Adaptive Day-Ahead Prediction of Resilient Power Distribution Network Partitions

Chinmay Shah ¹ and Richard Wies ²

¹University of Alaska Fairbanks
²Affiliation not available

October 30, 2023

Abstract

The conventional power distribution network is being transformed drastically due to high penetration of renewable energy sources (RES) and energy storage. The optimal scheduling and dispatch is important to better harness the energy from intermittent RES. Traditional centralized optimization techniques limit the size of the problem and hence distributed techniques are adopted. The distributed optimization technique partitions the power distribution network into sub-networks which solves the local sub problem and exchanges information with the neighboring sub-networks for the global update. This paper presents an adaptive spectral graph partitioning algorithm based on vertex migration while maintaining computational load balanced for synchronization, active power balance and sub-network resiliency. The parameters that define the resiliency metrics of power distribution networks are discussed and leveraged for better operation of sub-networks in grid connected mode as well as islanded mode. The adaptive partition of the IEEE 123-bus network into resilient sub-networks is demonstrated in this paper.
Abstract—The conventional power distribution network is being transformed drastically due to high penetration of renewable energy sources (RES) and energy storage. The optimal scheduling and dispatch is important to better harness the energy from intermittent RES. Traditional centralized optimization techniques limit the size of the problem and hence distributed techniques are adopted. The distributed optimization technique partitions the power distribution network into sub-networks which solves the local sub problem and exchanges information with the neighboring sub-networks for the global update. This paper presents an adaptive spectral graph partitioning algorithm based on vertex migration while maintaining computational load balanced for synchronization, active power balance and sub-network resiliency. The parameters that define the resiliency metrics of power distribution networks are discussed and leveraged for better operation of sub-networks in grid connected mode as well as islanded mode. The adaptive partition of the IEEE 123-bus network into resilient sub-networks is demonstrated in this paper.

Index Terms—Resiliency, Adaptive System, Graph Partitioning, Spectral Clustering Analysis, Active Distribution Network, Distributed Energy Resources.

I. INTRODUCTION

Power distribution networks are undergoing a great transformation due to high penetrations of distributed energy resources (DERs) and energy storage [1]. The apt use of these DERs and energy storage can improve the network resiliency as well as reduce dependence on the conventional resources to meet the demand and thereby reduce the generation cost. The objective of optimal power flow (OPF) - economic dispatch is to minimize the total generation cost while satisfying the distribution network constraints and device operating limits [2]. Solving the OPF problem in a traditional centralized manner limits the size of the problem that can be solved since it becomes computationally intensive. Several distributed optimization algorithms have been developed in the recent years to solve the OPF problem and are discussed in [3] - [7]. In order to solve the distributed optimization problem, the power distribution network is partitioned into sub-networks which solve their local sub-problems and exchange information with the adjacent sub-network for the global update. The convergence speed of the distributed optimization algorithm is dependent on the low-coupling, synchronization, and load balance between the sub-networks.

Several methods have been proposed over the years for optimal partitioning of the power distribution network. In [8], the spectral clustering and mixed integer programming methods have been used for partitioning. The graph laplacian for the spectral clustering is tabulated from the admittance matrix of the power grid. The graph partitioning method based on graph reduction and spectral clustering for power distribution networks is discussed in [9]. The authors in [10] have integrated a self organizing map algorithm with spectral clustering with a goal of tackling problems like optimal number of clusters, multi-objective partitioning, and big-data. The multi-objective graph partitioning with a goal of minimizing the active and reactive power imbalance within each sub-network is presented in [11], and [12]. The criteria for the size of the cluster/sub-network is not taken into consideration in these papers. Minimizing the deviation in the size of sub-networks is important to maintain synchronization while solving the distributed optimization problem. The hybrid K-means/evolutionary algorithm in [13], and particle swarm optimization algorithm in [14] based on electrical distance considers the cluster size index in the fitness function for partitioning the power distribution network. The authors in [15] introduces the power mismatch index (PMI) in K-nearest neighbor clustering algorithm based on electrical distance for power grid partitioning to improve the system resiliency. The mixed integer linear program is formulated to maximize the critical loads to be picked up while satisfying the operational constraints for power grid partitioning in [16] to form resilient sub-networks.

The power distribution networks are assumed to be static for all the partitioning algorithms discussed above. In the real world, power distribution networks are dynamically changing due to the high penetration of intermittent renewable energy resources, energy storage, and unbalanced non-linear loads. As a result the power distribution network needs to be re-partitioned periodically in order to maintain the optimal partitions. The quota based iterative vertex migration algorithm for optimal re-partitioning of dynamic graphs discussed in [17], and [18] is adopted in this paper for adaptive partitioning of the power network.
distribution network. The main aim of adaptive partitioning in this research work is to make optimal power distribution network partitions that are low coupling, can synchronize with each other while solving the distributed optimal power flow problem, and are more resilient.

For this research work, the initial partitioning of the power distribution network is conducted via spectral clustering. After initial partitioning, the boundary nodes/buses migrates from current position to the neighboring partitions at each iteration to obtain the equivalent power balance ratio (PBR) for each partition while keeping the computational load of the partition balanced. The contributions of this paper are as follows:

1) The proposed algorithm calculates the day-ahead PBR for each power distribution network partition with one hour time resolution based on load and renewable energy resource generation forecast.

2) The proposed algorithm migrates nodes from one power distribution network partition to the adjacent one based on iterative vertex migration in such a way that each partition has an equivalent PBR while keeping the partitions computationally balanced. The balanced partition will help with synchronization while solving the power flow problem in a distributed fashion. The partitions are generated for day ahead with one hour resolution.

3) The proposed algorithm in this research work also quantifies resiliency of each power distribution network partition to ensure it is able to adapt to abrupt and new operating conditions.

The rest of the paper is structured as follows: The Section II describes in brief about the spectral clustering analysis, power balance ratio (PBR), iterative vertex migration, and quantifying resiliency. The implementation of the proposed algorithm is explained in Section III, followed by case study and simulation results in Section IV. Section V concludes the paper stating the benefits of the proposed method.

II. PROBLEM FORMULATION

In this section, the definitions and methods are explained in brief before the implementation of the adaptive partitioning algorithm is presented.

A. UNNORMALIZED SPECTRAL CLUSTERING

The power distribution network is presented as an undirected graph \( G = (V, E) \) with \( V = \{v_1, v_2, ..., v_n\} \) as a set of buses/vertices and \( E \) as a set of branches/edges. The graph is assumed to be weighted and hence weighted adjacency matrix \( W = (w_{ij})_{i,j=1,2, ..., n} \), weight \( w_{ij} \geq 0 \) [19]. The weighted adjacency matrix \( W \) for the power distribution network is tabulated from the admittance matrix \( Y \) as:

\[
\begin{bmatrix}
0 & Y_{1,2} & \cdots & Y_{1,n} \\
Y_{1,2} & 0 & \cdots & Y_{2,n} \\
\vdots & \vdots & \ddots & \vdots \\
Y_{n,1} & Y_{n,2} & \cdots & 0
\end{bmatrix}
\]

The admittance matrix \( Y \) represents how tightly the buses are connected to each other. If the buses are not connected then the admittance is zero \( (Y_{ij} = 0) \). Now, the degree of a vertex \( v_i \) is defined as

\[
d_i = \sum_{j=1}^{n} w_{ij}
\]

The degree matrix \( D \) is the diagonal matrix with degrees \( d_i \) defined in (2) on the diagonal. The unnormalized graph laplacian \( L \) is defined as

\[
L = D - W
\]

The eigenvectors and eigenvalues of the Laplacian matrix \( L \) are tabulated by the following equation

\[
Lv_i = \lambda_i v_i
\]

\( L \) has \( n \) non-negative, real-valued eigenvalues \( 0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n \). The last step of the unnormalized spectral clustering method is to extract the cluster of buses using the k-means algorithm. The \( k \) number of clusters can be derived by supplying eigenvectors corresponding to the first \( k+1 \) smallest eigenvalues thereby generating the k-area partitioning \( \Pi_{k} \) of the power distribution network.

B. POWER BALANCE RATIO

The resiliency of the real power grid is primarily dependent on the capacity of the power generation to satisfy the demand. The power balance ratio (PBR) is defined as the ratio of total load demand to the overall power generation capacity of the grid (5).

\[
PBR = \frac{\sum_{i=1}^{n} (P_L)_i}{\sum_{i=1}^{n} (P_{GE})_i}
\]

\( (P_L)_i \) is the average hourly load at bus \( i \), \( (P_{GE})_i \) is the total power generation capacity and available energy storage at bus \( i \), and \( n \) is the total number of buses in a power distribution network.

The day ahead PBR prediction of each initial power distribution network partition or sub-network with time resolution of one hour can be calculated from (5) and is given by

\[
PBR(t) = \frac{\sum_{i=1}^{n} P_{Li}(t)}{\sum_{i=1}^{n} P_{GEi}(t)} \quad t = 1, 2, 3, \cdots , 24h
\]

\( P_{Li}(t) \) is the average hourly load forecast at time \( t \), \( P_{GEi}(t) \) is the total power generation forecast for DERs and available energy storage at time \( t \).

C. ITERATIVE VERTEX MIGRATION

The power distribution network is a dynamic graph in which the characteristics of the vertices (buses) can change over time based on load and generation forecast. Let \( \Pi(t) \) be the set of power distribution network partitions at time \( t \) comprising of the vertices/buses \( V \). \( \Pi_x(t) \subseteq \Pi(t) \) is an individual partition \( x \) such that \( \Pi_x(t) \cap \Pi_y(t) = \phi \) for \( x \neq y \).

At time \( t = 0 \), the power distribution network is loaded with the initial partition \( \Pi_0 \) such that each bus in the network has an assigned partition. On each iteration at time \( t > 0 \) after the initial partitioning, each bus/vertex will either remain in the current partition or migrate to a neighboring partition based
on the set of predefined objectives. The objectives that need to be satisfied are:

1) Balanced computational load among the local controller of all the partitions.
2) Minimized coupling between the partitions.
3) Minimized difference between the PBR of each partition \( PBR_x \) and the entire network \( PBR_{sys} \).

The power distribution network partitioning is said to be computationally balanced if

\[
\sum_{x=1}^{k} \frac{\nu(\Pi_x(t))}{k} - \varepsilon \leq \nu(\Pi_x(t)) \leq \sum_{x=1}^{k} \frac{\nu(\Pi_x(t))}{k} + \varepsilon \quad (7)
\]

where \( \nu(\Pi_x(t)) \) is the number of buses/vertices in the partition \( \Pi_x(t) \), \( \varepsilon \) is the user-defined imbalance tolerance, and \( k \) is the total number of partitions of the power distribution network. To achieve balanced partitioning, we carry out quota-based bus/vertex migration allowing only a limited number of buses/vertices to be migrated from the overloaded partition \( \Pi_x(t) \) to the neighboring underloaded ones \( \Pi_y(t) \) and is given by

\[
\text{quota}[x][y] = \max\{0, \min\{Q(\Pi_x(t)), -Q(\Pi_y(t))\}\} \quad (8)
\]

where

\[
Q(\Pi_x(t)), Q(\Pi_y(t)) = \nu(\Pi_x(t)) - \nu_{\max}(\Pi_x(t))
\]

\[
\nu_{\max}(\Pi_x(t)) = \max\{\sum_{v=1}^{k} \nu(\Pi_v(t))\}. \quad (9)
\]

The algorithm while migrating the number of buses/vertices \( q \) from \( \Pi_x(t) \) to \( \Pi_y(t) \) will attempt to minimize the difference between the PBR of each overloaded or underloaded partition and the entire network and is defined by

\[
\min \sum_{i=1}^{T} [PBR_{sys}(t) - PBR_{x,y}(t)], \quad (10)
\]

s.t. \( PBR_{x,y}(t) = \frac{\sum_{i=1}^{n} P_{Li}(t) + P_{load}}{\sum_{i=1}^{n} P_{GEi}(t) + P_{gen}}, \quad (11) \)

\[
t = 1, 2, 3, \ldots, T = 24h \quad (12)
\]

\[
0 < P_{load} \leq \sum_{i=1}^{q} P_{Li}(t) \quad (13)
\]

\[
0 < P_{gen} \leq \sum_{i=1}^{q} P_{GEi}(t) \quad (14)
\]

The algorithm will iterate until all the partitions are balanced, i.e. satisfy (7).

D. Resiliency

There are numerous definitions of resiliency for power distribution networks. But the resiliency as a function of time duration of an event and the number of loads affected by an event defined in [20] and [21] is considered in this research work. The resiliency metric to capture these two factors in power distribution network partitions for the forecasted weather or load surge event of duration \( 10^3 \)s is defined in Fig. 1, where \( A \) is the variable that stores the resilience value \( R \). The measured resilience metric, \( R \), will indicate how well the system is prepared for an upcoming event.

\[
R = 10^3 \text{ Seconds}
\]

To calculate the resilience metric of the system, the fraction of the load \( FL \) in a power distribution network partition that is unaffected by the forecasted event is given by

\[
FL = \frac{\text{Load unaffected by the forecasted event (kW)}}{\text{Total load of the partition (kW)}} \quad (16)
\]

The second step is to compute the unscaled resiliency metric \( A' \) for an event that lasts for \( \tau \) seconds. The equation to compute \( A' \) is

\[
A' = b(\tau + e^{FL})(1 + FL) \quad (17)
\]

where \( b \in \{0, 1\} \) is a binary variable that indicates whether an event happened or not and, \( FL \) is the fraction of unaffected load by the event. The minimum and maximum values of \( A' \) during an event for each hour will be 4.6 and 12.6, corresponding to all of the load or none of the load affected, respectively [22]. The scaled resilience metric \( A \), corresponds to the values of unscaled resiliency metric \( A' \), shown in TABLE I. If there is no event forecasted, the resilience metric \( A \) will be forced to 0.

<table>
<thead>
<tr>
<th>TABLE I</th>
</tr>
</thead>
<tbody>
<tr>
<td>RESILIENCE METRICS</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>Low Resilience</td>
</tr>
</tbody>
</table>

III. ALGORITHM IMPLEMENTATION

The distributed optimal power flow problem is solved by partitioning the power distribution network into sub-networks, where the local controller solves the problem for each sub-network and coordinates with the neighboring sub-networks to solve for the global update. The power distribution grid is partitioned initially by the unnormalized spectral clustering algorithm. The data relating to each partition/sub-network is assigned to the corresponding local controller as shown in Fig. 2.

Once the initial partitions are assigned to the local controllers, the re-partitioning vertex migration algorithm is executed to make the day ahead partition prediction with a
time resolution of one hour and is updated every hour. The algorithmic flowchart is presented in Fig. 3 and the steps are shown below.

**Algorithm:** Day ahead power distribution network partition prediction

1. Obtain the admittance matrix of the network at $t = 0$
2. Obtain initial partitions using spectral clustering.
3. Calculate the day ahead PBR of the entire network using eq. (6)
4. For $t \leq T = 24h$
5. if Algorithm identifies a partition pair then:
   6. Obtain the number of buses and the PBR of both the partitions $\nu(\Pi_x(t))$ & $\nu(\Pi_y(t))$ in a partition pair and go to Step 9.
   7. else:
      8. Calculate the resiliency of each partition and go to Step 14
   9. if both the partitions in the pair satisfy eq. (7) OR one partition satisfies eq. (7) and the other partition is overloaded or underloaded with PBR equivalent to the network, then:
      10. Go to step 5 for a new partition pair.
   11. else:
      12. Calculate the number of buses quota $x[y] = \max\{0, \min\{Q(\Pi_x(t)), -Q(\Pi_y(t))\}\}$ to be migrated from partition $\Pi_x(t)$ to $\Pi_y(t)$.
      13. Migrate the buses to the new partition and than update the partitions and go to Step 9. If both partitions are either overloaded or underloaded go to Step 5.
      14. $t = t + 1$
      15. Go to Step 4.
6. End

Once the day ahead prediction is made by the algorithm, the vertices/buses are actually migrated to the new partition as shown in Fig. 4. The migration of the vertices/buses is performed only once at the end of each hour. The proposed algorithm is tested for the IEEE 123-bus system and is presented in the next section along with the results.
of the PV and battery energy storage is shown in TABLE II. Initially three area partitioning of the IEEE 123-bus network is obtained by the unnormalized spectral clustering algorithm as shown in Fig. 5.

Now the algorithm calculates the day ahead PBR for the entire IEEE 123-bus network based on the load and solar forecast, and availability of energy storage. Next, the algorithm identifies all the partition pairs from the initial three partitions generated by spectral clustering and calculates the number of buses and PBR in each partition for the first hour (t = 1h). Initial partitions Π₁(t) and Π₂(t) have 35 buses and Π₃(t) has 53 buses and PBR is 1. For each partition pair, the algorithm checks if both partitions satisfy (7) and if the PBR is approximately equal to that of the entire IEEE 123-bus network. If any one or both of the partitions do not satisfy this criteria, the algorithm calculates the quota of vertices/buses that need to be migrated and is given by:

\[
\text{quota}(\Pi_1(t))|\Pi_2(t)| = \max\{0, \min\{-6, 6\}\} = 0 \quad (18)
\]

\[
\text{quota}(\Pi_3(t))|\Pi_1(t)| = \max\{0, \min\{6, 6\}\} = 6 \quad (19)
\]

\[
\text{quota}(\Pi_3(t))|\Pi_2(t)| = \max\{0, \min\{6, 6\}\} = 6 \quad (20)
\]

For partition pair (Π₁(t), Π₂(t)), it can be seen from (18) that none of the buses will be migrated since both the partitions are underloaded. A maximum of six buses can be migrated from Π₃(t) to Π₁(t) and Π₂(t) each for the partition pair (Π₁(t), Π₃(t)) and (Π₂(t), Π₃(t)) as calculated in (19) and (20), respectively. Based on the results generated by the optimization problem defined by (10), all 6 boundary buses are migrated to Π₁(t) and Π₂(t) and the partitions are updated accordingly. So at t = 1h the prediction is to migrate buses \{91, 92, 93, 94, 95, 96\} from Π₁(t) to Π₂(t) and buses \{300, 111, 112, 113, 114, 115\} from Π₃(t) to Π₂(t). The number of buses in the new partition prediction is \(\nu(\Pi_1(t)) = 41\), \(\nu(\Pi_2(t)) = 41\), and \(\nu(\Pi_3(t)) = 41\). The partition prediction of the IEEE 123-bus test system for t = 1h is shown in Fig. 6. The algorithm iterates for t = 1h to t = 24h and generates day ahead partition predictions for the IEEE 123-bus test network. The partition prediction remains the same until hour 8, i.e. t = 8h. During the 9th hour, the PV on buses 3, 8, and 13 in partition Π₁(t) do not generate any power because of cloud coverage. As a result, in partition Π₁(t) the load is more than the PV and the available energy storage and PBR = 1.05. In order to reduce the PBR of partition Π₁(t) closer to the PBR = 1 of the entire IEEE 123-bus network, the algorithm generates a prediction to migrate the buses 89, and 90 from Π₁(t) to Π₂(t). The partitions are updated accordingly and the new partition prediction is shown in Fig. 7. The prediction remains the same until any changes are forecasted. The buses/vertices will physically migrate from one partition to the another based on the prediction only once at the end of each hour.

Once the partition prediction is generated, the algorithm will tabulate the resiliency of each partition using (16) and (17). The resilience metric R of the IEEE 123-bus network partition Π₁(t) generated by the proposed algorithm and the initial partition generated by spectral clustering analysis for the 24 hour period is shown in Fig. 8. It is clear from Fig. 8 that the resilience metric for the IEEE 123-bus network partition Π₁ is forced to zero when no event is forecasted. At hour 9 during the cloud coverage with reduced PV generation and at hour 19 during the load swelling, the partition Π₁ generated by the proposed algorithm is more

<table>
<thead>
<tr>
<th>Gen. Type</th>
<th>Buses</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV</td>
<td>3,8,13,14,15,18,21,23,25,26,27,250,36,40,44,54,57,61,67,72,78,81,89,91,93,97,101,105,108,110,300,450</td>
<td>3220 kW</td>
</tr>
<tr>
<td>Battery Energy Storage System</td>
<td>14,20,26,54,64,65,81,94,95,96</td>
<td>1855 kW and 7420 kWh</td>
</tr>
</tbody>
</table>
resilient than the initial partition $\Pi_1$ generated by spectral clustering analysis. The loads affected during hour 9 and 19 are fed by the power from the substation in grid connected mode and shed in an islanded mode. After a disastrous event, fault, or cyber-attack, if these partitions are operating in an islanded mode, the cloud coverage or load variation would affect the resiliency of the network. Therefore, the purpose of considering these events in this paper is to show that the partitions generated by the proposed algorithm are more resilient. Forming the power distribution network partitions and tabulating their resiliency when a disastrous event like a hurricane or earthquake is happening is not in the scope of this work. It will be presented in the future work.

The proposed algorithm is a part of an application, RAFTES (resiliency application for fault-tolerant energy system), which is being developed to integrate with an open-source ADMS (advanced distribution management system) platform GridAPPS-D [25].

V. CONCLUSION

In this paper, a vertex migration based adaptive spectral clustering algorithm was proposed to dynamically predict the day ahead power distribution network partition. From the results it is clear that the proposed algorithm generates the balanced partitions in terms of computational load as well as the active power. Hence it will help with the synchronization between the partitions while solving the distributed optimal dispatch problem, and also reduce the communication with the neighboring partitions to achieve active power balance. The algorithm also tabulates the resiliency of each partition. We can conclude from the results that the partitions generated by the proposed algorithm are more resilient to the forecasted event if operated in an islanded mode.

The future scope of this work is to generate more resilient power distribution network partitions in real time while a disastrous event is happening.

ACKNOWLEDGMENT

The contributions to this research work were achieved through the Grid Modernization Laboratory Consortium (GMLC). The GMLC was established as part of the U.S. Department of Energy’s Grid Modernization Initiative (GMI) to accelerate the modernization of the U.S. electricity infrastructure. The views expressed in the article do not necessarily represent the views of the U.S. Department of Energy or the United States Government.

The authors would like to thank Dr. Ron Melton, and Dr. Kevin Schneider at Pacific Northwest National Laboratory, Alaska Center for Energy and Power (ACEP), and the University of Alaska Fairbanks for providing the resources and the data needed for this research work.

REFERENCES


