\).

To further demonstrate the capability of the proposed dynamic bound to accommodate different vehicle properties, another set of results is generated for a vehicle that has the same total weight as for previous results, but has its center of mass (CoM) at twice the height in the vehicle frame. The vehicle is again placed at a 30 degree nose down orientation. The resulting iso-probability bounds comparison between the two vehicles are shown in Figure 7, again for the two extreme accelerations. For the vehicle with the original CoM height, the location and size of its bounding ellipse are the same as previously shown in Figure 5. For the vehicle with higher CoM, not only does its nominal ZMP location change due to the property of the ZMP formulation, its bounding ellipses become larger as well.

The observed changes in the nominal ZMP location and distribution are aligned with the common conception that the higher a vehicle’s CoM, the more likely it is to experience a rollover. The reason for the \(6\, \text{m/s}^2\) results showing a smaller nominal ZMP discrepancy is due to the direction of acceleration being aligned with the downhill component of the gravitational acceleration. As shown in Figure 8, the coverage rate for the high CoM vehicle is again consistent at an average of 86% for the range of accelerations considered, as a result of the proposed method being able to accommodate different vehicle and terrain parameters. For comparison, the coverage rate plot of the proposed dynamic bound for the low CoM vehicle shown in Figure 6 is included in Figure 8 to show how similar they are.

It is worth pointing out that the expected coverage rate for an ideal nonlinear bound should be, by definition, 90% while the approximate dynamic bound has a coverage rate of 86%. This discrepancy is due to the bounding ellipse being a linearized approximation of (12) using (14). Considering the linearization allows (4) to be readily solvable by simplifying the nonlinear (4d), the slight loss in coverage rate is tolerable nonetheless.

B. Demonstration with Experimental Data

To validate the proposed idea of terrain uncertainty-induced rotation vector distribution and its variation for different terrain roughness, a demonstration with experimental data obtained from driving an unmanned vehicle in real forests is presented. The data set available online\(^1\) is obtained from a previous work [31], where a Clearpath Husky A200 mobile robot equipped with an Xsense MIT-30 IMU was driven through different types of forests to demonstrate 3D mapping and tree diameter estimation algorithms. From the data set, the IMU data with orientation information parameterized with quaternions for the ‘Young’ and ‘Maple’ forests are of interest here. As described in [31], the ‘Young’ forest is characterized by young balsam firs and a rough forest floor made of mossy soil. The ‘Maple’ forest is characterized by tall maple trees and a smooth forest floor. Both forests are considered to be nominally flat since the ‘Young’ site is a small section of \(30\, \text{m} \times 35\, \text{m}\) that does not show elevation change on topographical maps \(^2\) and the ‘Maple’ site is a flat \(100\, \text{m} \times 100\, \text{m}\) area on campus at Laval University [31].

\(^1\)https://norlab.ulaval.ca/research/montmorencydataset/
\(^2\)https://goo.gl/maps/M3lf86Xk6t8RWzL98
The orientation error caused by terrain uncertainty is found by computing the difference between the measured orientations and the nominal orientations. Since accurate heading angle tracking is assumed, the nominal orientation is defined by the corresponding yaw-angle rotation so that its heading direction is the same as that of the measured orientation. Finally, all orientation errors are converted into axis angle representation defined by $\epsilon$. The distributions of its components $\epsilon_1$, $\epsilon_2$, $\epsilon_3$, and the corresponding ZMPs of Young and Maple forests are shown in Figures 9 and 10, respectively.

It can be observed from Figures 9 and 10 that the distributions of the rotation vector’s components of both forests can be reasonably approximated with Gaussian distributions, those overlaid in red. For the two forests, the rotation vector distribution’s covariance matrices are, respectively:

$\begin{align*}
Q_{\text{Young}} &= \begin{bmatrix}
0.014 & 0.001 & -0.001 \\
0.001 & 0.021 & 0.001 \\
-0.001 & 0.001 & 0.000
\end{bmatrix}, \\
Q_{\text{Maple}} &= \begin{bmatrix}
0.003 & 0.000 & -0.000 \\
0.000 & 0.006 & 0.000 \\
-0.000 & 0.000 & 0.000
\end{bmatrix},
\end{align*}$

The near diagonal covariance matrices and each component’s distribution shows that Assumption III-A on the rotation vector components distributions is relevant for real-world rough terrain applications. It can also be observed that the distribution of ZMPs in the ‘Young’ forest has more spread than that in the ‘Maple’ forest. Consequently, as shown in the iso-probability bounds comparison in Figure 11, the chance constraint is stricter in the ‘Young’ forest since its bounding ellipse is larger under the same condition. It can also be observed that, on nominally flat terrain, the uncertainty affects the roll axis more than the pitch axis in both forests which agrees with the results in [32].
C. Path Planning with 90% Chance Constraint

To present how different terrain uncertainty compositions can affect path planning on the same terrain surface geometry, the vehicle is placed on a simulated terrain constructed by the sinusoidal function:

\[ s_z = 10 \cos\left(\frac{\sqrt{s_x^2 + s_y^2}}{10}\right), \quad (24)\]

which is integrated into the vehicle’s kinematics (1). The terrain is symmetric in the \( s_x - s_y \) plane with respect to the \( s_x \) axis. To illustrate the obstacle avoidance capability of the formulation, cylindrical stump-like obstacles with 0.2m diameter are placed in a uniform grid on the terrain at 20m apart from each other. Three different uncertainty compositions are tested: ‘No Uncertainty’ (\( Q_{11} = 0 \cdot I_3, Q_{r1} = 0 \cdot I_3 \)), overall ‘Low Uncertainty’ with milder left hand side (LHS) terrain (\( Q_{21} = 0.005 \cdot I_3, Q_{r2} = 0.05 \cdot I_3 \)), and overall ‘High Uncertainty’ with milder RHS terrain (\( Q_{31} = 0.01 \cdot I_3, Q_{r3} = 0.001 \cdot I_3 \)). For each terrain, subscripts \( l \) and \( r \) correspond to the LHS and RHS terrain, respectively.

![Fig. 12: Paths planned under different uncertainties. For each plot, transparent red lines are the planned paths and the solid red line is the average path found by averaging the planned paths along normalized waypoint index.](image)

D. Trajectory Planning with 90% Chance Constraint

To showcase the overall performance of the proposed chance-constrained trajectory planner, the vehicle is placed on the same terrain with the same obstacles as introduced in the previous section. The LHS of the terrain plane is assigned a milder 3-D Gaussian noise \( \epsilon \sim \mathcal{N}(0, 0.005 \cdot I_3) \) \( \forall s_x \leq 0 \), representative of the uncertainty level of the ‘Maple’ forest; the RHS is assigned \( \epsilon \sim \mathcal{N}(0, 0.05 \cdot I_3) \) \( \forall s_x > 0 \), more similar to the uncertainty level of the ‘Young’ forest. The vehicle is initially located at coordinate \((s_x=0, s_y=0, s_z=10)\) and is required to relocate to \((s_x=0, s_y=67, s_z=10)\), as indicated in red font in the top view of the terrain in Figure 13. We note that the initial and final vehicle base heading angles can be set arbitrarily as the machine has a skid-steered base.

In Figure 13, we indicate the RRT nodes by black crosses and as can be seen, they are more densely distributed in the left half-plane even though the nominal terrain is symmetrical about \( s_x = 0 \). This shows that the ZMP chance constraint is more likely to be satisfied on the smoother terrain, i.e., with low orientation uncertainty. As a result, when all other variables are the same, a connected path is more likely to pass through the low uncertainty areas. The final path that ensures 90% ZMP constraint satisfaction is indicated with a red line and it is comprised of 25 path-parametrized segments. The initial configuration for the vehicle’s crane is \( q(t_0) = [0, -\pi/6, -2\pi/3, \pi/6, -\pi/2]^T \) and corresponds to that of the RRT result where no crane reconfiguration is required for this test scenario. The RRT path was generated in 1.32 seconds.

The complete path solution is leveraged to warm-start the trajectory generation NOCP as described in [8]. For the trajectory generation, both the vehicle and the crane are specified to start and finish at rest, with the final crane uncertainty while they are more evenly divided for the case of equal uncertainty on both sides in the ‘No Uncertainty’ case. The same trend can be observed more intuitively from Figure 12.

![TABLE II: Path planning data of three terrains.](image)

<table>
<thead>
<tr>
<th>Uncert. Comp.</th>
<th>( t_c(\sigma) ) (Seconds)</th>
<th>Waypoint % Left</th>
<th>Waypoint % Right</th>
<th>Avg. # of Reconf.</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>1.33 (0.49)</td>
<td>55.17</td>
<td>44.83</td>
<td>0</td>
</tr>
<tr>
<td>Low</td>
<td>1.98 (2.32)</td>
<td>94.95</td>
<td>5.05</td>
<td>0</td>
</tr>
<tr>
<td>Hi</td>
<td>53.20 (27.92)</td>
<td>0.27</td>
<td>99.73</td>
<td>4.45</td>
</tr>
</tbody>
</table>
configuration left free. The constraints on the vehicle base’s linear velocity $v$, acceleration $a$, and angular rates and accelerations are:

$$0 \leq v \leq 1 \text{ m/s}, \quad -1 \leq a \leq 1 \text{ m/s}^2,$$

$$-1 \leq \dot{\psi} \leq 1 \text{ rad/s}, \quad -1 \leq \ddot{\psi} \leq 1 \text{ rad/s}^2.$$ 

The time optimal trajectory that brings the vehicle from start to finish is shown in Figure 14. The figure shows that the vehicle’s linear and angular velocity and acceleration profiles appear to be time optimal for the specific path as the linear velocity of the vehicle is kept at maximum except when the other states and the dynamically margined ZMP constraints are nearing their limits. The motion’s corresponding ZMP trajectory is shown in Figure 15 where the red dashed lines represent the boundaries margined with (22) to accommodate the chance constraint; these deviate non-trivially from the black dashed lines representing the fixed deterministic support polygon boundaries. Both figures show that the vehicle’s state and ZMP trajectories stayed within their corresponding limits. Each of the 25 segments of the whole 88.52 seconds of trajectory took an average of 0.47 seconds to compute. Since the NOCP trajectory planner employs a multi-segment structure, the problem can be easily implemented to solve the NOCP segment-by-segment for faster and more robust online computation.

VI. CONCLUSION

In this paper, a novel chance-constrained NOCP formulation for dynamically stable motion planning of reconfigurable vehicles under 3-D terrain uncertainty has been presented. To solve the NOCP efficiently, the geometrical properties of the linearized $\text{SO}(3)$ disturbance resulting from terrain uncertainty are exploited to construct the proposed dynamic bound. The dynamic bound lifted the computational burden of the original chance-constrained NOCP. By comparing the proposed dynamic bound against the previously proposed static bound and the naive i.i.d assumption through Monte Carlo simulations, the proposed bound is shown to capture the ZMP distribution during dynamic motions better and it is more capable of emulating the general chance constraint. A demonstration is carried out with experimental data in real forests to show that Assumption III-A and the proposed model are applicable in the real world. Finally, the chance constraint is implemented in a trajectory planning algorithm to showcase its purpose and performance. The results demonstrate the state and chance stability constraint satisfaction and the computational times required to plan the trajectory are suitable for online implementation.

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