Impact of the core deformation on the tidal heating and flow in Enceladus’ subsurface ocean

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Key Points:

• The heat production in the ocean depends on the ocean thickness and the material properties of the core.
• Dissipation is likely to be negligible for ocean thicknesses > 1 km but can exceed 20 GW if the ocean is thin and the core is easy to deform.
• The shallow water approach, widely used in previous studies, can lead to incorrect results regardless of ocean thickness.

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Abstract

We present a novel approach to modeling the tidal response of icy moons with subsurface oceans. The problem is solved in the time domain and the flow in the ocean is calculated simultaneously with the deformation of the core and the ice shell. To simplify the calculations, we assume that the internal density interfaces are spherical and the effective viscosity of water is equal to or greater than 100 Pa s. The method is used to study the effect of an unconsolidated core on tidal dissipation in Enceladus’ ocean. We show that the partitioning of tidal heating between the core and the ocean strongly depends on the thickness of the ocean layer. If the ocean thickness is significantly greater than 1 km, heat production is dominated by tidal dissipation in the core and the amount of heat produced in the ocean is negligible. In contrast, when the ocean thickness is less than about 1 km, tidal heating in the core diminishes and dissipation in the ocean increases, leaving the total heat production unchanged. Extrapolation of our results to realistic conditions indicates that tidal flow is turbulent which suggests that the linearized Navier-Stokes equation may not be appropriate for modeling the tidal response of icy moons. Finally, we compare our results with those obtained by solving the Laplace tidal equations and discuss the limitations of the two-dimensional models of ocean circulation.

Plain Language Summary

The origin of the heat powering Enceladus’ geological activity and preventing its ocean from freezing has been debated since the discovery of a plume of icy particles above Enceladus’ south pole in 2005. Here, we evaluate the heat generated by tides in Enceladus’ ocean assuming that the internal density interfaces are spherical and the flow in the ocean is primarily driven by the deformation of Enceladus’ unconsolidated core. We find that the heat production in the ocean can explain only a small fraction of Enceladus’ heat budget under the present day conditions (i.e., for an ocean thickness of about 40 km) but can be as high as 25 GW if the thickness of the ocean layer is less than about 1 km. Analysis of the flow field suggests that the simplifying assumptions often used in previous studies may not be appropriate. In particular, we show that, regardless of ocean thickness, the dissipation rate obtained by solving the shallow water equations corrected for the dampening effect of the ice shell can be significantly different from that obtained by solving the three-dimensional Navier-Stokes equations.
1 Introduction

Enceladus is a small icy moon with a global subsurface ocean. The ocean is likely to be maintained by tidal dissipation but the data tell us little about how and where the dissipation takes place. The total internal power of Enceladus is estimated to be between 10 and 50 GW (for a review, see Nimmo et al., 2018). Analysis of the stability of the ice shell suggests that the viscosity of ice at the contact with the ocean is higher than $3 \times 10^{14}$ Pa s (Čadek et al., 2019a). If this estimate is correct, then the heating rate due to tidal dissipation in the ice shell is $\sim 1$ GW (Souček et al., 2019), which is much less than the heat loss due to conduction ($\approx 40$ GW, Čadek et al., 2019a). This indicates that the heat necessary to maintain the ocean at the melting temperature is produced in the core or the ocean itself.

Since the interior of Enceladus is likely to have never been exposed to high temperatures and pressures, the core may be made of unconsolidated, easily deformable, porous rock (Travis & Schubert, 2015; Roberts, 2015). Tidal deformation of such a material is associated with rearrangements of rock fragments, producing frictional heat that may explain Enceladus’ heat budget (Choblet et al., 2017; Rovira-Navarro et al., 2022).

The amount of tidal heat generated within the ocean is still debated. The prevailing view is that heating caused by ocean tides is negligible at present but may have been significant in the past when dissipation could have been enhanced by resonance effects and/or the thickness of the ocean was strongly reduced (e.g., Matsuyama et al., 2018; Hay & Matsuyama, 2019). In addition to tides, dissipation in the ocean can be caused by the turbulent flow response to the longitudinal libration of the ice shell. Whether this mechanism can provide enough heat to explain Enceladus’ heat production, as suggested by, e.g., Lemasquerier et al. (2017) and Wilson and Kerswell (2018), will require further examination.

The first models of tidal dissipation in a subsurface ocean were based on the solution of the Laplace tidal equations (LTE). The studies pointed out the possible role of inertial waves in the thermal evolution of icy moons (Tyler, 2008, 2009, 2014, 2020) and provided the first estimates of the dissipative heat generated by oceans on Enceladus, Europa and other icy satellites (Chen et al., 2014; Matsuyama, 2014; Hay & Matsuyama, 2017). The limitation of this approach was that it was based on two simplifying assumptions: First, the thickness of the ocean, $D$, was assumed to be small compared to the outer
radius of the ocean, \( R_{i/o} \), and second, the surface of the ocean was assumed to be free to move. In reality, neither of these assumptions is true: The ocean on Enceladus is not shallow (\( D/R_{i/o} \approx 0.15 \), see, e.g., Čadek et al. (2016), Beuthe et al. (2016) or Hemingway and Mittal (2019)) and its surface is covered with a 20–25 km thick layer of solid ice (Thomas et al., 2016). Beuthe (2016) showed that the ice shell has a strong stabilizing effect on the tidal flow in the ocean, leading to a significant reduction of tidal dissipation. In the years that followed, the approach based on the solution of LTE, but including the dampening effect of the ice shell, was used to obtain more accurate estimates of dissipative power generated by subsurface oceans (Matsuyama et al., 2018; Hay et al., 2020) and to study the impact of non-linear bottom drag and ocean thickness variations on tidal dissipation (Hay & Matsuyama, 2019; Rovira-Navarro et al., 2020).

To date, there are only two studies (Rovira-Navarro et al., 2019; Rekier et al., 2019) that have examined the tidal dissipation in a subsurface ocean by solving the three-dimensional Navier-Stokes equation (NSE). In both studies, the non-linear term in NSE (\( \mathbf{v} \cdot \nabla \mathbf{v} \) where \( \mathbf{v} \) is the flow velocity) is neglected and the problem is transformed into the frequency domain. The deformation of the ice shell is assumed not to be affected by the flow of water and is imposed as a boundary condition at the surface of the ocean. The latter study also takes into account the deformation of the core and includes the effects of self-gravitation and libration. The studies carefully analyze the role of inertial modes and agree that the heat production in Enceladus’ present-day ocean is much smaller than the heat flux observed by Cassini (Spencer et al., 2018).

This paper offers an alternative approach to modeling the tidal response of a planetary body with a subsurface ocean. The approach is different from that used by Rovira-Navarro et al. (2019) and Rekier et al. (2019) in two respects: First, the tidal response is evaluated not only in the ocean but also in the crust and the core. In our approach, the equations governing the deformation in different parts of the body are linked by the boundary conditions guaranteeing the continuity of the velocity and traction vectors. This allows us to assess the effect of mechanical and gravitational coupling between the layers on the tidal dissipation process and specifically to investigate the interaction between the ocean and Enceladus’ weak core. Second, the problem of tidal deformation is solved directly in the time domain as an initial value problem, similarly as in the case of thermal convection (except that the buoyancy force is replaced by the tidal force, see, e.g., Appendix A in Kvorka and Čadek (2022)). Although this approach is computationally
more demanding than the solution in the frequency domain, it makes it possible to as-
109 sess, at least in principle, the relative importance of non-linear effects and resonant in-
110 teractions. These effects cannot be investigated by standard frequency domain methods
112 because the response of a non-linear system to a periodic loading is not necessarily pe-
113 riodic in time. Our approach is also more general than that of Beuthe (2016) and Matsuyama
114 et al. (2018) who coupled the flow in the ocean with the deformation of the ice shell and
115 the core but used the LTE to calculate the tidal response of the ocean.

Modeling the coupled tidal deformation of an icy moon with a subsurface ocean
117 and a plastic core is challenging and, therefore, several simplifying assumptions have been
118 made to make the problem more amenable to solution. The most important one is the
119 assumption that the lower boundary of the ice shell is spherical. This assumption is mo-
120 tivated by numerical convenience and is widely used in studies of subsurface ocean dy-
121 namics that employ numerical tools based on the spectral decomposition of the govern-
122 ing equations. While the assumption of a constant ice shell thickness may be approx-
123 imately satisfied in the case of large, slowly rotating icy moons, such as Titan, it is not
124 valid for Enceladus where the thickness of ice varies laterally by tens of kilometers (e.g.,
125 Hoolst et al., 2016; Beuthe, 2016; Hemingway & Mittal, 2019; Čadek et al., 2016, 2019a).
126 Although these variations are likely to affect the flow in the ocean (Rovira-Navarro et
127 al., 2020, 2023), most studies addressing Enceladus’ ocean dynamics do not include them
128 because of the numerical difficulties arising from the implementation of irregular bound-
129 aries in spectral methods (e.g., Beuthe, 2016; Matsuyama et al., 2018; Rovira-Navarro
130 et al., 2019; Rekier et al., 2019; Soderlund, 2019). One way to solve problems with com-
131 plex geometry is to use the finite element method. However, even though this method
132 has proven to be effective in modeling the tidal deformation and viscous flow in Encel-
133 audus’ ice shell (Souček et al., 2016, 2019; Běhounková et al., 2017; Čadek et al., 2019a,
134 2019b; Berne et al., 2023), its application to the tides in the ocean is still prohibitively
135 expensive and out of reach of current computational algorithms. The reason for this is
136 that the thickness of the boundary layers in the ocean is much smaller than the char-
137 acteristic length scale in the ice shell and, therefore, simulations have to be performed
138 with a spatial resolution that is significantly higher than that usually considered in the
139 ice shell.

Another limitation of our approach is that the viscosity of water used in our model
140 is likely to be higher than the effective viscosity in the real ocean. The lowest value of
the dynamic viscosity, $\eta$, that can be achieved in our numerical simulations is 100 Pa s,
which is five orders of magnitude more than the molecular viscosity of water. Time-domain
simulations with $\eta < 100$ Pa s place extreme demands on the numerical resolution needed
to represent the velocity field and will require algorithms that achieve a higher level of
parallelization and memory management. The lowest viscosity used in the 3D frequency-
domain simulations by Rovira-Navarro et al. (2019) and Rekier et al. (2019) is about 70
Pa s and 0.2 Pa s, respectively. The effective viscosity of water may be significantly higher
than the molecular viscosity due to the effect of turbulence (e.g., Pope, 2000). On Encel-
adus, the turbulent flow may occur because of the convective heat transfer in the ocean
driven by the tidal heating in the core. The effect of turbulence may be further enhanced
in the bottom boundary layer as a result of the hydrothermal circulation in the core (Choblet
et al., 2017) and the friction at the interface. In any case, the effective viscosity is likely
to vary in space and time depending on the flow conditions and the degree of turbulence.
The model with a constant viscosity should therefore be viewed as a first-order approx-
imation of the tidal flow in the ocean.

While, for the reasons outlined above, the values of the dissipation rate predicted
by our simulations should be interpreted with caution, the trends inferred here are gen-
erally valid and provide context for further studies in this area. In particular, we focus
on the following questions: How much is the dissipation in the ocean affected by the de-
formation of the core? Does the deformation of the core depend on the thickness and
viscosity of the ocean? How does the velocity field in the ocean change as a function of
model parameters? Is the radial component of the flow velocity significant? What are
the validity limits of the LTE approach, which has been widely used to estimate heat
production in subsurface oceans?

Based on the previous studies (e.g., Beuthe et al., 2016; Čadek et al., 2019a), we
assume that the radius of Enceladus’ core is about 190 km, and we vary the thickness
of the ocean to investigate its role in different stages of Enceladus’ evolution. Particu-
lar attention is paid to thin ($\approx 1$ km thick) ocean models that may be important for un-
derstanding the early stage of ocean formation. The results presented here may also help
to clarify the role of tidal heating in the hypothetical oceans on Mimas (Tajeddine et
al., 2014) and Dione (Beuthe et al., 2016; Zannoni et al., 2020) whose cores may have
material properties similar to those expected on Enceladus.
2 Method

We investigate the tidal deformation of an icy moon whose size and internal structure are similar to those of Enceladus. We assume that the moon consists of three layers: a solid ice shell, a liquid water ocean, and a solid core. The tidal deformation is solved simultaneously for all three layers, while the mechanical coupling between the layers is achieved by imposing continuity of traction and velocity at the boundaries. For simplicity, we assume that the boundaries are spherical, the material forming the moon is incompressible, and the tidal flow in the ocean is not affected by convective heat transport. The effect of tidal dissipation on material parameters is neglected. Under these conditions, the governing equations take the form

\[
\begin{align*}
\nabla \cdot \mathbf{v} &= 0, \\
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) &= \nabla \cdot \mathbf{\sigma} - 2\rho \omega \times \mathbf{v} - \rho (\nabla V_t + \nabla V_g). 
\end{align*}
\]

Here, \(\mathbf{v}\) is the velocity, \(\rho\) is the density, \(t\) is time, \(\mathbf{\sigma}\) is the incremental Cauchy stress tensor, \(\omega\) is the angular velocity and \(V_g\) is the gravitational potential due to tidal deformation (for details of the calculation, see Appendix A in Čadek et al., 2021),

\[
V_g(t, r, \theta, \phi) = G \int \int \left( \int_{R_t}^{R_t + u_t} \frac{\rho_i}{|r - r'|} r'^2 dr' + \int_{R_{i/o}}^{R_{i/o} + u_{i/o}} \frac{\rho_o - \rho_i}{|r - r'|} r'^2 dr' \\
+ \int_{R_{o/c}}^{R_{o/c} + u_{o/c}} \frac{\rho_c - \rho_o}{|r - r'|} r'^2 dr' \right) \sin \theta' d\theta' d\phi' \tag{3}
\]

where \(G\) is the gravitational constant, \(r, \theta, \phi\) are the spherical coordinates, \(R_t, R_{i/o}\) and \(R_{o/c}\) are the radii of the density interfaces (see Figure 1) and \(u_t, u_{i/o}\) and \(u_{o/c}\) represent the shape changes of the boundaries. Finally, \(V_t\) is the eccentricity tidal potential valid to first order in eccentricity (Kaula, 1964),

\[
V_t(t, r, \theta, \phi) = r^2 \omega^2 e \\
\left\{ \frac{3}{2} P_2^0 (\cos \theta) \cos \omega t - \frac{1}{4} P_2^2 (\cos \theta) [3 \cos \omega t \cos 2\phi + 4 \sin \omega t \sin 2\phi] \right\}, \tag{4}
\]

where \(e\) is the eccentricity, and \(P_2^0\) and \(P_2^2\) are the associated Legendre functions. The effect of obliquity tides on the deformation of Enceladus' ocean is likely to be small (Baland et al., 2016) and is neglected. The dimensional analysis of Equation (2) shows that the Coriolis \((2\rho \omega \times \mathbf{v})\) and inertial \((\rho \partial \mathbf{v}/\partial t + \rho \mathbf{v} \cdot \nabla \mathbf{v})\) forces can be neglected in the ice shell and the core. In the ocean, Equation (2) can be simplified by neglecting the nonlinear
Figure 1. A sketch of the computational domain. Equations (1)–(5) are solved in a spherical domain made of three shells representing the solid ice crust, the liquid water ocean, and the solid silicate core. The radius of the core, $R_{o/c}$, is fixed at 194.1 km and the thickness of the ocean, $D$, is varied from 10 m to 50 km. We assume that the ice shell behaves as an elastic solid, while three different constitutive models of the core are considered (A–rigid, B–elastic, and C–viscoelastic). For details regarding the material parameters and the boundary conditions, see Table 1 and Equations (6) and (7), respectively.

The mechanical behavior of all three layers is described by a single constitutive equation (e.g., Čadek et al., 2017),

$$\frac{1}{\mu} \frac{\partial \sigma^d}{\partial t} + \frac{1}{\eta} \sigma^d = \nabla v + (\nabla v)^T,$$

(5)
where \( \sigma^d \) is the deviatoric part of tensor \( \sigma \). Depending on the choice of the shear modulus \( \mu \) and the viscosity \( \eta \), Equation (5) describes either an elastic solid (\( \eta \to \infty \)), a Newtonian fluid (\( \mu \to \infty \)) or a Maxwell viscoelastic body. Note that since the conservation laws, Equations (1) and (2), are formulated in terms of the velocity, Hooke’s law, and the Maxwell constitutive equation must be expressed in terms of the strain-rate tensor.

### Table 1. Parameters of the model.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_t )</td>
<td>Radius of the moon</td>
<td>252.1</td>
<td>km</td>
</tr>
<tr>
<td>( R_{i/o} )</td>
<td>Radius of the ice/ocean interface</td>
<td>194.11 − 244.1</td>
<td>km</td>
</tr>
<tr>
<td>( R_{o/c} )</td>
<td>Radius the ocean/core interface</td>
<td>194.1</td>
<td>km</td>
</tr>
<tr>
<td>( e )</td>
<td>Eccentricity</td>
<td>0.0047</td>
<td></td>
</tr>
<tr>
<td>( \omega )</td>
<td>Angular velocity</td>
<td>( 5.31 \times 10^{-5} )</td>
<td>rad s(^{-1} )</td>
</tr>
<tr>
<td>( \rho_i )</td>
<td>Density of the ice shell</td>
<td>925</td>
<td>kg m(^{-3} )</td>
</tr>
<tr>
<td>( \rho_o )</td>
<td>Density of the ocean</td>
<td>1000</td>
<td>kg m(^{-3} )</td>
</tr>
<tr>
<td>( \rho_c )</td>
<td>Density of the core</td>
<td>2350 − 2424</td>
<td>kg m(^{-3} )</td>
</tr>
<tr>
<td>( \eta_o )</td>
<td>Viscosity of the ocean</td>
<td>( 10^2 )−( 10^6 )</td>
<td>Pa s</td>
</tr>
<tr>
<td>( \eta_c )</td>
<td>Viscosity of the core</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Model A</td>
<td>( \to \infty )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Model B</td>
<td>( \to \infty )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Model C</td>
<td>( 6 \times 10^{11} )</td>
<td>Pa s</td>
</tr>
<tr>
<td>( \mu_i )</td>
<td>Shear modulus of the ice</td>
<td>( 3.3 \times 10^9 )</td>
<td>Pa</td>
</tr>
<tr>
<td>( \mu_c )</td>
<td>Shear modulus of the core</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Model A</td>
<td>( \to \infty )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Model B</td>
<td>( 10^9 )</td>
<td>Pa</td>
</tr>
<tr>
<td></td>
<td>Model C</td>
<td>( 2 \times 10^7 )</td>
<td>Pa</td>
</tr>
</tbody>
</table>

Boundary conditions are imposed on the reference spheres of radius \( R_t \), \( R_{i/o} \), and \( R_{o/c} \). Since the deformed surface of the moon is stress-free, the boundary condition at
\( r = R_t \) can be expressed as follows (Souček et al., 2019):

\[
\sigma \cdot e_r = -\rho u_r g e_r = -\rho g e_r \int_0^t v_r dt',
\]

(6)

where \( u_r \) is the radial component of the displacement, \( v_r \) is the radial component of the velocity, \( g \) is the gravitational acceleration, and \( e_r \) is the radial unit vector. At the internal interfaces, the velocity and traction vectors are required to be continuous:

\[
[v]_+^- = 0,
\]

\[
[\sigma \cdot e_r]_+^- = \Delta \rho u_r g e_r = \Delta \rho g e_r \int_0^t v_r dt',
\]

(7)

where \([ \cdot ]_+^-\) represents the jump of the enclosed quantity across the interface and \( \Delta \rho \) is the density difference \((\rho_o - \rho_i \text{ or } \rho_c - \rho_o)\). The assumption that the ice/ocean interface is spherical is obviously a simplification. Analysis of the gravity data shows that the thickness of ice on Enceladus varies laterally by tens of kilometers (e.g., Beuthe, 2016; Hemingway & Mittal, 2019; Čadek et al., 2016, 2019a). Although these variations are likely to affect the flow in the ocean (Rovira-Navarro et al., 2020, 2023), they are not included in our model because of the numerical difficulties arising from the implementation of irregular boundaries in spectral methods.

The total heat production of the moon is calculated using the following formula (e.g., Tobie et al., 2008):

\[
H = \frac{1}{P} \int_{t_0}^{t_0+P} \int_V \frac{\sigma^d : \sigma^d}{2\eta} dV dt = \frac{1}{P} \int_{t_0}^{t_0+P} \int_V 2\eta \dot{\varepsilon} : \dot{\varepsilon} dV dt,
\]

(8)

where \( P \) is the rotation period \((P = 2\pi/\omega)\), \( t_0 \) is an arbitrary time, and \( V \) is the volume of the moon and \( \dot{\varepsilon} = \frac{1}{2}(\nabla \varepsilon + (\nabla \varepsilon)^T) \). In section 3, the lateral variations of tidal dissipation in the ocean are characterized by the heat production integrated over the radius (Souček et al., 2019):

\[
q(\theta, \varphi) = \frac{1}{P} \int_{t_0}^{t_0+P} \frac{1}{R_{i/o}^2} \int_{R_{i/o}}^{R_{i/o}} \frac{\sigma^d : \sigma^d}{2\eta} r^2 dr dt.
\]

(9)

Data collected by the Cassini spacecraft suggest that the ocean is at a depth of about 20 km below the surface on average (Thomas et al., 2016; Hoolst et al., 2016) and that the mean ocean thickness is about 40 km (Beuthe, 2016; Čadek et al., 2016, 2019a). It is likely, however, that the thickness of the ocean has changed in the past and will change in the future. In order to understand the role of ocean tidal dissipation in Enceladus' evolution, we vary the thickness of the ocean from 10 m to 50 km. The lower bound of
the interval corresponds to the case where the ocean is almost completely frozen but it
is mechanically decoupled from the core by a thin layer of liquid water or partially molten
ice. In all simulations, we assume that the radius of the core is 194.1 km (Čadek et al.,
2019a) and we change the thickness of the ocean by changing the radius of the ice/ocean
interface.

We assume that the ice shell behaves as an elastic solid. As shown by Souček et
al. (2019), the effect of viscosity on tidal deformation of Enceladus’ ice shell is minor and
can be neglected in the first approximation. Unfortunately, there are presently no ob-
servations to constrain the material parameters of Enceladus’ core. In most of the pre-
vious studies on the ocean tides, the core was assumed to be rigid and its effect on the
flow in the ocean was neglected. However, if the core has never been exposed to high tem-
peratures and pressures, the core material may still be in an unconsolidated, fragmented
state and may effectively behave as a low rigidity, low viscosity viscoelastic body (Travis
& Schubert, 2015; Roberts, 2015; Choblet et al., 2017; Rovira-Navarro et al., 2022). If
this is the case, the tidal deformation of the core may affect the flow in the ocean in a
similar way as the tidal deformation of the ice shell. Here we consider three models of
the core: A - the rigid core, B - the elastic core with $\mu_c = 10^9$ Pa (cf. Roberts, 2015),
and C - the viscoelastic core with $\mu_c = 2 \times 10^7$ Pa and $\eta_c = 6 \times 10^{11}$ Pa s. The param-
eters of model C are chosen so that the total heat production of the core is roughly 25
GW (cf. Figure 1a in Choblet et al., 2017). The parameters of models A–C are summa-
ized in Table 1.

Equations (1)–(7) are solved in the time domain using a pseudo-spectral method
similar to that used by Kvorka and Čadek (2022). The unknown functions $v$ and $\sigma$ are
represented in terms of the generalized spherical harmonic expansions (e.g., Golle et al.,
2012), truncated at degree $\ell_{\text{max}}$. The cut-off degree $\ell_{\text{max}}$ is chosen so that $V(\ell_{\text{max}})/V(2) <
10^{-4}$, where $V(\ell)$ denotes the $L_2$ norm of the velocity field at degree $\ell$, and ranges from
10 to 220 depending on the thickness and viscosity of the ocean. The coefficients in the
harmonic series are discretized in the radial direction on an uneven grid consisting of 800
Chebyshev nodes (for an example of the radial resolution, see Figure S1 in the Support-
ing Information). The radial resolution varies from 1 m in the ocean boundary layers to
1.5 km in the ice shell and the core. The time derivatives in Equations (2) and (5) are
discretized using the implicit Euler method, while the explicit second-order Adams-Bashforth
method is applied to the Coriolis and non-linear terms. Each simulation is started from
the initial condition \( \mathbf{v} = 0 \), and the equations are integrated in time until a periodic solution is reached (cf. Tobie et al., 2008; Běhounek et al., 2010). At each time step, the accuracy of the method is assessed by evaluating the energy balance (Patočka et al., 2017). The numerical method was coded in Fortran 90 and tested against the results by Beuthe (2016) and Rekier et al. (2019), see section 5 and Figure S2 in the Supporting Information.

3 Tidal Dissipation in the Ocean and the Core

![Graphs showing heat production in Enceladus’ ocean and core](image)

**Figure 2.** Heat production in Enceladus’ ocean (a–c) and core (d) as a function of ocean thickness evaluated for different values of viscosity \( \eta_o \). Heat production in the core is shown only for model C since the core in models A and B is treated as non-dissipative (\( \eta_c \to \infty \)). The solid lines represent models where the deformation equations are solved in the whole domain (including the solid parts, see section 2), while the colored dashed lines correspond to models where the flow is solved only in the ocean and the deformation of the ice shell the core is implemented through boundary conditions (Rekier et al., 2019). The vertical black dashed lines indicate the current thickness of Enceladus’ ocean.
Figure 2 shows the tidal dissipation in the ocean (panels a–c) and the core (d) as a function of ocean thickness for different values of ocean viscosity. Comparison of panels a–c reveals two general trends: (i) The maximum heat production in the ocean is independent of the viscosity of the ocean but depends on the material properties of the core ($H = 4$ GW for model A but 20 GW for model C). The value of $D$ at which the maximum is reached in less than 2 km and decreases with decreasing viscosity. (ii) When the ocean thickness is larger than 5 km, the curves are similar in shape to those obtained by Rekier et al. (2019) and Rovira-Navarro et al. (2019). The curves are characterized by several resonance peaks whose amplitudes increase with decreasing viscosity. The dissipation in the ocean is less than 0.1 GW and thus insufficient to explain the heat output (> 10 GW) of present-day Enceladus ($D = 38$ km).

The maximum heat production obtained for model A (stiff core) is about twice higher than for model B (elastic core). This somewhat counterintuitive result is due to the fact that, in the elastic case, the upper and lower boundaries move in phase, and consequently, the radial deformation of the fluid layer is smaller than in the case of the rigid core. The parameters of model C are chosen such that the deformation of the ocean/core boundary is significantly larger than that of the elastic shell. The Deborah number of the core ($\tau_M/P$ where $\tau_M = \eta_c/\mu_c$ is the Maxwell time) is about 0.25, implying that the tidal response of the core is dominated by viscous flow. This, together with the fact that the response of the core to the tidal force is delayed, results in an enhancement of the heat production in the ocean (Figure 2c). The maximum heat production is about 20 GW and is attained for a slightly larger value of $D$ than in models A and B (70 m to 2.5 km, depending on $\eta_o$). This suggests that tidal dissipation in a thin ocean may be sufficiently high to prevent the ocean from freezing.

The thickness of the ocean influences not only the dissipation in the ocean itself but also in the viscoelastic core (Figure 2d). When the thickness of the ocean drops below a critical value, $D_{\text{crit}}$, the heat production in the core starts to decrease rapidly. The thickness of the ocean at which the drop occurs roughly corresponds to the thickness at which the dissipation in the ocean reaches a maximum value (cf. Figure 2c and 2d). Note that the heat production in the core for $D > D_{\text{crit}}$ is the same as the maximum heat production in the ocean, suggesting that at $D \approx D_{\text{crit}}$, the role of the core as the main producer of heat is taken over by the ocean. When the ocean thickness is further decreased, the heat production in the ocean goes to zero, and the total heat flow of the moon ap-
Figure 3. Surface distribution of ocean tidal heating averaged over the tidal cycle, Equation (9), for different thicknesses (columns) and viscosities (rows) of the ocean. All heat flux maps are computed for model C and shown in the Mollweide projection with the sub-Saturnian point at the center. For the sake of comparison of the heating patterns, each map is scaled to its own maximum (indicated above the map in mW/m²).

As already mentioned in section 2, our approach is different from the one used by Rekier et al. (2019) in that the deformation equations are solved simultaneously in the whole domain (including the solid parts) while Rekier et al. (2019) first calculate the deformation of the ice shell and the core and then use it as a boundary condition to compute the flow in the ocean. Comparison of the solid and dashed lines in Figure 2 (panels a–c) shows that the two approaches give the same heating rates for $D > D_{\text{crit}}$, implying that the flow in the ocean has little impact on the deformation of the ice shell and the core if the ocean thickness is larger than about 2 km. However, the approach where the deformation of the solid parts is not self-consistently included grossly overestimates the tidal dissipation in the ocean for $D < D_{\text{crit}}$. It is likely that Rekier et al. (2019) anticipated this behavior and therefore only considered models where $R_{o/c}/R_{i/o} < 0.98$. 

$\eta_{o} = 10^{5}$
Figure 4. As in Figure 3 but in North pole stereographic projection.
(corresponding to $D \approx 4$ km). The large discrepancy between the two solutions sug-
gests that the assumption that the deformation of the ice shell and the core is indepen-
dent of the tidal flow in the ocean is not valid for $D < D_{\text{crit}}$. This implies that the me-
chanical coupling between the ocean and the solid parts of the moon must be considered
when evaluating dissipation in a thin ocean.

The flow induced by eccentricity tides in Enceladus’ ocean yields a wide variety of
heating patterns depending on the thickness of the ocean and its viscosity (Figures 3 and
4). For some combinations of parameters $\eta_o$ and $D$, the heat flux distribution is simi-
lar to those obtained using the shallow water approximation (see Figure 4 in Tyler et
al. (2015) or Figure 7 in Matsuyama et al. (2018)). In most cases, however, the heat flux
distribution is rather complex, indicating that dissipation in the ocean is strongly influ-
enced by three-dimensional flow effects. The heat flux distribution obtained for $D = 47$
km and $\eta_o \leq 10^4$ Pa s, characterized by a system of concentric circular fringes, is com-
patible with the presence of inertial waves discussed by Rovira-Navarro et al. (2019) and
Rekier et al. (2019).

The spatial distribution of the tidal heat produced in the core is independent of
the viscosity and thickness of the ocean and only differs by magnitude. Similar to the
results of Roberts (2015, Figure 3b) and Choblet et al. (2017, Figure 1c), the maximum
heat flux is concentrated over the polar regions, reaching the value of about 70 mW/m$^2$
for $D > D_{\text{crit}}$ and 0.66 mW/m$^2$ when $D \to 0$ (see also Figure S3 in the Supporting
Information).

### 4 Velocity Field in the Ocean

A number of studies of ocean tides have been based on the assumption that the
horizontal scale of flow is much larger than the depth of the ocean. Under this assump-
tion, the trajectories of water parcels are nearly horizontal, the radial velocity is small
compared to the horizontal velocity and the Navier-Stokes equations can be reduced to
a two-dimensional form, called the Laplace tidal (or shallow water) equations, in which
the horizontal velocity is a function of only $\vartheta$ and $\varphi$. The equations do not contain the
radial component of velocity and the effect of radial flow on the viscous, inertial, and Cori-
olis force is neglected (Tyler, 2008). In this section, we will address the limitations of this
approach and demonstrate the complex structure of the tidal flow in Enceladus’ ocean.
We will follow up on the studies by Rovira-Navarro et al. (2019) and Rekier et al. (2019) that have investigated the tidal response of a subsurface ocean by solving the three-dimensional Navier-Stokes equations. However, unlike these studies, we will present the flow field distribution not in the frequency domain but in the time domain, and we will examine the importance of the non-linear term in the momentum equation, which has been neglected in previous research.

\[ a) \ D = 0.1 \text{ km} \]
\[ v_r^{\text{max}} = 0.2 \quad v_\theta^{\text{max}} = 82.5 \quad v_\phi^{\text{max}} = 46.2 \]

\[ b) \ D = 1 \text{ km} \]
\[ v_r^{\text{max}} = 0.1 \quad v_\theta^{\text{max}} = 3.1 \quad v_\phi^{\text{max}} = 19.2 \]

\[ c) \ D = 5 \text{ km} \]
\[ v_r^{\text{max}} = 0.3 \quad v_\theta^{\text{max}} = 2.1 \quad v_\phi^{\text{max}} = 3.7 \]

\[ d) \ D = 10 \text{ km} \]
\[ v_r^{\text{max}} = 2.5 \quad v_\theta^{\text{max}} = 2.4 \quad v_\phi^{\text{max}} = 2.9 \]

\[ e) \ D = 38 \text{ km} \]
\[ v_r^{\text{max}} = 1.6 \quad v_\theta^{\text{max}} = 1.0 \quad v_\phi^{\text{max}} = 2.3 \]

\[ f) \ D = 47 \text{ km} \]
\[ v_r^{\text{max}} = 2.6 \quad v_\theta^{\text{max}} = 2.9 \quad v_\phi^{\text{max}} = 11.6 \]

**Figure 5.** Velocity fields (components \( r, \theta \) and \( \phi \)) at time \( t = 0 \) obtained for \( \eta_o = 100 \text{ Pa s} \) and \( D \) ranging form 0.1 to 47 km. The fields are normalized to 1 and plotted on a meridional cross-section at longitude \( \phi = 0 \). The maximum values in mm/s are given on the left-hand side of each cross-section. While the ratio \( D/R_o/c \) varies from \( 5.2 \times 10^{-4} \) in panel a to 0.24 in panel f, the thickness of the ocean in the visualization is fixed at \( D/R_o/c = 0.25 \) for clarity.

In Figure 5, we show the velocity field at time \( t = 0 \) for models with \( \eta_o = 100 \text{ Pa s} \) and \( D \) ranging from 0.1 to 47 km. Note that for clarity, the thickness of the ocean in the visualization is fixed at \( D = 0.25R_o/c \approx 50 \text{ km} \). Inspection of the figure shows that the assumption of shallow water is well satisfied for \( D \lesssim 1 \text{ km} \) but breaks down when \( D \gtrsim 5 \text{ km} \). When \( D = 38 \text{ km} \) (corresponding to the average ocean thickness at present), the radial and tangential components are the same size, and the horizontal scale...
Figure 6. Horizontal component of the flow velocity, $v - (v \cdot e_r)e_r$, at a depth of $D/10$ below the ocean surface computed in different orbital phases ($t = 0, 0.2$ and $0.4 \ P$) for the same models as in Figure 5. The horizontal velocity is represented by streamlines with colors representing the flow speed. All maps are shown in the North pole stereographic projection.

of the flow is comparable with the thickness of the ocean, suggesting that the shallow water approach is not suitable for modeling Enceladus’ ocean tides.

Figure 6 shows the same flow models as Figure 5 but in North pole stereographic projection. The velocity fields represented by surface streamlines are shown at a depth of $D/10$ below the ice/ocean boundary. A long-wavelength flow pattern typical of shallow water models is found only for $D \leq 5 \ km$. For larger values of $D$, the velocity field is dominated by small-scale concentric flow loops rotating in the azimuthal direction that cannot be obtained using the standard shallow water approach.

Previous studies of tidal flow in subsurface oceans have assumed that the non-linear term in the momentum equation ($\rho v \cdot \nabla v$) can be neglected or its effect can be simulated by a global increase in viscosity (i.e., by using the eddy viscosity instead of the molecular viscosity). The problem of tidal flow is then linear in $v$, and its solution shows a marked resonant behavior characterized by a significant increase in the kinetic energy of the flow (e.g., Tyler, 2008). The question arises as to how much this behavior would be affected by the non-linear term and, in general, what role turbulence plays in tidal dissipation. One way to assess whether the flow is turbulent or not is to evaluate the Reynolds num-
Figure 7. a) The Reynolds number as a function of ocean thickness evaluated for different values of viscosity \( \eta_o \). b) Comparison of the terms in the Navier-Stokes equation (10) near the top and bottom boundaries and in the middle of the ocean for the model with \( D = 38 \) km and \( \eta_o = 100 \) Pa s. The numbers above the plots indicate the maximum values in N/m³.

The Reynolds number, \( Re = \rho_o |v|D/\eta_o \), where \( |v| \) is the \( L_2 \) norm of the velocity field. The Reynolds number is a dimensionless quantity that is used to predict the transition from laminar to turbulent flow. Turbulent flow occurs when \( Re > Re_c \) where \( Re_c \) is the critical Reynolds number (\( Re_c = 10^3 \)–\( 10^4 \), depending on the geometry of the system). As shown in Figure 7a, all models presented in this study (\( \eta_o \geq 100 \) Pa s, \( D \leq 50 \) km) are characterized by low Reynolds numbers (\( Re \lesssim 1000 \)), suggesting the dominant role of laminar flow. This conclusion is supported by a direct comparison of the terms in the Navier-Stokes equation, which is obtained by substituting \( \sigma \) from Equation (5) into Equation
(2) and setting $\mu^{-1} = 0$,
\[
\rho \left( \frac{\partial v}{\partial t} + v \cdot \nabla v \right) = -\nabla p + \eta \nabla \cdot \nabla v - 2\rho \omega \times v - \rho (\nabla V_t + \nabla V_g),
\]  
(10)

where $p$ is the pressure. Figure 7b shows that the non-linear term $(\rho v \cdot \nabla v)$, which is responsible for turbulence, is small compared to other terms in Equation (10). Extrapolation of the results shown in Figure 7a indicates that the transition from laminar to turbulent flow occurs when the viscosity $\eta_o$ drops to about 10 Pa s, presumably leading to an increase of the Reynolds number to $10^3 - 10^4$. We are currently unable to perform time domain simulations for $\eta_o \ll 100$ Pa s because they place extreme demands on the numerical resolution required to represent the velocity fields. The problem of whether the turbulent flow can influence the resonant states predicted by the linearized model thus remains unresolved and will require further improvements of the numerical method.

5 Comparison with the LTE approach

In this section, we compare our approach with that of Beuthe (2016). To couple the flow in the ocean with the deformation of the ice shell, Beuthe (2016) formulated the Laplace tidal equations (LTE) as follows:
\[
\frac{\partial u}{\partial t} = -\frac{1}{\rho_o} \nabla_p + 2\omega \times u + F(u),
\]  
(11)
\[
\frac{\partial h}{\partial t} + D \nabla_s \cdot u = 0,
\]  
(12)
\[
p = \Delta \rho gh - \rho_o (V_t + V_g) - \sigma_{rr}.
\]  
(13)

Here, $\mathbf{u} = \mathbf{u}(\vartheta, \phi)$ is the depth-averaged velocity, $\nabla_s$ is the surface gradient, $h$ is the radial displacement at the boundary between the ice shell and the ocean, $\sigma_{rr}$ is the radial component of the traction vector at the base of the ice shell, and $F(u)$ represents the dissipative stress
\[
F(u) = \frac{\eta_o}{\rho_o} \Delta_s u - \alpha u,
\]  
(14)

where $\alpha$ is the linear drag coefficient, and $\Delta_s$ is the horizontal Laplace operator. Unlike the previous studies (e.g., Matsuyama, 2014; Chen et al., 2014; Tyler, 2014), the approach by Beuthe (2016) includes the stabilizing effect of the overlying shell, which damps the ocean tides and reduces the tidal heating in the ocean (see also Matsuyama et al., 2018). In the following, Equations (11)–(14) are referred to as the modified LTE.

The modified LTE approach differs from our approach, Equations (1)–(7), in two ways. First, as already mentioned above, the modified LTE approach does not include
the radial component of velocity and assumes that the horizontal velocity is a function of only $\vartheta$ and $\varphi$ (i.e., $\partial u/r = 0$). Second, the modified LTE approach guarantees that the radial components of the displacement and traction do not change across the ice/ocean boundary (Equation 13), but, for understandable reasons, it imposes no restriction on the horizontal components (in other words, the boundary is free-slip). In contrast, in our approach, we assume that the boundary is no-slip and both radial and horizontal components of the displacement and traction are continuous (Equation 7).

To facilitate the comparison of the methods, we replace the no-slip boundary condition in our model with the free-slip one and omit the last term in Equation (14) because the drag coefficient $\alpha$ is not included in our 3D model. The modified LTE method has been coded in Fortran 90 and tested against the results of M. Beuthe (personal communication). Unlike the original method (Beuthe, 2016) where the layer overlying the ocean is treated as a thin shell, the stress and displacement in the ice are obtained by solving the standard equations for thick shells (cf. Matsuyama et al., 2018). The total heat production of the 3D model can be calculated from Equation (8). For the sake of comparison with the modified LTE approach, we replace the velocity $v$ in Equation (8) by the depth-averaged velocity, $\overline{v}$:

$$H = \frac{2\eta D}{P} \int_{t_0}^{t_0 + P} \int_S \mathbf{\hat{e}} : \mathbf{\hat{e}} dS dt,$$

(15)

where $S$ is the surface of the sphere of radius $R_{i/o}$ and $\mathbf{\hat{e}} = \frac{1}{2}(\nabla_s \mathbf{v} + (\nabla_s \mathbf{v})^T)$. Note that $\overline{v} = u$ in the case of the modified LTE method.

The heat production obtained for the two methods is compared in Figure 8a. Inspection of the figure shows that the level of agreement between the methods depends not only on the thickness of the ocean but also on the viscosity of water and on how the dissipation rate of the 3D model is calculated. Our results indicate that for the low values of the viscosity ($\eta \lesssim 100$ Pa s), the modified LTE method is only applicable to models with very thin ocean ($D \lesssim 2$ km). On the other hand, when the viscosity is high ($\eta = 10^6$ Pa s), the modified LTE method and the 3D method show good agreement over the whole range of ocean thickness.

The agreement between the LTE and 3D solutions is generally better when the heat production of the 3D model is calculated from Equation (15). When depth variations in velocity are included, i.e., when $H$ is calculated from Equation (8), the heat production increases and the difference between the modified LTE solution and 3D solution is
Figure 8.  a) Dissipation as a function of the ocean thickness computed using the modified LTE approach by Beuthe (2016, thick solid lines) and the 3D approach (this study) in which the no–slip boundary conditions were replaced by the free-slip ones (dashed and dash-dotted lines). Dissipation in the 3D models is obtained using either Equation (8) or (15). The violet, blue and red colors represent different values of viscosity $\eta_0$. b) Comparison of 3D free-slip (dash-dotted lines) and 3D no-slip solutions (thin solid lines) obtained for model A (see Table 1). Colors are the same as in panel a.
Figure 9. Radial variation of the mean flow velocity (a) and heat production (b) in the ocean for $D=100$, $\eta_o=100$ Pa s and $t=0$. The violet and green lines represent the 3D models with the free-slip and no-slip conditions, respectively. The mean velocity is calculated as $\sqrt{\int_S |\mathbf{v}(r, \theta, \varphi)|^2 dS}$ while the mean heat production is given by $2\eta_o S^{-1} \int_S \mathbf{e} : \mathbf{e} dS$.

significant even for ocean thicknesses as small as 10 m. The difference is more pronounced for models with a low viscosity ocean, ranging from a factor of 2 at $D \leq 100$ m to a factor of $10^5$ at $D = 50$ km for $\eta = 100$ Pa s. This indicates that depth variations in velocity play an important role in calculating the tidal heat production even when $D/R_{i/o} \ll 1$ (see Figure S4 in the Supporting Information). These variations are inherently absent in the modified LTE solution, casting doubt on the applicability of the LTE method for determining the dissipation rate in the ocean. It is also worth noting that the dissipation curves computed for 3D models where the internal interfaces are treated as free-slip boundaries are significantly different from those computed for 3D models with no-slip boundaries. To allow for a better comparison between the free–slip and no–slip solutions, the two sets of curves are shown together in Figure 8b.

No-slip is usually considered to be the appropriate boundary condition for a viscous fluid in contact with a solid impermeable surface. More generally, the boundary condition can be expressed as a weighted combination of no-slip and free-slip boundary con-
ditions:

\[ v_r = 0, \]  
\[ (\sigma \cdot e_r)_\tau = -\alpha v, \]  (16)  (17)

where the subscripts \( r \) and \( \tau \) denote the radial and tangential components, respectively, and \( \alpha \) is the drag coefficient, which may depend on the velocity. The boundary condition (17) reduces to no-slip boundary condition for \( \alpha \to \infty \) and to the free-slip boundary condition for \( \alpha = 0 \). No-slip and free-slip can therefore be considered as extreme cases.

Unlike the free-slip model, the no-slip model is characterized by the presence of velocity boundary layers, the thin layers of fluid in the vicinity of the boundaries where the flow velocity monotonically increases with the distance from the boundary (Figure 9a). Although the velocity far from the boundary may not be affected by the type of the boundary condition (i.e., by the presence or absence of the boundary layer), the dissipation, which depends on the gradient of velocity, is usually higher for no-slip models compared to free-slip models. Nearly all of the tidal dissipation in the no-slip models is concentrated in the boundary layers (Figure 9b) and even though the thickness of the boundary layers is small, the total heat production can be up to six orders of magnitude higher than in the corresponding free-slip models.

6 Conclusion

In this study, we have presented a novel approach to modeling the tidal response of icy moons with subsurface oceans. The problem of tidal deformation is solved in the time domain and the flow in the ocean is calculated simultaneously with the deformation of the core and the ice shell. The main advantage of this approach is that it makes it possible, at least in principle, to investigate the non-linear effects associated with turbulent flow. However, this advantage is also its biggest limitation: Since the time required to reach a stable periodic or quasi-periodic solution is rather long (\( \sim 100-1000 P \)), the calculations are time-consuming and at present are only feasible for viscosities that are several orders of magnitude higher than the molecular viscosity of water.
The method developed here is used to study the tidal flow in the ocean of Enceladus. In particular, we focus on the role of a highly deformable and dissipative core in ocean dynamics and investigate the mechanical coupling between the core and the ocean. We show that the amount of tidal heat generated in the core and the ocean strongly depends on the thickness of the ocean layer. If the ocean thickness is significantly greater than 1 km, heat production is dominated by tidal dissipation in the core and the amount of heat produced in the ocean is negligible. In this case, tidal deformation of the core and crust is unaffected by tidal currents in the ocean, implying that tidal flow in the ocean can be investigated using the approach by Rekier et al. (2019) where the deformations of the crust and core are computed beforehand and imposed as boundary conditions. In contrast, when the ocean thickness is less than about 1 km, tidal heating in the core diminishes and dissipation in the ocean increases, leaving the total heat production unchanged. The maximum heat production in the ocean depends only on the material properties of the core and may exceed 20 GW if the core is highly deformable.

We demonstrate that the flow pattern and the distribution of tidal dissipation strongly depend on the thickness of the ocean. The radial component of the velocity is found to be negligible only if the ocean thickness is less than about 5 km, implying that the shallow water approximation (Tyler, 2009, 2020) is not applicable to present-day Enceladus. The amount of small-scale content in the velocity field increases with decreasing viscosity and increasing ocean thickness, leading to complex patterns of heat flux at the surface of the ocean.

Comparison of our results with those obtained using the LTE approach by Beuthe (2016) suggests that the shallow water equations corrected for the dampening effect of the ice shell give a reasonable estimate of the horizontal flow velocity but they fail to predict the correct values of the dissipation rate. The reason for this is that the solution of these equations cannot, in principle, provide information about the radial changes of the velocity vector. We show that these changes play an important role in calculating the dissipation and may be significant even in the case of a thin ocean.

Extrapolation of our results to realistic (low viscosity) conditions indicates that the heat production in Enceladus’ present-day ocean is likely to be less than 0.1 GW, corresponding to 0.25% of Enceladus’ total heat loss. This suggests that the tidal dissipation in the ocean plays a minor role in Enceladus’ heat budget, which is in agreement...
with most of the recent studies, except the one by Tyler (2020). It is unclear, however, whether such an extrapolation is justified because the models of ocean tides presented here are characterized by laminar flow (Reynolds number $\lesssim 10^3$) while the flow in the real ocean is likely to be turbulent. Based on our results, we estimate that the onset of turbulence occurs when the model viscosity drops below 10 Pa s, i.e., at a value that is still rather high compared to the molecular viscosity of water. The reader should be aware that the results presented here may be affected by the fact that our model does not include the effect of ocean thickness variations, which can significantly enhance the dissipative processes in the ocean.

The question of to what extent the non-linear effects and possible resonant interactions can influence tidal dissipation in the ocean has not been addressed in previous studies. Some of these studies (e.g., Tyler, 2014; Matsuyama, 2014; Rovira-Navarro et al., 2019; Rekier et al., 2019) have demonstrated that the solution of the linearized Navier-Stokes equation exhibits strong resonance peaks occurring when the system is able to store the tidal energy and transfer it into kinetic energy. However, the increase in the kinetic energy would likely to result in the onset of turbulence, violating the assumption of linearity, enhancing the damping properties of the system, and possibly reducing the amplitude of flow oscillations. Understanding these effects will require a thorough analysis of tidal flow at small spatial scales. This goal can hardly be achieved without the development of new numerical tools that will make it possible to perform high resolution simulations in the time domain. We believe this study is a first step towards this goal.

Open Research Section

The data regarding the figures are available for download from Zenodo (Aygün & Čadek, 2023).

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