Unambiguous 2-D Angle-of-Arrival Estimation Receiver based on a 16-port Interferometer

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Abstract

An unambiguous 2-D angle-of-arrival (AoA) estimation receiver (Rx) based on a 16-port interferometer is proposed and demonstrated in this paper. It has three main components: eight antennas, a 16-port interferometer, and a signal processor. The antennas are in the same plane in a concentric-square shaped configuration. The eight input signals are superposed on eight output port signals in the interferometer. First, an AoA estimation algorithm for the proposed 16-port interferometer is described. The algorithm transposes the phase differences between the antenna elements into the power relationship of the output signals. Then, a method for reducing angle ambiguity is introduced. The method allows for the distance between the antenna elements to be larger than $\lambda/2$, and the angle detection range is theoretically $-90^\circ$ to $90^\circ$. Then, the Rx error models are established to obtain the systematic errors. An Rx operating at 31 GHz was fabricated. The measurements demonstrate that the system operates as expected, with AoAs ranging from $30^\circ$ to $30^\circ$ and a mean squared error of approximately $0.4^\circ$. 
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Index Terms—Angle modulation, direction of arrival estimation, interferometry, multipath channels, receivers.

I. INTRODUCTION

INTERFEROMETERS have a wide range of applications in direction-finding algorithms [1], [2], such as sun fringe observations [3], airborne single observer passive locations [4], and threat determination signal detection [5]. Interferometric receivers (Rx)s estimate the angle of arrival (AoA) based on the phase relationships between the signals received by the antenna elements at several different positions [6]. If the impinging wave is planar, the AoA can be specified by the direction of the Poynting vector [7].

In most cases, for interferometer Rx,s, multielement arrays are required to achieve both good angular resolution and a low probability of ambiguity for realistic values of receiving channel phase errors [8]. This means that two or more phase comparators are needed to measure the phase differences between each element [1].

Multipath interferometer (MPI) techniques can be used to decrease the number of phase comparators, as discussed in [9], [10]. MPI techniques have advanced since the six-port reflectometer was developed in the early 1970s [11], [12]. The six-port reflectometer has two input ports and four output ports that superimpose the two input signals with four different relative phases. At the time, the six-port reflectometers were a simple and accurate current, voltage, power and impedance measurement setup. This concept was also used to determine the relative permittivity of a material [13], adaptive load- and source-pulling [14] and antenna near-field characterization [15]. The six-port concept was first used in Rx(s) in [16], [17]. After that, the concept gained widespread attention, such as in [18]-[22], and evolved into MPI techniques.

MPIs are a passive component network. In an MPI, incoming signals are superposed in the analog domain with multiple output port signals. MPI techniques handle high-power signals better than other techniques [23], allowing the setup of large-signal analysis systems for power amplifiers or semiconductor circuits [24], as well as precise phase measurements [25].

For MPI-based Rxs (MPRs), the output signals from the MPI have a specific amplitude and phase distribution to passive diode detectors for power reading [10]. There are many emerging MPRs in different applications, such as power sensors [26], software-defined radio techniques [27], tunable cognitive radio platforms [28], quadrature amplitude modulation (QAM) modulators [29], [30], millimeter-wave complementary metal oxide semiconductor (CMOS) receiver chips [31], frequency-modulated continuous wave (FMCW) modulators [32], highly accurate ranging sensors [33], and AoA detection Rx(s) [34].

However, a six-port interferometer can only estimate one dimension of the AoA because it has a maximum of two receiving channels. Some solutions for obtaining azimuth and elevation (2-D) AoAs have been proposed: incorporating down-converted mixers [35], measuring the time difference of arrival (TDOA) [36], and adding more signal channels (at least two six-port junctions) [37], [38]. However, these are not attractive schemes in terms of cost and size [39].

In 2017, [39] showed a 2-D AoA estimation system based on an eight-port interferometer for the first time through specific theoretical analyses and simulations. It combined the signals received by an area array with four antennas through one interferometer. However, in this system, the unambiguous angle range in the elevation and azimuth planes is narrow. Multipath interferometer techniques are based on phase analysis,
where the phase is defined as the phase within half a wavelength. Ambiguity issues occur when the measured distance is larger than half the wavelength. The ambiguity is due to the periodic repetition of the distance variation. This causes the distance between the antenna elements to be short, which limits the shape and gain of the antenna.

The most common method for broadening the unambiguous detection range is to measure the same dimensional angle information at different relative distances between the receiving antennas, with each distance referred to as the baseline. Reference [40] proposed an easy, wide-unambiguous-range antennas, with each distance referred to as the baseline.

In this paper, an unambiguous 2-D AoA estimation receiver based on a new 16-port interferometer is proposed.

The three major goals of this paper are to

1) Introduce the proposed 16-port interferometer and develop the corresponding 2-D AoA estimation algorithm when the antenna elements are arranged in the same plane in a concentric-square-shaped configuration (Section II).

2) Resolve the phase ambiguities of the designed 2-D AoA estimation receiver by extending the 1-D method from [40] to 2-D by adding four more antennas (Section III).

3) Establish a mathematical model for calibrating the system error to improve the accuracy (Section IV).

The prototyping receiver is fabricated in Section V, and the measurements verify the theory.

II. AOA ESTIMATION ALGORITHM

A. 16-port Interferometer

The antenna spacing for a signal frequency six-port interferometer Rx must be less than half the wavelength to cover a wide unambiguous detection range since the phase difference can only be measured in modulo $2\pi$ [1, 39]. To overcome this limitation and achieve a 2-D AoA estimation, at least three antenna elements are required on the horizontal axis and the vertical axis. Thus, a 16-port interferometer with eight input and eight output ports is proposed in this paper.

The topology of the 16-port interferometer is shown in Fig. 1. The 16-port interferometer consists of eight $3\,\text{dB}$ $180^\circ$ RRCplrs and four $45^\circ$ phase shifters. $P_{in}$ represents the input port connected to the receiving antenna. $P_{out}$ represents the output port connected to the detector. The indices $\text{in}$ and $\text{out}$ range over (1, 2, $\cdots$, 8) and (9, 10, $\cdots$, 16), respectively. $a_{in} = [a_1, a_2, \cdots, a_{16}]^T$ represents the independent power waves that enter port $P_{in}$. $b_{out} = [b_9, b_{10}, \cdots, a_{16}]^T$ represents the dependent power waves that exit port $P_{out}$.

The S-parameter network of the 16-port interferometer is represented by the matrix relationship shown in (1), where $a_{\text{OUT}} = [a_9, a_{10}, \cdots, a_{16}]^T$ represents the independent power waves that enter port $P_{\text{OUT}}$, and $b_{\text{IN}} = [b_9, b_{10}, \cdots, a_{16}]^T$ represents the dependent power waves that exit port $P_{\text{IN}}$.

$$b_{\text{IN}} \cdot b_{\text{OUT}} = S \cdot a_{\text{IN}}$$

The 16-port interferometer’s scattering matrix $S$ can be divided into four submatrices, as shown in (2). The subscripts $(8 \times 8)$ and $(16 \times 16)$ indicate that the matrices have 8 or 16 rows, respectively, and the number of rows equals the number of columns.

$$S = \begin{bmatrix} S_{I1(I8 \times 8)} & S_{I2(I8 \times 8)} \\ S_{II(I8 \times 8)} & S_{II(I16 \times 16)} \end{bmatrix}$$

The transfer coefficient matrix $S_{H1,1}$ shown in (3) is a solely concerned submatrix that explains the phase and amplitude changes as the input signals from the antennas travel through the passive network.

$$S_{H1,1} = \begin{bmatrix} S_{9,1} & S_{9,2} & S_{9,3} & 0 & 0 & 0 & 0 & 0 \\ S_{10,1} & S_{10,2} & S_{10,3} & 0 & S_{10,5} & S_{10,6} & S_{10,7} & 0 \\ S_{11,1} & S_{11,2} & S_{11,3} & 0 & S_{11,5} & S_{11,6} & S_{11,7} & 0 \\ 0 & 0 & 0 & 0 & S_{12,5} & S_{12,6} & S_{12,7} & 0 \\ S_{13,1} & S_{13,2} & S_{13,3} & 0 & S_{13,5} & 0 & 0 & 0 \\ S_{14,1} & S_{14,2} & 0 & S_{14,4} & S_{14,5} & S_{14,6} & 0 & S_{14,8} \\ S_{15,1} & S_{15,2} & 0 & S_{15,4} & S_{15,5} & S_{15,6} & 0 & S_{15,8} \\ 0 & 0 & 0 & 0 & S_{16,1} & S_{16,2} & 0 & S_{16,4} \end{bmatrix}$$

$S_{H1,1}$ satisfies two conditions: the amplitude relationships and the phase relationships, shown in Table I and Table II, respectively. These two tables take the amplitude and phase of the transmission parameters $S_{H1,1}$, $|S_{H1,1}|$, and $\angle S_{H1,1}$ as the reference parameters. The row headers indicate the serial number of the output ports, and the column headers indicate the input ports.

B. Topology of the Antenna Elements

The antenna elements shown in Fig. 2 are arranged in the same plane in a concentric-square-shaped configuration. $\beta$ is the Poynting vector of the incoming wave. Taking the 7th element as the reference element, the element locations in the $x$-$y$-$z$ coordinate system can be expressed as
The Poynting vector is an algebraic column unit vector $\mathbf{u} = [\cos \theta_0 \sin \theta_0, \cos \theta_0 \cos \theta_0, \sin \theta_0]$, where $\theta_0$ and $\theta_1$ are the altitude and azimuth, respectively [7, 39]. The antenna element phases are

$$\varphi = \beta_n Lu = \frac{2\pi}{\lambda} Lu = [\varphi_1, \varphi_2, \ldots, \varphi_n]^T$$ \hspace{1cm} (5)

Let $F(\theta_0, \theta_1)$ and $A$ be the pattern factor of the antenna and the amplitude of the incoming wave, respectively [41]. The wave incident on the input port of the 16-port interferometer can be expressed as $a_{in} = AF \exp(j\varphi_{in})$. It should be noted that $\varphi_{in} = 0$. Therefore, $a_f = AF$.

**TABLE I**

<table>
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<th>$\theta$</th>
<th>1</th>
<th>2</th>
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<tr>
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<td>1</td>
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<td>$</td>
<td>S_{4,1}</td>
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<td>S_{6,1}</td>
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<td>$</td>
<td>S_{7,1}</td>
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</tr>
</tbody>
</table>

$|S_{0,1}|$ is the reference parameter.

e.g.: $|S_{0,1}| = |S_{1,1}| = |S_{2,1}| = |S_{3,1}| = \sqrt{2}$ $|S_{4,1}| = |S_{5,1}| = 1/\sqrt{2}$ $|S_{6,1}| = |S_{7,1}| = 1/\sqrt{2}$ $|S_{8,1}|$.

**TABLE II**

<table>
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<tr>
<th>$\theta$</th>
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<td>$7\pi/4$</td>
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<td>$\pi$</td>
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<tr>
<td>$3\pi/2$</td>
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<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

$|S_{0,1}|$ is the reference parameter. $n$ is an integer number.

e.g.: $|S_{0,1}| = |S_{0,2}| = 2n\pi$, $|S_{0,1}| = |S_{0,3}| = 2n\pi + \pi/2$.

**C. 2-D AoA Estimation**

After the mathematical modeling of the interferometer and the received signals, the output waves of the 16-port interferometer can be calculated as

$$b_{out} = \sum_{i=1}^{16} S_{out,i} a_{in} = AF \sum_{i=1}^{16} S_{out,i} \exp(j\varphi_{in})$$ \hspace{1cm} (6)

Let $B_{out} = |b_{out}|^2$ be the output power. The output powers can be expressed by (13) in Appendix A.

The relationship in (7) can be obtained immediately because the sum of each column of $E$ in (16e) except the first column is zero. This means that the total power of the output is equal to the total power of the input, and when the power received by each antenna unit is equal, the total power of the output is eight times that of the power received by each unit. This also means that there is no need to consider the pattern factor of antenna $F(\theta_0, \theta_1)$ in theory.

$$B_{sum} = B_{sum1} + B_{sum2} = \sum_{i=9}^{12} B_{out} + \sum_{i=13}^{16} B_{out} = 8a_7^2$$ \hspace{1cm} (7)

when $a_1 = a_2 = a_3 = a_5 = a_6 = a_7 = a_8 = a_9$.

After some algebraic operations, the four phase differences can be solved by

$$\varphi_{\alpha} = \cos^{-1} \left[ \frac{2(B_{sum1} - B_{sum2})}{B_{sum}} \right]$$

$$\varphi_{\alpha} - \varphi_{\beta} = \sin^{-1} \left[ \frac{2 \{B_{sum1} - B_{sum} - 4B_2 + 4B_1\}}{B_{sum}} \right]$$

$$\varphi_{\alpha} + \varphi_{\beta} = \sin^{-1} \left[ \frac{2 \{B_{sum1} - B_{sum} + 4B_2 + 4B_1\}}{B_{sum}} \right]$$

$$\varphi_{\beta} = \sin^{-1} \left[ \sqrt{K^2 + M^2 \{B_{sum1} - B_{sum} + 4B_1 - B_2\}} / B_{sum} \right]$$

$$- \tan \left( M / K \right)$$ \hspace{1cm} (8a)

where

$$K = 2^{1/2} \left( \cos(\varphi_{\alpha} - \varphi_{\beta}) - \cos(\varphi_{\alpha} + \varphi_{\beta}) \right)$$

$$M = 2 - 2^{1/2} \left( \sin(\varphi_{\alpha} - \varphi_{\beta}) - \sin(\varphi_{\alpha} + \varphi_{\beta}) \right)$$ \hspace{1cm} (8b)

*Fig. 2.* Concentric-square layout of the 8 antennas and the relationship to the incoming wave.
and \( \varphi_{1z} = \beta_0 d_1 \sin \theta_E, \varphi_{2z} = \beta_0 d_2 \sin \theta_E, \varphi_{1x} = \beta_0 d_1 \cos \theta_E \sin \theta_H, \varphi_{2x} = \beta_0 d_2 \cos \theta_E \sin \theta_H \).

Let \( \varphi_{2z0} \) and \( \varphi_{2x0} \) be the unambiguous phase differences in the vertical and horizontal directions on the baseline \( d_2 \), respectively. They can be solved using the angle ambiguity-resistant method shown in Section III and the four phase differences calculated from (8).

Finally, the 2-D unambiguous AOAs are

\[
\theta_E = \sin^{-1} \left( \frac{\varphi_{2z0}}{(\beta_0 d_2)} \right) \quad \text{and} \quad \theta_H = \sin^{-1} \left( \frac{\varphi_{2x0}}{(\beta_0 d_2 \cos \theta_E)} \right).
\]

### III. ANGLE AMBIGUITY-RESISTANT METHOD

The algorithm shown in the last section can only be used to measure a narrow range of angles because of angle ambiguity [39]. In this paper, to suppress the ambiguity, the 1-D method from [40] is extended to 2-D by adding four more antennas, which establish two baselines, \( d_1 \) and \( d_2 \), as shown in Fig. 2.

The two baselines are related by \( d_2 = v d_1 \), where \( v \) is a noninteger positive number; this relationship improves the unambiguous range. The unambiguous range could extend to \((-90^\circ, 90^\circ)\). The values of \( d_1 \) and \( d_2 \) are flexible, as long as the condition for \( v \) is satisfied and \( d_1 > \frac{\lambda}{2} \) to leave enough space for the antennas.

#### A. Unambiguous Altitude \( \theta \)

With \( d_1 = 9 \text{ mm} \) and a signal frequency \( f = 31 \text{ GHz} \), there is still an unknown parameter \( v \). This parameter was selected based on the step-like phase difference, which will be introduced later, and its value is almost arbitrary as long as it is a noninteger. Here, \( v \) was chosen to be 3.33 to describe the step-shaped phase difference.

When the real azimuth \( \theta_E \) ranges over \((-90^\circ, 90^\circ)\) and no method to improve angle ambiguity is used, the detected azimuth \( \theta_{Ei} \) under different baselines is

\[
\sin^{-1} \left[ \frac{\varphi_{i}}{(\beta_0 d_i)} \right], \quad i = 1, 2,
\]

as shown in Fig. 3 (a) and Fig. 3 (b). It can be seen from the figure that \( \theta_{Ei} \) is equal to \( \theta_E \) only in the small range of the red lines, indicating that only a narrow range of AOAs can be measured in the system.

However, the difference \( \theta_{E1} - \theta_{E2} \) constantly changes [40]; when plotted, it had unambiguous step-like sections with the observation range of \( \theta_E \) shown in Fig. 3(b). For each section, a static offset was defined. There are nine ‘steps’ in total, numbered from left to right, and the specific data are shown in Table III. Based on this, \( \varphi_{2z} \) can be divided into nine segments,

#### Table III

<table>
<thead>
<tr>
<th>Step</th>
<th>Range of ( \theta_{E1} - \theta_{E2} ) (°)</th>
<th>Range of ( \theta_E ) (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[6.38, 6.53]</td>
<td>[–90, –56.5]</td>
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<tr>
<td>2</td>
<td>(25.73, 27.33)</td>
<td>(–56.5, –34)</td>
</tr>
<tr>
<td>3</td>
<td>(–40.09, –39.66)</td>
<td>(–34, –30.5)</td>
</tr>
<tr>
<td>4</td>
<td>(–20.41, –19.19)</td>
<td>(–30.5, –9.2)</td>
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<tr>
<td>5</td>
<td>0</td>
<td>(–19.2, 9.5)</td>
</tr>
<tr>
<td>6</td>
<td>(19.19, 20.41)</td>
<td>(9.5, 30.5)</td>
</tr>
<tr>
<td>7</td>
<td>(39.66, 40.09)</td>
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</tr>
<tr>
<td>8</td>
<td>(–27.33, –25.73)</td>
<td>(34, 56.5)</td>
</tr>
<tr>
<td>9</td>
<td>(–6.53, –6.38)</td>
<td>(56.5, 90)</td>
</tr>
</tbody>
</table>

![Fig. 3](image3.png)

**Fig. 3.** Step-like phase difference in the vertical direction. a) \( \theta_{E1} \), b) \( \theta_{E2} \), c) \( \theta_{E1} - \theta_{E2} \).

![Fig. 4](image4.png)

**Fig. 4.** Step-like phase difference in the horizontal direction when a) \( \theta_E = 20^\circ \), b) \( \theta_E = 40^\circ \), c) \( \theta_E = 58^\circ \), and d) \( \theta_E = 75^\circ \).
there are at most 9 steps when $\theta_i$ ranges from $-33.5^\circ$ to $33.5^\circ$, and there are at least 3 steps when $\theta_i$ ranges from $-90^\circ$ to $-60^\circ$ and $60^\circ$ to $90^\circ$, as shown in Table IV. However, $\varphi_{2x0}$ can be restored with (10) regardless of the range for $\theta_i$.

$$
\varphi_{2x0} = \begin{cases} 
\varphi_{2x} + 2 \left( \frac{S_{\text{at}}}{2} - \frac{S_{\text{at}}}{2} + 1 \right) \pi, & S_{\text{at}} \in \left[ \frac{S_{\text{at}}}{2}, \frac{S_{\text{at}}}{2} - t \right], \\
\varphi_{2x} + 2 \left( \frac{S_{\text{at}}}{2} - \frac{S_{\text{at}}}{2} - 1 \right) \pi, & S_{\text{at}} \in \left[ \frac{S_{\text{at}}}{2} + t, \frac{S_{\text{at}}}{2} \right]. 
\end{cases}
$$

(10)

with $h = 1, 2, \ldots, S_{\text{at}},$ where $S_{\text{at}}$ indicates the total number of ‘steps’ for a specific $\theta_i$, and $S_{\text{at}}$ indicates the step number to which $\theta_{12} - \theta_{11}$ belongs for a specific $\theta_i$.

IV. CALIBRATION TECHNIQUE

Calibration is an essential step in the measurement system because the impact of imperfections in various components on the measurement results can be catastrophic. Fortunately, the errors caused by nonideal devices are systematic errors. Thus, these errors are constant and measurable [39].

A. Systematic Errors Mathematical Model

The AoA detection system in this paper, shown in Fig. 5, consists of eight antennas, a 16-port interferometer, eight Schottky detectors, eight operational amplifiers (OpAmps) and a digital processing unit with a micro control unit (MCU) and a PC interface. $\gamma_{\text{in}}$ and $\gamma_{\text{out}}$ are the amplitude and phase deviations caused by the antennas and the transmission line between the antenna and the interferometer, respectively. $\gamma_{\text{out}}$ and $\gamma_{\text{out}}$ indicate the deviation from the ideal amplitude and phase difference in the 16-port interferometer from $P_{\text{in}}$ to $P_{\text{out}}$, respectively. $K_{\text{out}}$ denotes the voltage gain of the detector. $G_{\text{out}}$ is the gain of the operational amplifier.

A receiver with LNAs would improve the noise figure and the signal-to-noise ratio (SNR), and hence improve measurement precision [42]. However, this receiver needs at least eight LNAs, increasing the system’s cost and volume. In this paper, a method that improves the power of an analog signal generator was selected to ensure that the OpAmps detect the signal, and a system error mathematical model was established to improve measurement precision.

The error model can be determined from the main algorithm:

Table IV

<table>
<thead>
<tr>
<th>Range of $\theta_i$ (°)</th>
<th>$\theta_i$ (°)</th>
<th>Step $\varphi_{2x,h}$ Range of $\varphi_{2x,h}$ (°)</th>
<th>Range of $\theta_i$ (°)</th>
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<td>[-33.5, 33.5]</td>
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<td>No. 2</td>
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<td>[-62.5, -36.5]</td>
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<td>[-36.5, -32.5]</td>
</tr>
<tr>
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<tr>
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<td>[10.0, 21.9]</td>
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<td></td>
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<td></td>
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<tr>
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<td>No. 3</td>
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<tr>
<td></td>
<td>No. 5</td>
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<td>[-12.5, 41.0]</td>
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<td></td>
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<td>[41.0, 46.5]</td>
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</tr>
<tr>
<td></td>
<td>No. 3</td>
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<td>(36.7, 52.2]</td>
<td>[18.0, 70.5]</td>
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<tr>
<td></td>
<td>No. 5</td>
<td>(89.2, 105)</td>
<td>[70.5, 90.0]</td>
</tr>
<tr>
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</tr>
<tr>
<td>(60, 90]</td>
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<td>[-40.5, 40.0]</td>
</tr>
<tr>
<td></td>
<td>No. 3</td>
<td>(80.2, 106.7)</td>
<td>[40.0, 90.0]</td>
</tr>
</tbody>
</table>

and the unambiguous phase difference $\varphi_{2x,0}$ can be calculated by (9).

$$
\varphi_{\text{at}} = \begin{cases} 
\varphi_{2x} + 2 \left( \frac{S_{\text{at}}}{2} - \frac{S_{\text{at}}}{2} + 1 \right) \pi, & S_{\text{at}} \in \left[ \frac{S_{\text{at}}}{2}, \frac{S_{\text{at}}}{2} - t \right], \\
\varphi_{2x} + 2 \left( \frac{S_{\text{at}}}{2} - \frac{S_{\text{at}}}{2} - 1 \right) \pi, & S_{\text{at}} \in \left[ \frac{S_{\text{at}}}{2} + t, \frac{S_{\text{at}}}{2} \right]. 
\end{cases}
$$

(9)

with $e = 1, 2, \ldots, S_{\text{at}},$ where $S_{\text{at}}$ indicates the total number of ‘steps’, $S_t$ indicates the step number to which $\theta_{11} - \theta_{12}$ belongs, and $t$ is a positive integer related to $v$: $t = 1$ when $v$ ranges from 1 to 3 and $t = 2$ when $v$ ranges from 3 to 5. The symbol ‘$<$’ denotes rounding.

B. Unambiguous Azimuth $\theta_i$

The step-like diagrams in the horizontal direction are different at different vertical angles $\theta_i$:

$$
\theta_i = \sin^{-1} \left[ \frac{\sin \theta_i}{\sin \phi_i \cos \theta_i} \right]
$$

is the detected horizontal azimuth when no angle-ambiguity-improving approach is used. The steps in the horizontal direction $\theta_{12} - \theta_{11}$ for some typical values of $\theta_i$ (20°, 40°, 58°, 75°) are shown in Fig. 4. There are four kinds of step numbers based on different $\theta_i$:
for angle detection in Section II. C. The parameters that consider systematic error are represented by adding ‘~’ to the symbols:

$$\bar{b}_{\text{out}} = AF_{\text{out}}K_{\text{out}} \sum_{i=1}^{n} a_{\text{out},i} e^{j(\beta_{\text{out},i}+\gamma_{\text{out},i})} S_{\text{out},i} \tag{11}$$

Then, the output power $\bar{B}_{\text{out}}$ after the OpAmps with error factors can be expressed as (12).

$$[\bar{B}_{\text{out},1}, \ldots, \bar{B}_{\text{out},n}]^T = E X \tag{12}$$

where $X$ is the phase vector shown and $E = [\bar{e}_{9}, \bar{e}_{10}, \ldots, \bar{e}_{16}]^T$ is the error matrix. Each element in $E$ is a different combination of system errors: $\alpha_{\text{in}}$, $\gamma_{\text{in}}$, $\alpha_{\text{out},\text{in}}$, $\gamma_{\text{out},\text{in}}$, $G_{\text{out}}$, and $K_{\text{out}}$. Given that they are constant errors, the combinations could be ignored and every element of $E$ is treated as an unknown number here.

Parameters in (12) are shown in (14) in Appendix B.

The values for the elements of $E$ can be obtained after several calibration measurements when the angle of arrival is known based on the least squares technique (LS). For example, the estimated error vector $\bar{e}_{9}$ of the 9th channel would be obtained after several measurements when the angles of each premeasured value form a phase matrix $\bar{X}_{9}=[X_{9,1}, X_{9,2}, \ldots, X_{9,n}]^T$ for $P_{9}$, where $n$ is the number of calibration measurements. $\bar{X}_{9}$ and $\bar{B}_{9,1}, \bar{B}_{9,2}, \ldots, \bar{B}_{9,n}$ are known parameters.

After estimating the error vector, $\phi_{1x}$, $\phi_{2x}$, $\phi_{1z}$, and $\phi_{2z}$ can be obtained by solving (12).

V. EXPERIMENTAL RESULTS

A prototype that achieved unambiguous 2-D AoA estimations was fabricated, as shown in Fig. 6. This prototype was an eight-layer board with four 0.254 mm Rogers RO4350B substrates.

A. Antenna

The patch antennas shown in Fig. 6(a) were fabricated on the first copper layer (L1). The numbers next to the elements correspond to the port of the interferometer; for example, antenna ① is connected to input port $P_{1}$ of the interferometer. Eight meandering lines with the same length were used to maintain the same phase difference between the receiving signal from the antenna element and the associated input port of the multiport network. The antenna simulation and measured results are shown in Fig. 7. A −10 dB bandwidth was achieved from 30.6 to 31.2 GHz. The radiation at 31 GHz in the E-plane was larger than that in the H-plane: a 120° 3 dB beamwidth was achieved in the E-plane, while a 100° 3 dB beamwidth was achieved in the H-plane. To obtain high reliability data, the system measured the AoA from −30° to 30°.

B. 16-port Interferometer

The 16-port interferometer shown in Fig. 8(b) was fabricated on the 5th copper layer (L5). The 3 λ / 4 section of the 180° RRCplr was arranged inside the coupler for miniaturization, which is different from the classic structure. Fig. 9 shows the simulation and measured results of the amplitude and phase components of the 16-port interferometer scattering matrix. The phase differences at 30.5 GHz were approximately 45°, 90°.
135°, 180°, 225°, 270°, and 315°. For example, \(\angle S_{4,13} - \angle S_{1,9} = 45°\), \(\angle S_{1,11} - \angle S_{1,9} = 90°\), \(\angle S_{1,10} - \angle S_{1,9} = 135°\), \(\angle S_{3,10} - \angle S_{1,9} = 180°\), \(\angle S_{3,9} - \angle S_{1,9} = 225°\), \(\angle S_{1,11} - \angle S_{1,9} = 270°\), \(\angle S_{1,13} - \angle S_{1,9} = 315°\).

C. Detector

A SMS7630-061 silicon, zero bias Schottky detector diode was used for the detector circuit, as shown in Fig. 10. As shown in Fig. 11, the simulation and the measurement produced similar results. The detected voltage was linear for input powers ranging from –30 dBm to –10 dBm. Thus, this detector was acceptable to be used in the proposed system.

Fig. 8. (a) Top view of the PCB’s interferometer area. (b) Structure of the layer stack. L1–L8 are all copper foil layers. (c) Layer stack of the interferometer part and the stripline-to-microstrip line transition. The two metallized via-holes beside the stripline were used to form a quasi-coaxial section, resulting in better impedance matching [43]. The 45° phase shifters here are realized by \(\lambda/4\) transmission lines.

Fig. 9. Simulation and measured results of the 16-port interferometer. (a) Return loss and transmission characteristics; (b) Eight kinds of phase differences between different pairs of input signals at each of the eight output ports. Other S-parameters not shown here have similar characteristics due to the symmetry of the interferometer. The simulation was carried out in HFSS.

Fig. 10. Detector circuit based on a SMS7630-061 silicon, zero bias Schottky detector diode.

Fig. 11. Simulated and measured detected voltage \(v_d\) versus RF power at 30 GHz: the y-coordinates of the blue lines have a base-10 logarithmic scale; the y-coordinates of the red lines have a linear scale.
D. Other Parts

The remaining parts were on the 8th copper layer (L5). A THS4551 OpAmp was used to achieve 90 V/V signal amplification with signal-ended differential gain. A STM32L476 microcontroller was used to process the received signal, which includes analog-to-digital conversion and simple calculations. The FT230X is a USB-to-serial UART interface that was used to transmit data from the MCU to the laptop. The whole board, including the antennas, was designed by the author.

E. Link Budget

The minimum input voltage of the OpAmps in this prototype was 0.1 mV. This means that differences in detector output voltages greater than 0.1 mV would be favorable. In that case, based on a system-level simulation by Keysight ADS, the signals entering the 16-port interferometer input ports should be greater than -30 dBm. The free space loss is calculated by: 
\[
-57.8 \text{ dB} = 92.4 + 20 \log f + 20 \log R \,[44].
\]
where \( R = 6 \times 10^{-4} \) is the distance between the receiver and the Tx antenna in kilometers. Therefore, a 5 dBm transmitted signal before a 25 dBm horn Tx antenna is sufficient for measurement.

F. Measurement

The test bed is shown in Fig. 12. The best system performance upon calibration was obtained at 31 GHz, which is higher than the designed frequency of 30 GHz. This is mainly because the dielectric constant and thickness of the multilayer substrate after the actual production process are lower than the design values. A KEYSIGHT E8257D PSG analog signal generator was used to generate a 31 GHz signal with a 5 dBm analog signal. A RIGOL DP832 power supply was used to supply a 3.3 V voltage. A HENGDA MICROWAVE HD-320SGAH25 K horn antenna with a 25 dB gain was used as the Tx antenna, which was connected to an FUYU three-axis linear workbench to simulate signals in any direction.

The unambiguous AoA detection range is –90° to 90° in theory. Given that the beamwidth of the antennas used here is narrow, the highly reliable beamwidth of this systems is 60°. Thus, in the range of –30° to 30°, the measurement was performed at 1° step intervals. The test bed moved both horizontally and vertically.

The measurement process is shown in Fig. 13. Some calibration measurements were performed first to obtain the error matrix \( \mathbf{E} \), which was calculated with the PC. The calibration must be done once and then be used for ever because of the constant system errors. In the formal test, the DC raw data at the output of the detectors were amplified by the OpAmps and received by the ADCs of the STM32 microcontroller. The MCU solved (13) and (14) to obtain \( \theta_E \), \( \phi_1x \), \( \phi_2x \), and \( \phi_1z \). After that, the unambiguous azimuth \( \theta_E \) and altitude \( \theta_H \) were estimated by (8), (9), and (10).

Finally, the measurement results and the mean squared error are shown in Fig. 14. (a) Altitude of unambiguous 2-D AoA system; (b) Azimuth of unambiguous 2-D AoA system.
In this table, ‘±’ means the range, e.g. ‘±3’ where cut from 3.8° to 0.4° by calibration. This demonstrates the results after calibration are blue lines, and the solid red lines the incoming wave angle with the different phase of fset algorithm is described first. Then, an ambiguity-resistant the measured results are approximately 0.4°. Table V compares the beamwidth of the receiving antenna. The MSE values of directions. The system measures the range from –30° to 30° due

<table>
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<th>Frequency (GHz)</th>
<th>[34]</th>
<th>[37]</th>
<th>[39]</th>
<th>This paper</th>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Port number of interferometer</td>
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<td>6</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>Enabled AoA dimension</td>
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<td>2</td>
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<tr>
<td>Number of baselines</td>
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<td>4</td>
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<td>11.2/37.2</td>
<td>5.53/9.30</td>
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<td>±60</td>
<td>±5</td>
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<tr>
<td>MSE</td>
<td>/</td>
<td>/</td>
<td>0.2</td>
<td>0.4</td>
</tr>
</tbody>
</table>

In this table, ‘±’ means the range, e.g. ‘±3’ means ‘from –3 to 3.’

(VI). Conclusion

An unambiguous 2-D AoA estimation receiver operating at 31 GHz is demonstrated in this work. The receiver determines the incoming wave angle with the different phase offset characteristics of the ports, which are caused by the 16-port interferometer proposed in this paper. The 2-D AoA estimation algorithm is described first. Then, an ambiguity-resistant method is introduced. This allows for greater flexibility in antenna spacing and size; theoretically, the unambiguous angle range is from –90° to 90° in both the horizontal and vertical directions. The system measures the range from –30° to 30° due to the beamwidth of the receiving antenna. The MSE values of the measured results are approximately 0.4°. Table V compares

the receiver presented in this work with AoA estimation MPRs in the main references.

In future work, an interferometer with more ports and a wider bandwidth can be designed, and the emission source can be integrated into the system to achieve multitarget AoA estimation. This kind of multiple distributed MPR can be used to perform more complex functions, such as multitarget tracking [45]. In addition, a wideband interferometer can provide absolute measurements of the target distance.

APPENDIX A

The output powers of the 16-port interferometer without the error factor can be expressed as

\[ \mathbf{B} = \begin{bmatrix} B_1, B_2, \ldots, B_{16} \end{bmatrix} = a^2 \mathbf{E} \mathbf{X}, \quad \mathbf{X} = [I, \mathbf{x}_1, \mathbf{x}_2]^T \]  

(13a)

where

\[ \mathbf{x}_1 = \begin{bmatrix} \cos \Delta \phi_1, \cos \Delta \phi_2, \ldots, \cos \Delta \phi_{13} \end{bmatrix} \]

\[ \mathbf{x}_2 = \begin{bmatrix} \sin \Delta \phi_1, \sin \Delta \phi_2, \ldots, \sin \Delta \phi_{13} \end{bmatrix} \]

(13b)

\[ \mathbf{\phi} = \begin{bmatrix} \phi_1, \phi_2, \ldots, \phi_8 \end{bmatrix} = \begin{bmatrix} \beta_1 \sin \theta_e, & \beta_2 \sin \theta_e, & \ldots, & \beta_8 \sin \theta_e \end{bmatrix} \]

(13c)

\[ \mathbf{B} = \begin{bmatrix} B_1, B_2, \ldots, B_{16} \end{bmatrix} = a^2 \mathbf{E} \mathbf{X}, \quad \mathbf{X} = [I, \mathbf{x}_1, \mathbf{x}_2]^T \]  

(13e)

APPENDIX B

The output powers after the OpAmps with error factor can be expressed as

\[ \mathbf{\tilde{B}} = \begin{bmatrix} \tilde{B}_1, \tilde{B}_2, \ldots, \tilde{B}_{16} \end{bmatrix} = \mathbf{\tilde{E}} \mathbf{X} \]  

(14a)

where

\[ \mathbf{\tilde{E}} = \begin{bmatrix} \tilde{e}_1, \tilde{e}_2, \ldots, \tilde{e}_{16} \end{bmatrix} \]

\[ \mathbf{\tilde{e}} = \begin{bmatrix} [1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] \end{bmatrix}^T \]

(14b)
and the symbol ‘o’ indicates the Hadamard product.

Each unit in $\tilde{E}$ is a different combination of system errors $a_m$,

\[ \tilde{e}_{1,n} = \tilde{e}_{2,n} = \tilde{e}_{3,n} = \tilde{e}_{4,n} = \tilde{e}_{5,n} = \tilde{e}_{6,n} = 0, \]

and $\tilde{e}_{i,n}$ is not exist, \( i = 1, 2, \ldots, 6 \).

\[ \tilde{e}_{7,n} = 4, \tilde{e}_{8,n} = 0, \tilde{e}_{9,n} = \cos(\gamma_1 - \gamma_2 - \gamma_3), \]

\[ \tilde{e}_{10,n} = \tilde{e}_{11,n} = \tilde{e}_{12,n} = \tilde{e}_{13,n} = \tilde{e}_{14,n} = \tilde{e}_{15,n} = \tilde{e}_{16,n} = 0. \]

\[ (14c) \]

REFERENCES


