Width Confinement in 3D Dielectric Waveguides and Comparison to 2D Analytical Models

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Abstract

Two-dimensional (2D) analytical models are only approximations for 3D structures where one cross-sectional dimension is much larger than the other. This paper uses the finite-difference time-domain method (FDTD) to perform numerical experiments on fully 3D dielectric waveguide structures to compute the wave-impedance and the propagation-constant for finite-width dielectric waveguides. These data are used to determine the width required to achieve good correlation against 2D analytical models. Results show that width \([?]\) 10x height is the limit for good approximation.
Width Confinement in 3D Dielectric Waveguides and Comparison to 2D Analytical Models

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Abstract—Two-dimensional (2D) analytical models are only approximations for 3D structures where one cross-sectional dimension is much larger than the other. This paper uses the finite-difference time-domain method (FDTD) to perform numerical experiments on fully 3D dielectric waveguide structures to compute the wave-impedance and the propagation-constant for finite-width dielectric waveguides. These data are used to determine the width required to achieve good correlation against 2D analytical models. Results show that width \( w \geq 10 \times \) height is the limit for good approximation.

I. INTRODUCTION

Nano-scale dielectric waveguides are a critical component in modern integrated circuit design for both signal/power integrity and ultra high-speed data transfer. Analytical formulation of dielectric waveguides has historically been limited to the spatially two-dimensional (2D) canonical form of the slab waveguide [1]–[7]. However, realistic dielectric waveguides are fully three-dimensional (3D) structures. Despite the dimensional discrepancy, 2D analytical models are often used as approximations, provided that there is only tight confinement in one of the two cross-sectional dimensions, where length is functionally infinite along the longitudinal-section in both directions. 3D dielectric waveguides with width much larger than height are spatially inefficient. Therefore, in this paper we conduct numerical experiments in FDTD to determine the limit for width where 2D analytical models may be considered a good approximation of 3D dielectric waveguides. The 2D analytical models for wave-impedance \( Z_w \) (Ohm) and the propagation-constant \( \beta \) (rad/m) are used for comparison.

II. FORMULATION

The waveguides of interest are fully 3D and have step index contrast between core and cladding regions with refractive indices \( n_1 = 3.5 \) and \( n_2 = 1.5 \) (corresponding to Si/SiO\(_2\)), respectively, where the core is surrounded uniformly on all sides by a cladding which is assumed to be infinite in extent. The core region geometry is shown in Fig. 1, where we note the top and bottom walls of the waveguide are separated by the height \( \delta \), the left and right walls are separated by the width \( w \), and all walls are smooth.

The 2D analytical \( Z_w \) model is defined in (1) [4] may be used as an approximation for comparison with 3D simulations data,

\[
Z_w^{TE} = \frac{E_y}{H_z} = -\frac{j\omega \mu_0}{\gamma}, \quad (\Omega) \tag{1}
\]

where \( j = \sqrt{-1}, \gamma = \sqrt{\beta^2 - n_2^2 k_0^2} \) (rad/m), \( \mu_0 \) is the free-space magnetic permeability (H/m), angular frequency \( \omega = 2\pi f \) (rad/s), cyclic frequency \( f \) (Hz), and field components \( E_y \) (V/m) and \( H_z \) (A/m) are time-harmonic frequency-domain phasors, and \( Z_w \) is purely imaginary for a smooth dielectric waveguide with no added loss mechanism (such as surface roughness or lossy material).

The propagation constant can be evaluated from the relation of E-field components using the imaginary component of the complex logarithm in (2) [8]

\[
\beta = \frac{1}{\ell} (\text{arg}(E_1) - \text{arg}(E_2)) = n_{eff} k_0, \quad (\text{rad/m}) \tag{2}
\]

where \( k_0 \) is the free-space wave number at the source wavelength \( \lambda_0 \), \( n_{eff} \) is the effective refractive index as calculated from the Effective Index Method [2, §D.C], \( \log(E_1/E_2) = \alpha + j\beta, \alpha = 0 \) in the absence of any additional loss mechanism, and \( \ell \) is the distance between \( E_2 \) and \( E_1 \). The arg function returns the complex angle including any and all additional \( 2\pi m \) turns with \( m \in \mathbb{Z} \).

III. RESULTS AND DISCUSSION

For computing \( Z_w \) using (1), each numerical experiment uses \( \lambda_0 = 1.54 \mu\text{m} \) and \( \delta = 200 \text{nm} \), and reported values are evaluated at \( f = 194.8 \text{ THz} \). FDTD field data are collected at the midpoints along \( \ell \) and \( w \), with a three-cell offset from the bottom of the waveguide core region.

These data are collected over widths varying from 200 nm to 5 \( \mu\text{m} \). Those results are shown in Fig. 2, where \( \Im\{Z\} \) is the imaginary component of the complex number \( Z \). We see therein that \( Z_w \) calculated with FDTD approaches the 2D analytical model and saturates at \( \approx 800 \text{ nm} \). There is a noticeable offset between the 3D FDTD data and the 2D analytical approximation below that point but little variation.
between FDTD and analytical calculations above that point. In this case, the 2D analytical model is a good approximation for \( w \geq 4 \delta \).

Sample FDTD \( Z_w \) data are shown with all discrete cells in the \( w \times \ell \) cross-section in Fig. 3. The boundary of the region directly below the waveguide core is shown as dotted lines. Within the below core region there is minimal variation along length. \( Z_w \) settles to a stable value within 2 \( \mu m \) from the source location at \( z = 0 \), and the variations along \( w \) appear only near the region boundary. Outside the boundary there are several null points appearing periodically along length. The length interval between nulls seems to be inversely proportional to waveguide width. The null points are likely the result of 3D multi-modal behavior as more propagating modes exist in the waveguide with increasing \( w \).

IV. CONCLUSION

The experiments conducted in this paper showed that fundamental parameters \( Z_w \) and \( \beta \) have a strong dependence on the \( w/\delta \) ratio, where in this case, good correlation to 2D analytical model is achieved for \( w \geq 10 \delta \); the data suggests an order-of-magnitude difference would be sufficient. Further experiments should be conducted for additional parameters, e.g., \( \alpha \), in both smooth and rough waveguides.

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REFERENCES