Benford's law as mass movement detector in seismic signals

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July 20, 2023

Abstract

Seismic instruments placed outside of spatially extensive hazard zones can be used to rapidly sense a range of mass movements. However, it remains challenging to automatically detect specific events of interest. Benford's law, which states that first non-zero-digit of given datasets follow a specific probability distribution, can provide a computationally cheap approach to identifying anomalies in large datasets and potentially be used for event detection. Here, we select raw seismic signals to derive the first-digit distribution. The seismic signals generated by debris flows, landslides, lahars, and glacier-lake-outburst floods follow Benford's law, while those generated by ambient noise, rockfalls, and bedload transports do not. Focusing on debris flows, our Benford's-law-based detector is comparable to an existing random forest method for the Illgraben, Switzerland, but requires only single station data and three non-dimensional parameters. We suggest this computationally cheap, novel technique offers an alternative for event recognition and potentially for real-time warnings.

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12 Key Points:

- The first-digit distribution of seismic signals generated by high-energy mass movements
 and fluvial processes follows Benford's law
- When Benford's law appears, raw seismic signals tend to increase exponentially and converge to a power law distribution with exponent one
- A computationally cheap and novel detector based on Benford's law is developed for
 debris-flow events

19 Abstract

Seismic instruments placed outside of spatially extensive hazard zones can be used to rapidly 20 sense a range of mass movements. However, it remains challenging to automatically detect 21 specific events of interest. Benford's law, which states that first non-zero-digit of given datasets 22 follow a specific probability distribution, can provide a computationally cheap approach to 23 identifying anomalies in large datasets and potentially be used for event detection. Here, we 24 select raw seismic signals to derive the first-digit distribution. The seismic signals generated by 25 debris flows, landslides, lahars, and glacier-lake-outburst floods follow Benford's law, while 26 those generated by ambient noise, rockfalls, and bedload transports do not. Focusing on debris 27 flows, our Benford's-law-based detector is comparable to an existing random forest method for 28 the Illgraben, Switzerland, but requires only single station data and three non-dimensional 29 30 parameters. We suggest this computationally cheap, novel technique offers an alternative for event recognition and potentially for real-time warnings. 31

32 Plain Language Summary

Natural hazards, such as debris flows and landslides, pose a significant threat to the exposed 33 34 communities. Seismic instruments as seen as effective tools for detecting these hazardous processes and may be used in early warning systems. However, the difficulty lies in identifying 35 the events of interest concisely and objectively. Our study explores Benford's law, a probability 36 distribution of the first-non-zero digit. We collected seismic data generated by various hazard 37 events and compared the observed first-digit distribution with their agreement with Benford's 38 law. We found seismic signals of high-energy mass movements follow Benford's law during the 39 running phase, while ambient noise and other small mass movements do not. In order to explain 40 why Benford's law is followed, we argue that raw signals increase exponentially and fit a power 41 law distribution with exponent as one. Our detector, based on Benford's law and designed for 42 debris flow, which is a computationally cheap and novel model, performs similar to a machine 43 learning algorithm previously used in the study site. Our work illustrates a new approach to 44 detecting events and designing warning systems, which can be used in different regions. 45

Keywords Environmental seismology, mass movement, Benford's law, event detector, debris
 flow, early warning system.

48 **1 Introduction**

Mass movements (e.g., landslide and debris flow) and extreme fluvial processes (e.g., 49 50 flash floods and glacier-lake-outburst flood) are of significant concerns in populated areas, as they can cause huge loss of life and damage to civil infrastructure each year (Holub & Hübl, 51 2008; Merz et al., 2021; Regmi et al., 2015). Classification criteria for mass movements may 52 vary depending on the focus of interest (Coussot & Meunier, 1996; Nemčok et al., 1972). Yet, 53 the most widespread and destructive mass movements are generally considered to be debris 54 flows, landslides, and rockslides (Dowling & Santi, 2014; Froude & Petley, 2018). Despite 55 56 extensive efforts to mitigate their hazard through risk assessment and structural measures (Dai et al., 2002; Fuchs et al., 2007; Huebl & Fiebiger, 2015), the intricate geological conditions and 57 dynamic processes of mass movements frequently pose challenges in preventing property 58 59 damage and fatalities (Fan et al., 2019; Kean et al., 2019; Tiwari et al., 2022).

60 Early warning systems are an established approach to mitigating the impact of mass 61 movements (Badoux et al., 2009; Guzzetti et al., 2020; Hürlimann et al., 2019). For example,

systems based on measured rainfall intensity and predefined thresholds for triggering alarms 62 (Baum & Godt, 2010; Marra et al., 2016) are among the most popular warning approaches. 63 However, maintaining rain gauges and obtaining accurate rainfall intensity data in real-time is 64 challenging for the operation of a warning system, especially for catchments with large elevation 65 differences. Inaccurate measurements and uncertainty in data interpolation lead to significant 66 errors in rainfall thresholds (Nikolopoulos et al., 2015). In addition, due to the variability in 67 geological and hydrological conditions, empirical thresholds for triggering debris flows and 68 landslides are not transferable between catchments (Gregoretti et al., 2016; Wilson & Wieczorek, 69 1995). Detecting specific events of interest from time-series signals is essential for releasing a 70 warning. Force plates, radar, laser, and video cameras are the most common sensors used for 71 monitoring in early warning systems (Comiti et al., 2014; McArdell et al., 2007). However, some 72 of these devices require a high-power supply and regular maintenance, and can be easily 73 destroyed by the hazard processes itself. 74

Continuous seismic and acoustic signals offer a new way to monitor mass movements 75 with high temporal resolution (Le Breton et al., 2021; Burtin et al., 2016; Cook & Dietze, 2022; 76 Farin et al., 2019; Schimmel et al., 2013). The instruments can be installed outside the zones 77 affected by the hazard and are thus in lesser danger of being destroyed. An array of seismic 78 79 stations can help to detect and locate extreme, high-energy events on a regional scale (Cook et al., 2021; Ekström & Stark, 2013; Hammer et al., 2012). However, a seismic station records all 80 ground vibration signals within its bandwidth, blending events of interest and those considered as 81 noise. Current seismology-based detectors of mass movements and fluvial processes, such as 82 seismic attributes-based methods (Dietze et al., 2022; Govi et al., 1993; Schimmel & Hübl, 2016; 83 Wei & Liu, 2020), short-term average to long-term average ratio (Coviello et al., 2019), random 84 forests (Hibert et al., 2019; Provost et al., 2017), and hidden Markov models (Dammeier et al., 85 2016; Hammer et al., 2012) require numerous waveform, spectral, network features or 86 parameters to be fed into the model to identify events. In addition, collecting and labeling the 87 data to parameterize or train such a model is time-consuming and requires experience. Applying 88 89 these existing approaches to other sites requires re-training the model or calibrating the parameters; worse, often no historical data are available for most new sites to do this. Before 90 warning systems can be constructed, implemented, and promoted, a convenient and portable 91 approach to event detection must be found. Compared to ambient noise and signals not 92 associated with extreme events in a natural environment, the temporal occurrence probability of 93 mass movements is relatively low. Therefore, detecting debris flows and other mass movements 94 95 in seismic time-series signals can be treated as an anomaly detection.

The Newcomb–Benford law (BL) or the first-digit law, which is widely used in fraud and data quality detection, is a probability distribution of the first digit of a dataset (Castañeda, 2011; Cho & Gaines, 2007; Ley, 1996). Newcomb (1881) stated that the probability of occurrence of the first digits is such that the mantissae of their logarithms are equally probable:

		$P(d) = \log_{10}(1 + d^{-1})$	(1)
100	where $P(d)$ is the theoretical	probability of the first none z	dero digits, $d = \{1, 2,, 9\}$. For
101	example, -0.01 and 100 share	one as the same first digit w	ith a likelihood of 0.301. Frank
102	Benford rediscovered this relati	onship tested it with twenty dif	ferent datasets. It was later named

Benford rediscovered this relationship, tested it with twenty different datasets. It was later named after him as Benford's law (Benford, 1938). BL has been used to several fields of the geosciences, such as in studying the homogeneity of natural hazard datasets and anomalies (Geyer & Martí, 2012; Joannes-Boyau et al., 2015). Earthquakes and Mars quakes were detected
 in seismic signals with BL (Díaz et al., 2015; Sambridge et al., 2010; Sun & Tkalčić, 2022). Due
 to the dimensionless and low computational cost of BL, it has the potential to be used to identify
 mass movements in seismic data at different catchments, perhaps even as detector in data
 loggers.

In this study, we compiled seismic data generated by various mass movements and fluvial processes, calculated the first-digit distribution of seismic signals, and investigated which processes or periods follow the BL. We explain why BL appears in seismic signals generated by some of the processes and not by others. Finally, we present a BL-based event detector for debris flows and compare its performance with a previously developed random forest model (Chmiel et al., 2021) for the same seismic network. This work shows a novel approach for detecting highenergy mass movements and the potential for establishing a real-time warning system using BL.

117 2 Data Source and Event Catalog

118 **2.1 Study Site and Data Source**

The Illgraben catchment near the village of Leuk, southwest Switzerland (Figure 1a) is 119 one of the most active debris flow catchments in the Alps. It covers an area of about 9.5 km² and 120 extends from the Rhône River at 610 m to the Illhorn Mountain, peaking at 2716 m (Badoux et 121 al., 2009). The annual rainfall is concentrated from May to October, and the Illgraben catchment 122 roughly experiences three to five debris flows and several floods each year, mainly triggered by 123 short-duration convective storms (McArdell et al., 2007). To mitigate the risk of debris flows and 124 floods, a warning system has been implemented at the Illgraben that triggers an alarm when the 125 126 impulse of in-torrent ground vibration sensors exceeds empirically determined thresholds (Badoux et al., 2009). 127



Figure 1 Study area and debris flow at Illgraben catchment. (a) Location and distribution of seismic stations (red stars, Table S1 for details). (b) to (d) are the spectrograms of the vertical component for a debris flow event between 14:40 and 18:00, 12th July 2014.

Considering that seismic data are not available for all stations for the whole year and the complexity of signals from stations far from the spatially propagating event, we mainly selected the IGB02 station for this study (same location as ILL02/ILL2 deployed by Swiss Federal Institute for Forest, Snow and Landscape Research WSL), which is closest to the channel and far from the nearby residential area of Leuk.

137 **2.2 Event Catalog**

To calculate the first-digit distribution of seismic signals and quantify which processes or 138 139 periods follow BL, we examined 24 debris flows (one of which may be a flood event) that occurred 2013-2014 and 21 debris flows that occurred 2017-2019 (Tables S2-S3) in the Illgraben 140 catchment. For the 2013-2014 debris flows, ten of the 24 events were recorded by local warning 141 systems (WSL events), and we manually labeled an additional 14 debris flows based on the event 142 duration of waveforms and the 1-50 Hz features of the spectrogram (GFZ events, Text S1). One 143 example of the debris flow that occurred on 12th July 2014, with the WSL label and high signal-144 to-noise ratio SNR (about 20 based on IGB02), is shown in Figures 1b-1d. 145

To complement the data with events from other locations and instruments for calculating the first-digit distribution, we added seismic signals from other mass movements and fluvial processes (Table S4), such as a 2013 rockfall event in Illgraben, Switzerland (Burtin et al., 2016), a 2015 rockfall event in Lauterbrunnen, Switzerland (Dietze et al., 2017), a 2014 landslide in Askja, Iceland (Schöpa et al., 2018), a 2015 hurricane-induced lahar in Volcán de Colima, Mexico (Capra et al., 2018), a 2016 glacial-lake-outburst flood GLOF in Bhotekoshi, Nepal (Cook et al., 2018), and a 2019 bedload transport event in Liwu catchment, Hualien.

153 **3 Methods**

154 **3.1 Data Preparation**

Processing seismic signals using demeaning, detrending, filtering, or deconvolution may 155 alter the first-digit distribution and obscure the difference between BL in ambient noise and the 156 event phase (Figure S1). Therefore, we use the raw vertical-component seismograms (units are 157 counts) to calculate the first-digit distribution and check whether the observed distribution 158 adheres to BL. We choose a one-minute moving window (no overlap) to calculate the probability 159 distribution of digits one to nine to avoid statistical errors associated with a small dataset. The 160 number of data points (n) for each window is equal to the moving window length (W_{l} , units are 161 seconds) multiplied by the sampling frequency (f_S , units are Hertz, Table S1 for details): 162

				$n = W_L * f_S$		(2)
	 -	-			 	

163 For each window, data points with a raw amplitude equal to zero are discarded.

164**3.2 First-digit Distribution and Benford's Law**

We used two established statistical methods, the Chi-squared test (Geyer & Martí, 2012;
 Patefield, 1981) and the Kolmogorov-Smirnov test (Kaiser, 2019; Feller, 1948), to validate

167 whether the observed first-digit distribution follows BL. The hypothesis is that the frequency of 168 the observed first digits is not distinct from the theoretical BL values, or both represent the same 169 distribution. We define that a *p*-value greater than 0.95 for any test means acceptance of the 170 hypothesis. The observed first-digit distribution is considered consistent with BL if the 171 hypotheses of two tests are accepted. In addition, the goodness of fit φ introduced by Sambridge

et al. (2010) is used to evaluate the difference between the observed distribution and BL:

$$\varphi = \left(1 - \left(\sum_{D=1}^{9} \frac{(f_{obs_d} - f_{BL_d})^2}{f_{BL_d}}\right)^{1/2}\right) \times 100\%$$
(3)

173 where f_{obs} and f_{BL} are the observed digit frequency and theoretical probability of BL, $d=\{1, 2, ..., 9\}$. A value of φ closer to one means that the distribution is closer to the theoretical BL value.

To investigate the occurrence of BL in seismic signals, we examine the relationship between the time series of seismic signals and their corresponding first-digit distribution. This analysis allows us to understand the underlying factors contributing to the emergence of BL during specific processes or periods.

For the processes or periods that follow BL, we examine the relationship between the time series of seismic signals and their first-digit distribution to investigate why BL appears. In the time domain, the raw seismic signal S(t) is a function of time (t) and can be described with an interquartile range *iq* as magnitude changes of the measurements. Here, the seismic signals before the optimal goodness of fit $\varphi_{optimal}$ were selected to fit an exponential curve for the increased parts (Text S3 for details):

$S(t) = a * e^{b^*t} + c$	(4)
$iq = Q_{75} - Q_{25}$	(5)

where S is the seismic signals (units are counts), t is time (units are second), and a, b, and c are the coefficients of the exponential function. Q_{75} and Q_{25} are the upper and lower quartile of the data for each window.

Previous studies have demonstrated that datasets with a power law relationship (exponent one) in data pairs satisfy BL, such as the data on many hydrological phenomena (Nigrini & Miller, 2007). In this study, we assume that the seismic data in all one-minute moving windows have this power law distribution, then the data of each window were selected and sorted from smallest to largest (rank order) to calculate α by Equation (6-7) based on its magnitude (Newman, 2005). We subsequently examine whether the seismic data follow the power law with exponent one when BL appears:

$p(x) = C * x^{-\alpha}$	(6)
$\alpha = 1 + n \left(\sum_{i=1}^{n} \ln \frac{x_i}{x_{\min}} \right)^{-1}$	(7)

where p(x) is the seismic data in any one-minute moving windows. *C* and α are the coefficients of the power law function. x_i , and x_{\min} are the *i*-th data and minimum data in the dataset of length *n*.

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3.3 Data Preparation Debris Flow Detector Implementation

To demonstrate the application of BL, we developed a two-fold debris flow classification 199 detector using seismic data from Illgraben. There are three non-dimensional parameters in our 200 classification model to reduce uncertainty (Figure S2): the ratio between the interquartile range 201 iq at time i and its average value of the previous 20 minutes (R_{ia}) , the power exponent at time i 202 $(\alpha_i, \text{Equation 6})$, and the averaged power exponent for d minutes after time i (α_d) . We define the 203 debris flow (positive events) by manual interpretation of the seismic data (Text S1 and S4). With 204 a one-minute moving window, all events with all three non-dimensional variables are scanned, 205 and the detector returns either a positive (debris flow) or negative (not debris flow) labels. 206 207 Finally, in order to test the sensitivity of moving window size or the number of data points for each window, a varying window length from 1s to 600s in a one-second interval was chosen to 208 test the variation of the power law exponent. 209

Our dataset includes 14 manually marked debris flow events out of a total of debris flow 210 24 events, we divided the available dataset into a training dataset (24 events, 2013-2014) and a 211 validation dataset (21 events, 2017-2019). The details to define positive and negative cases for 212 training and validation are described in Text S4. We used a confusion matrix to evaluate our 213 detector performance (Beguería, 2006; Staley et al., 2013). The definition of true positive (TP), 214 215 true negative (TN), false positive (FP), false negative (FN), F1 score (F1), and Threat Score (TS) are given in Text S4. The detector model is considered the best when F1 is one or closest to one. 216 We compare the validation results with an existing random forest model trained with data from 217 2017 to 2019 recorded by the same seismic network using more than 70 seismic features (Chmiel 218 et al., 2021). 219

220 4 Results

4.1 Benford's Law and Seismic Signals

BL was observed in 38 out of the 45 debris flows events, while it was absent for two 222 events from the 2013-2014 GFZ dataset and five events from the 2017-2019 WSL dataset 223 (Figures 2f-2g, S3-S5 and Tables S2-S3). In the debris flow events that follow BL, we observed 224 that both Chi-squared and Kolmogorov-Smirnov tests only accept the hypothesis during the 225 running phase. For example, the debris flow event on 12th July 2014 (Figures 1b and 2a-2d) 226 exhibits $\varphi_{optimal}$ of 87.97% and suggests that the first-digit distribution of seismic signals follows 227 BL. Moreover, the first-digit distributions of station IGB03 and IGB04 are similar to what was 228 observed for this event (Figure 2e). In addition, for other mass movements, the first-digit 229 distribution of the seismic signal generated by the landslide (Figure S8) and the lahar (Figure S9) 230 231 also exhibit BL. However, our two rockfall cases failed to follow BL (Figures S6-S7).



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For fluvial processes, the 2014 flood case in Illgraben only exhibits a one-minute window obeying BL (Figure S3), but the GLOF obeys BL for a much longer period of 8 minutes (Figure S10). In contrast, the bedload transport cannot be distinguished by BL (Figure S11). Interestingly, the first-digit distribution of seismic waveform generated by long-period seismic signals and when amplitude is close to zero counts fluctuation (named LP0), also follows BL (Figure S12).

243 **4.2 Empirical Analysis for BL**

As the debris flow front approaches, both the *iq* and φ rapidly increase in the time series domain (Figure 3a). We observed a good fit between seismic signals *S* and time *t* using the exponential function. The kernel density and coefficient of determination R^2 were used to show the exponential fitting difference between the event ($\varphi_{optimal}$ period) and noise (manually labeled event start time period). The averaged R^2 of 2013-2014 WSL and 2017-2019 WSL label events are 0.853 and 0.840, respectively, and the averaged R^2 of 2013-2014 GFZ label events is 0.649, however, the raw amplitude data of the noise period could not be fitted with an averaged R^2 of 0.434 (Figure S13a-b). For instance, event 2014-07-12 has a R^2 of 0.944 (Figure 3a). The same fitting method did not yield an exponential curve for the noise period data of event 2014-07-12 (Figure S14).

In addition, we found that the α for the debris flow, lahar, landslide, and GLOF is 1.10-1.13 (Figure 3b). However, the exponent of two rockfall cases, bedload transport case, and most ambient noise is much higher than one. Values of α close to one could also be observed for ambient noise generated by LP0. The kernel density of α of all 38 BL-obeying debris flow events

is much closer to one than the noise period (Figure S13c-d).



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Figure 3 Correlation between seismic signals and BL, and BL-based debris-flow detector performance. (a) *iq* changes in the seismic signals for a debris flow event and exponential fitting (Exp. fitting). (b) Power law relationship in raw amplitude for different events. (c) F1 score for debris-flow event detectors (Figure S15 for details). (d) Confusion matrix of the best detector for training and output of validation dataset under the same parameters.

4.3 Debris-flow Detector Based on BL

The power exponent, which is calculated from the ten-minute average value when the 266 debris flows front approaches station IGB02, converges with an increasing moving window 267 (Figure S16a). During the training procedure, the performance of the detector was examined by 268 the 2013-2014 dataset (Figures 3c and S15-S16). Results show that the power law exponent 269 270 $(\alpha_i=1.25)$, interquartile range ratio $(R_{ia}=4)$, and event duration $(\alpha_d=20 \text{ minutes})$ produce the best detector, yielding an F1 score of 0.884 (TS=0.792, Figure 3c). Under this set of parameters, there 271 are five false-negative events in the training dataset (two events do not follow BL, Figure S17-272 S21). These three optimal parameters ($\alpha_i=1.25$, $R_{iq}=4$, $\alpha_d=20$) obtained from the training 273 procedure were tested with the validation dataset. We found that the detector produced a higher 274 TPR=0.905 (F1=0.704, TS=0.543) than the training procedure (Figure 3d). However, two false-275 negative cases for the validation dataset were observed, 14 cases were mislabeled as positive 276 events (false positive) in the validation catalog (Figure 3d). 277

278 **5 Discussion**

279 **5.1 Why Do Some Seismic Datasets Follow BL**

280 The results show that BL is an efficient approach for detecting high-energy mass movements and some fluvial processes with seismic signals. The processes that do follow BL 281 (debris flow, landslide, lahar, and GLOF) usually contain more kinetic energy during the running 282 process than cases that do not follow BL (ambient noise, rockfall, and bedload transport). 283 Interestingly, when the raw waveform is close to zero fluctuation (e.g., one-minute amplitude 284 data between -227 and 342 counts, Figure S12), the observed first-digit distribution could also 285 follow BL. For rockfall and other events that do not follow BL, we argue that the highly 286 attenuated signals or low SNR make it difficult to distinguish between an event and ambient 287 noise in the raw waveform domain. Generally, the low SNR is due to geometric spreading and 288 289 anelastic attenuation, the energy and amplitude of the signals dissipate during propagation, especially for high-frequency waves (Battaglia, 2003; Tsai & Atiganyanun, 2014). 290

For seismic datasets, Sambridge et al. (2010) first stated that a sufficient dynamic range 291 may lead the first-digit distribution (e.g., seismic signals generated by an earthquake) to follow 292 BL. However, the claim that regularity and large spread imply BL is not always correct (Berger 293 294 & Hill, 2011). In theory, datasets crossing several orders of magnitude do not necessarily follow BL (Figure S22a); in practice, we found that the seismic signals that cross two orders of 295 magnitude within one minute follow BL (iq 119 counts, Figure S12). For teleseismic events and 296 297 local seismicity, Díaz et al. (2015) observed using both natural and artificial data that compliance to BL does not depend primarily on the dynamic amplitude range, but rather relates to changes in 298 frequency content. Yet, it is nearly impossible to obtain seismic data in the field that feature only 299 frequency changes without changes in the dynamic range. For seismic signals generated by high-300 energy mass movements or fluvial processes, signals usually have significant changes in both 301 302 magnitude (> two orders) and frequency (>1 Hz change in central frequency) when compared to an earthquake (Figures 2a and 3a). In theory, the sole change in the frequency domain does not 303 necessarily cause compliance to BL (Figure S22b). We propose two possible mathematical 304 explanations for the appearance of BL in seismic signals. Firstly, when data adheres to BL, it 305 follows Zipf's law (Newman, 2005). BL appears exactly when the scaling exponent $\alpha=1$ 306 (Pietronero et al., 2001), so empirical values of the exponent close to one in seismic data will 307

yield compliance to BL. Secondly, BL appears exactly for processes that rise or fall exponentially in time, which mathematically corresponds to a mapping from a linear to a logarithmic space (Cong et al., 2019; Engel & Leuenberger, 2003). Thus, processes that develop with exponential dynamics in time can be expected to follow BL (Figure 3a). This implies that the data from exponentially evolving processes will also follow Zipf's law with a scaling exponent of one.

We suggest that events with an exponentially rising signal can follow BL is primarily 314 caused by their spatial mobility. The amplitude of an approaching seismic source is controlled by 315 the ground quality factor (as an exponential term) and source-receiver distance (as 1 over the 316 square root of the distance) (Burtin et al., 2016). As long as the distance at which a process emits 317 sufficient energy to be detected by a seismometer is much larger than the channel-sensor distance 318 (Figure 1a), a fast-moving mass will produce a signal sufficiently close to exponential increase 319 (Dietze et al., 2022) to be in agreement with BL. In other words, BL is an efficient detector of 320 fast approaching seismic sources at the landscape scale. 321

322 **5.2** Application of BL as early warning tool

BL is a computationally cheap and novel approach to detect debris flow and establish 323 real-time warning systems. Since only the raw data need to be counted, the computation time for 324 parameter preparation and model evolution is strongly reduced, e.g., our validation process could 325 326 be completed in 113 seconds (Figure S2). We expect that BL can be applied to different sites without a change in parameter values, because of the predefined non-dimensional variables and 327 328 their general applicability. Therefore, we suggest that the dimensionless detector input parameters are independent of catchment geometry and seismic station characteristics. 329 Furthermore, our approach could be a simplified version of an early warning system for 330 triggering or turning on high-power supply and data transmission devices to catch events, such as 331 radar and laser, for full-scale warning. In practice, a BL-based early warning system can be 332 implemented using data from two or three seismic stations along the main flow path to detect and 333 cross-validate events. 334

Our purpose in this paper is to explore the potential of BL as a prerequisite to developing 335 an operational event detector or warning system for debris flows, which can be adapted to other 336 processes as well. An efficient real-time warning system requires the rapid detection of the event 337 of interest, and signal processing plays a critical role in validating seismology-driven warning 338 systems (Arattano et al., 2014, 2016; Coviello et al., 2015). By using BL, high-energy processes 339 lasting longer than a few minutes can be reliably distinguished from background noise. Without 340 changing input parameters, our detector achieves a detection accuracy of 0.905 and 0.982 for 341 both debris flows and non-debris-flow events in the validation catalog (Figure 3d). This is 342 similar to the detection accuracy from a random forest model calibrated in the same catchment 343 (Chmiel et al., 2021), which gives 0.83 and 0.94 for debris flows before or after check dam 1 344 (marked in Figure 1a) and 0.92 for non-debris-flow events, respectively. Any supervised 345 machine-learning-based model, including those based on random forests, requires a large 346 training dataset from multiple seismic stations and many seismic features. The efficacy of our 347 debris-flow detector is at least comparable to the random forest model, but does not require 348 recalibration of parameters. Furthermore, the false-positive example of our model in the 349 validation catalog can be filtered out using data from multiple seismic stations (Figure S23), and 350 a full seismic network can improve true positive detection accuracy (Figures S24-S25). 351

More mass movements are needed to help understanding the scope of application of BL and the detector proposed in this paper needs to be further explored. This study suggests that a single seismic station could efficiently detect events such as debris flow that move continuously in the channel. However, the BL approach neglects frequency domain information, which could be used to improve the identification of other high-energy mass movements or fluvial processes type.

358 6 Conclusion

Detecting events of interest from seismic signals to establish early warning systems is 359 critical for hazard mitigation. In this study, we demonstrated that the first-digit distribution of 360 seismic signals generated by some high-energy mass movements and fluvial processes follows 361 Benford's law. Our detector model offers a less computationally intensive and novel approach for 362 extracting anomalous energetic events, such as debris flows and landslides, from massive seismic 363 364 signals. Moreover, the high-energy mass movement detector provides a promising strategy for building warning systems using seismic signals to mitigate hazards. In the future, we will collect 365 more mass movements to calculate their first-digit distribution and develop a seismic network-366 367 based detector system and implement our method for real-time detection.

368 Acknowledgments

The authors thank Dr. Lucia Capra provide the lahar data. This study is funded by the State Key Laboratory of Geohazard Prevention and Geoenvironment Protection Open Fund (SKLGP2023K003) and National Natural Science Foundation of China (42120104002).

372 **Open Research**

The seismic data between 2013 and 2014 is available 373 at https://doi.org/10.14470/4W615776. Please refer to Chmiel, M., Walter, F., Wenner, M., Zhang, 374 375 Z., McArdell, B. W., & Hibert, C. (2021). Machine learning improves debris flow warning. Geophysical Research Letters, 48, e2020GL090874. https://doi.org/10.1029/2020GL090874 for 376 2017 to 2019 seismic data. Our figures were generated using R ESEIS, Obspy, Matplotlib and 377 Seaborn. All seismic data processing codes are available in https://github.com/Nedasd/Benfords-378 379 law-in-environmental-seismology.git

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manuscript submitted to Geophysical Research Letters

Supporting Information for

Benford's law as mass movement detector in seismic signals

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Introduction

This file includes supplementary text, figures, tables, and code for the manuscript titled *Benford's law as movement detector in seismic signals.*

Supplemental Text1 presents a comprehensive description of the methods employed to label debris flow events between the years 2013 and 2014. **Supplemental Text 2** illustrates the seismic data source and the catalog of debris flow events from 2017 to 2019. **Supplemental Text3** provides an elaborate explanation of the methods employed to fit the exponential curve as debris flow approaches the seismic station IGB02. Lastly, **Supplemental Text4** offers additional information regarding the definition of the training and validation classes, as well as details about the detector model elevation.

Figure S1 demonstrates the effects of different seismic data processing methods on Benford's Law (BL). **Figure S2** shows the input parameters of the debris flow detector model. **Figures S3** to **S4** and **Figure S5** display the BL results of training events between 2013 and 2014 and between 2017 and 2019, respectively, with a focus on partial event representations. **Figures S6** to **S9** present the BL results of different mass movements, and **Figures S10** to **S12** exhibit the BL results of various fluvial processes. **Figure S13** show the kernel density for exponential fitting and power law exponent of noise and 38 BL-followed events. **Figure S14** display an example of exponential fitting curve results. **Figure S15** demonstrates the sensitivity of the debris flow detector model to different input parameters. **Figure S16** show the variation of power law exponent with different moving window sizes and detectors receiver operating characteristic. **Figures S17** to **S21** depict false-negative events encountered during the training process. **Figure S22** illustrates the relationship between BL and dataset range/signal frequency. **Figures S23** to **S25** highlight the potential of using a seismic array to improve detection performance, reduce the accuracy of false negative and improve the accuracy of true positive.

Table S1 provides parameters of seismometer stations for 2013-2014 and 2017-2019. **Table S2** to **S3** offer detailed information on start and end times, and whether BL was followed for the 24 training and 21 validation events. **Table S4** presents the seismic data source for different mass movements and fluvial processes to examine compliance with BL. **Table S5** displays the exponential fitting coefficients for all 45 debris flow events.

Text S1.

Methods to label debris flow training events (2013-2014)

In order to build a training events catalog using data from 2013 to 2014, we utilized the data recorded by the seismic station IGB02 (located at coordinates 46.2863735252704, 7.62780978682399, elevation 933m, and 60m away from the trunk channel) to extract debris-flow events. The Swiss Federal Institute for Forest, Snow and Landscape Research (WSL) warning systems provides traces for labelling ten debris flow events that occurred during the year 2013-2014, and we manually selected the event start and end time based on the seismic signals of station IGB02 (Table S2).

For debris flow events that may have occurred but were not recorded by the warning systems, we used the data from the seismic station IGB02 to extract events. The data were filtered between 1 and 90 Hz, and the spectrograms were plotted on a daily basis. Then the spectrograms were manually analyzed to retrieve the events. Here we refer to the methodology described in Burtin et al. (2014, 2016), Chmiel et al. (2021), Belli et al., (2022) and mainly focus on two main features that (1) the event duration of waveforms (debris flow should last more than 20 minutes), and (2) frequency features of the spectrogram (the characteristic frequency between 1 and 50 Hz, and peak frequency is around 7 Hz). For debris flows, we do not qualify an event corresponding to the surge, one debris flow could have different surges, and all these surges were merged as one event. Please refer to Table S2 for detailed catalog information. Because we do not have other data to validate the manually extracted events, it may introduce uncertainty.

Text S2.

Data source of debris flow validation events (2017-2019)

Between 2017 and 2019, there were 22 events collected by WSL (Chmiel et al., 2021), while the event 2017-05-19 is not available for ILL02. This event does not be included in the validation dataset. For event details, please refer to the supporting information for "Machine Learning improves debris flow warning". <u>https://doi.org/10.1029/2020GL090874</u>



Locations of seismometers at the Illgraben catchment from 2017 (red), 2018-2019 (yellow).

Text S3.

Methods to fit the exponential curve

For each BL-followed debris flow event, we first identify the one-minute time window t_i corresponding to the optimal goodness of fit within the manually labeled start and end times. Since the speed of debris flows can vary, the time required to record their approach by the seismometer may differ. To account for this variability, we select different lengths of raw waveforms (1 to 5 minutes before t_i , by minute) to fit an exponential curve. We then calculate the interquartile range (*iq*) of the raw waveforms using a 1-second sub-window. Next, we extract the dataset **X** during and before t_i . In cases where *iq* decreases at some points, we feed the data before the optimal of the **X** dataset to the *scipy.optimize.curve_fit* algorithm to obtain the exponential curve (Equation 1) and correlation coefficient R². Finally, we select the optimal fitting curve (optimal R²) based on all fitting, and the results are listed in Table S5.

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where S is the seismic signals (unit by counts), t is time (unit by second), and a, b, and c are the coefficient of the exponential function. If b is smaller than zero, then R^2 is zero.

The codes are available in content **Code S1**.

Text S4.

Define training and validation class

Our dataset has two labels: positive (debris flow) and negative (not debris flow). During the training process, the 24 events (Tables S2) between 2013 and 2014 were labeled positive based on data from station IGB02. An additional 1200 negative cases were randomly selected from outside the event at random start times and durations (between

20 minutes and 6 hours). The ratio between the positive and negative cases (P2N) is 0.02. During the validation process, the 21 debris-flow events between 2017 and 2019 (Tables S3) were labeled positive based on stations ILL02 and ILL12. A further 1050 negative cases were randomly selected using the same method as the training procedure with the same P2N ratio. The validation dataset of 1071 events was processed using the optimal detector determined during training, and the validation results with an existing random forest model trained with data from 2017 to 2019 recorded by the same seismic network using more than 70 seismic features (Chmiel et al., 2021).

Define TP, TN, FP, and FN class

We define the confusion matrix as follows: A true positive (TP) is an event that is classified as a debris flow by observation and our detector. A true negative (TN) is an event that is not classified as a debris flow by either observation or our detector. An event is labeled false positive (FP) when it is labeled as a debris flow by our detector but considered as a non-debris-flow event in observation. An event is labeled as false negative (FN) when an observed debris flow is not classified as debris flow by the detector.

Evaluate detector model

A good classification model should maximize the number of true-positive predictions and minimize the number of false-positive and false-negative predictions. The overall performance of the classifier is quantified as a true-positive rate (*TPR*, recall rate or sensitivity), false-positive rate (*FPR*, fall-out rate), false-negative rate (*FNR*, miss rate), and true-negative rate (*TNR*, specificity) which are calculated as Equation 2-5:

$TPR = \frac{TP}{TP + FN}$	(2)
$FPR = \frac{FP}{FP + TN}$	(3)
$TNR = \frac{TN}{TN + FP}$	(4)
$FNR = \frac{FN}{FN + TP}$	(5)

where TP, TN, FP, and FN are the number of true-positive, true-negative, false-positive and false-negative events.

We calculated the F-score or F1 score (F1) and the Threat Score (TS), which are commonly used to represent the predictive capability of classification models (Equation 6-7). The F1 measures the accuracy of a binary classification model and is a harmonic mean of precision and recall. When F1 gets the highest possible value (one), it means that the precision and recall are perfect, and F1 receives the lowest value (zero) when either the precision or the recall is zero. The TS is a measure of the overall performance of the classification model. In a perfect model, the threat value would be equal to one, with each

false prediction (false-negative or false-positive event) decreasing the value of the threat score.

$F1 = \frac{2TP}{2TP + FN + FP}$	(6)
$TS = \frac{TP}{TP + FN + FP}$	(7)

The results of F1 and TS are displayed in Figure S15, the detector model is considered the best when F1 is maximum.



Figure S1. Effects of data processing methods on BL, coding by R ESEIS Dietze (2018) . The red dashed vertical lines are manually labeled start and end time. The codes are available in **4. Codes**.



Figure S2.	The input	dimensionless	parameters and	range of ever	nt detector.
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The input parameters of detector in Figure S2 are listed below:

$R_{iq} = iq_i / iq_{i20},$	(8)
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where iq_i is the parameter 1, interquartile range ratio of time *i* and iq_{i20} is the average interquartile range of 20 minutes before the time *i*.

Range of input parameters					
Interquartile range (<i>iq</i>)	Power law exponent	Event duration <i>d</i> (unit: minute)			
ratio	α_i	or α_d			
From 2 to 10	From 1.01 to 1.50	From 3 to 20			
Optimal parameters from training data					
<i>iq</i> = 4	$\alpha_i = 1.25$	<i>d</i> = 20			

The validation process with optimal parameters could be completed in 113 seconds (do not count parameters calculating time, and codes were operated with 1 node and 48 G memory) via GFZ Cluster.



Figure S3. Raw waveform (a) and spectrogram (f) generated by a **flood** (2013-07-29, training events, WSL label, IGB02 station). (b) and (c) results from the Kolmogorov-Smirnov test and the Chi-squared test. One means to accept the null hypothesis that observed first-digit distribution is similar to BL's theoretical value; otherwise, the value is zero. (d) the goodness of fit φ in different colors represents the leading first digit in each one-minute moving window. First-digit distribution (e) of BL theoretical and observed optimal periods. The red dashed vertical lines have manually marked the start and end times. The green dashed vertical line is the time for an optimal goodness of fit (e).



Figure S4. Raw waveform (a) and spectrogram (f) generated by a **debris flow** (2014-06-19, training events, GFZ label, IGB02 station). (b) and (c) results from the Kolmogorov-Smirnov test and the Chi-squared test. One means to accept the null hypothesis that observed first-digit distribution is similar to BL's theoretical value; otherwise, the value is zero. (d) the goodness of fit φ in different colors represents the leading first digit in each one-minute moving window. First-digit distribution (e) of BL theoretical and observed optimal periods. The red dashed vertical lines have manually marked the start and end times. The green dashed vertical line is the time for an optimal goodness of fit (e).



Figure S5. Raw waveform (a) and spectrogram (f) generated by a **debris flow** (2019-10-09, validation events, WSL label, ILL12 station). (b) and (c) results from the Kolmogorov-Smirnov test and the Chi-squared test. One means to accept the null hypothesis that observed first-digit distribution is similar to BL's theoretical value; otherwise, the value is zero. (d) the goodness of fit φ in different colors represents the leading first digit in each one-minute moving window. First-digit distribution (e) of BL theoretical and observed optimal periods. The red dashed vertical lines have manually marked the start and end times. The green dashed vertical line is the time for an optimal goodness of fit (e).



Figure S6. Raw waveform (a) and spectrogram (f) generated by the **Rockfall 1** (2013-07-22 IGB01 station). (b) and (c) results from the Kolmogorov-Smirnov test and the Chi-squared test. One means to accept the null hypothesis that observed first-digit distribution is similar to BL's theoretical value; otherwise, the value is zero. (d) the goodness of fit φ in different colors represents the leading first digit in each one-minute moving window. First-digit distribution (e) of BL theoretical and observed optimal periods. The green dashed vertical line is the time for an optimal goodness of fit (e).



Figure S7. Raw waveform (a) and spectrogram (f) generated by the **Rockfall 2** (2015-04-06, Funny Rain station). (b) and (c) results from the Kolmogorov-Smirnov test and the Chi-squared test. One means to accept the null hypothesis that observed first-digit distribution is similar to BL's theoretical value; otherwise, the value is zero. (d) the goodness of fit φ in different colors represents the leading first digit in each one-minute moving window. First-digit distribution (e) of BL theoretical and observed optimal periods. The first event was a small long-distance earthquake. The red dashed vertical lines have manually marked the start and end times. The green dashed vertical line is the time for an optimal goodness of fit (e).



Figure S8-2. Raw waveform (a) and spectrogram (f) generated by the **landslide** (2014-07-21, MOFO station). (b) and (c) results from the Kolmogorov-Smirnov test and the Chi-squared test. One means to accept the null hypothesis that observed first-digit distribution is similar to BL's theoretical value; otherwise, the value is zero. (d) the goodness of fit φ in different colors represents the leading first digit in each one-minute moving window. First-digit distribution (e) of BL theoretical and observed optimal periods. The green dashed vertical line is the time for an optimal goodness of fit (e).



Figure S9. Raw waveform (a) generated by the a **hurricane-induced lahars** (2015-10-24 01:00, (Capra et al., 2018)). (b) and (c) results from the Kolmogorov-Smirnov test and the Chi-squared test. One means to accept the null hypothesis that observed first-digit distribution is similar to BL's theoretical value; otherwise, the value is zero. (d) the goodness of fit φ in different colors represents the leading first digit in each one-minute moving window. First-digit distribution (e) of BL theoretical and observed optimal periods. The green dashed vertical line is the time for an optimal goodness of fit (e). *No instrument response information is available for spectrogram*.



Figure S10. Raw waveform (a) and spectrogram (f) generated by a **glacial lake outburst flood** (2016-07-05, Hindi (NEP08) station, Cook et al., 2018). (b) and (c) results from the Kolmogorov-Smirnov test and the Chi-squared test. One means to accept the null hypothesis that observed first-digit distribution is similar to BL's theoretical value; otherwise, the value is zero. (d) the goodness of fit φ in different colors represents the leading first digit in each one-minute moving window. First-digit distribution (e) of BL theoretical and observed optimal periods. The green dashed vertical line is the time for an optimal goodness of fit (e).



Figure S11. Raw waveform (a) and spectrogram (f) generated by a **bedload transport** (from 2021-10-10 to 2021-10-14, TA64 station). (b) and (c) results from Kolmogorov-Smirnov test and the Chi-squared test. One means to accept the null hypothesis that observed first digit distribution is similar to BL's theoretical value; otherwise, the value is zero. (d) the goodness of fit φ in different colors represents the leading first digit in each one-minute moving window. First digit distribution (e) of BL theoretical and observed optimal periods. The black dashed vertical line is the time for an optimal of the goodness of fit (e).



Figure S12. Raw waveform (a) and spectrogram (f) generated by a long-period seismic signals (LP) and when LP is close to zero counts fluctuation (from 2014-03-30 to 2014-04-01, IGB02 station, follow BL). (b) and (c) results from the Kolmogorov-Smirnov test and the Chi-squared test. One means to accept the null hypothesis that observed first-digit distribution is similar to BL's theoretical value; otherwise, the value is zero. (d) the goodness of fit φ in different colors represents the leading first digit in each one-minute moving window. First-digit distribution (e) of BL theoretical and observed optimal periods. The green dashed vertical line is the time for an optimal goodness of fit (e). The max and min is -227 counts and 342 counts, the *iq* is 119 counts at 2014-03-31 22:22 to 22:23.



Figure S13. Kernel density for exponential fitting and power law exponent. (a) and (b) are the kernel density (bandwidth is 0.5) of R² of the exponential fitting. The coefficients of the exponential fitting for all events are listed in Table S5. (c) and (d) are kernel density (bandwidth is 0.5) of power law exponent α . The data displayed in (a) to (d) correspond to the optimal goodness of fit during 38 BL-followed debris flow events. T GFZ and T WSL represent training event datasets during 2013-2014 with GFZ and WSL labels, respectively. V WSL represents the validation event dataset during 2017-2019.



Figure S14. Exponential fitting of event 2014-07-12 debris flow noise.



Figure S15. F1 score and Threat score for different debris-flow detector parameters.



Figure S16. Variation of power law exponent with different moving window sizes and receiver operating characteristic of different detectors. (a) Averaged power law exponent α with different moving windows. The black dashed line indicates the 60 seconds window used in this work. (b) Receiver operating characteristic ROC of detectors with different parameters. The best detector (F1 0.884) obtained from the 2013-2014 training sample corresponds to FPR is 0, TPR is 0.792, and outputs of this detector for the sample from 2017-2019 FPR is 0.013, TPR is 0.905.



Figure S17. Raw waveform (a) and spectrogram (f) generated by a **debris flow** (**false-negative**, 2014-04-08, training events, GFZ label, IGB02 station). (b) and (c) results from the Kolmogorov-Smirnov test and the Chi-squared test. One means to accept the null hypothesis that observed first-digit distribution is similar to BL's theoretical value; otherwise, the value is zero. (d) the goodness of fit φ in different colors represents the leading first digit in each one-minute moving window. First-digit distribution (e) of BL theoretical and observed optimal periods. The red dashed vertical lines have manually marked the start and end times. The green dashed vertical line is the time for an optimal goodness of fit (e).



Figure S18. Raw waveform (a) and spectrogram (f) generated by a **debris flow** (**false-negative**, 2014-04-29, training events, GFZ label, IGB02 station). (b) and (c) results from the Kolmogorov-Smirnov test and the Chi-squared test. One means to accept the null hypothesis that observed first-digit distribution is similar to BL's theoretical value; otherwise, the value is zero. (d) the goodness of fit φ in different colors represents the leading first digit in each one-minute moving window. First-digit distribution (e) of BL theoretical and observed optimal periods. The red dashed vertical lines have manually marked the start and end times. The green dashed vertical line is the time for an optimal goodness of fit (e).



Figure S19. Raw waveform (a) and spectrogram (f) generated by a **debris flow** (**false-negative**, 2014-04-30, training events, GFZ label, IGB02 station). (b) and (c) results from the Kolmogorov-Smirnov test and the Chi-squared test. One means to accept the null hypothesis that observed first-digit distribution is similar to BL's theoretical value; otherwise, the value is zero. (d) the goodness of fit φ in different colors represents the leading first digit in each one-minute moving window. First-digit distribution (e) of BL theoretical and observed optimal periods. The red dashed vertical lines have manually marked the start and end times. The green dashed vertical line is the time for an optimal goodness of fit (e).



Figure S20. Raw waveform (a) and spectrogram (f) generated by a **debris flow** (**false-negative**, 2014-05-02, training events, GFZ label, IGB02 station). (b) and (c) results from the Kolmogorov-Smirnov test and the Chi-squared test. One means to accept the null hypothesis that observed first-digit distribution is similar to BL's theoretical value; otherwise, the value is zero. (d) the goodness of fit φ in different colors represents the leading first digit in each one-minute moving window. First-digit distribution (e) of BL theoretical and observed optimal periods. The red dashed vertical lines have manually marked the start and end times. The green dashed vertical line is the time for an optimal goodness of fit (e).



Figure S21. Raw waveform (a) and spectrogram (f) generated by a **debris flow** (**false-negative**, 2014-08-13, training events, GFZ label, IGB02 station). (b) and (c) results from the Kolmogorov-Smirnov test and the Chi-squared test. One means to accept the null hypothesis that observed first-digit distribution is similar to BL's theoretical value; otherwise, the value is zero. (d) the goodness of fit φ in different colors represents the leading first digit in each one-minute moving window. First-digit distribution (e) of BL theoretical and observed optimal periods. The red dashed vertical lines have manually marked the start and end times. The green dashed vertical line is the time for an optimal goodness of fit (e).



Figure S22. Synthetic amplitude dataset. (a) Datasets $[1, 10, ..., 10^7]$ with a first-digit of 1 that span 7 orders of magnitude but do not follow BL. (b) Datasets with a first digit of 1 that has a period of 0.1 Hz (data index from 0 to 0.1), 1 Hz (data index from 0.1 to 1.1), and 10 Hz (data index from 1.1 to 11.1) but do not follow BL.

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Figure S23. BL features of seismic signal generated by **Actual negative** class (2018-08-13, validation case). It was labeled as **Predicted positive** by our detector **(False positive)**. Raw waveform (row a), Chi-squared test (row b) and Kolmogorov-Smirnov test (row c) results of first-digit distribution (one means to follow BL), power-law exponent (row d), and goodness of fit φ (row e) of different seismic stations (column 1-4). The red dashed vertical lines are event start and time (2018-08-13 07:47:00, 08:06:00) of our detector marker based on ILL12. The upstream ILL18 and downstream ILL11 do not capture the same trend, this process could be caused by instruments noise or a local event.



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Figure S24. BL features of seismic signal generated by **Actual positive** (2014-07-12, training case). It was labeled as **Predicted positive** by our detector **(True positive)**. Raw waveform (row a), Chi-squared test (row b) and Kolmogorov-Smirnov test (row c) results of first-digit distribution (one means to follow BL), power-law exponent (row d), and goodness of fit φ (row e) of different seismic stations (column 1-4). The red dashed vertical lines are manually labeled start and end time (2014-07-12 14:40 and 18:00). IGB02 are closer to channel, and more than one station can detect this event through BL.





Figure S25. BL features of seismic signal generated by **Actual positive** (2018-08-08, validation case). It was labeled as **Predicted positive** by our detector **(True positive)**. Raw waveform (row a), Chi-squared test (row b) and Kolmogorov-Smirnov test (row c) results of first-digit distribution (one means to follow BL), power law exponent (row d), and goodness of fit φ (row e) of different seismic stations (column 1-4). The red dashed vertical line is debris flow front arrive at CD1 (2018-08-08 17:49:25). From upstream ILL18 to downstream ILL11, it is clearly seen that BL is captured sequentially in the time series, which indicates an event of movement along the trench.

Year	Sensor	Data Logger	Signal Bandwidth	Sampling Frequency	Stations	Operated	Format
2013-2014	Trillium Compact TC120s	Omnirecs Cube 3ext with breakout box	1-100 Hz	200 Hz	IGB02	GFZ ¹	SAC
2017	Lennartz LE- 3D/5S	Nanometrics Centaur	1-50 Hz	100 Hz	ILL02	WSL ²	MiniSEED
2018-2019	Lennartz LE- 3D/5S	Nanometrics Centaur	1-50 Hz	100 Hz	ILL12	WSL ²	MiniSEED

 Table S1.
 Seismometer Parameters

¹GFZ, German Research Centre for Geosciences;

²WSL, Swiss Federal Institute for Forest, Snow and Landscape Research.

N	Source	Туре	Start time (UTC)	End time (UTC)	SNR	Available Station	BL
N1	WSL	F	2013-05-03 07:45	2013-05-03 08:45	1.3	01, 06, 07, 10	
N2	WSL	DF	2013-08-08 10:30	2013-08-08 12:30	1.7	01, 04, 07, 08, 10	
N3	WSL	DF	2013-08-24 13:30	2013-08-24 15:00	1.4	01, 04, 07, 08, 10	
N4	WSL	DF	2013-09-08 19:00	2013-09-08 21:00	1.7	01, 04, 07, 08, 10	
1	WSL	DF	2013-07-22 16:30	2013-07-22 18:30	4.9	01, 02 , 10	Y
2	WSL	F	2013-07-29 07:30	2013-07-29 15:00	4.2	01, 02 , 03, 07, 08, 10	Y
3	GFZ	DF	2014-04-08 08:00	2014-04-08 18:00	42.4	02 , 03, 05, 06, 07	Y
4	GFZ	DF	2014-04-26 22:30	2014-04-27 05:00	12.9	02 , 03, 05, 06	Y
5	GFZ	DF	2014-04-29 09:00	2014-04-29 15:00	5.9	02 , 05, 06, 07	Y
6	GFZ	DF	2014-04-30 10:00	2014-04-30 15:00	27.0	02 , 05, 06, 07	Y
7	GFZ	DF	2014-05-02 03:00	2014-05-02 13:00	0.1	02 , 03, 05, 06, 07	Ν
8	WSL	DF	2014-05-07 15:30	2014-05-07 18:00	15.2	02 , 03, 05, 06, 07	Y
9	GFZ	DF	2014-05-23 03:00	2014-05-23 15:00	57.0	01, 02 , 04	Y
10	GFZ	DF	2014-05-24 09:00	2014-05-24 15:00	16.1	01, 02 , 04, 05	Y
11	GFZ	DF	2014-05-27 05:30	2014-05-27 16:00	22.2	01, 02 , 03, 04, 05	Y
12	GFZ	DF	2014-06-19 14:00	2014-06-19 16:00	6.2	01, 02 , 04, 05	Y
13	GFZ	DF	2014-06-23 18:00	2014-06-23 20:00	10.1	01, 02 , 04, 05	Y
14	WSL	DF	2014-07-08 02:00	2014-07-08 14:00	10.1	01, 02 , 03, 04, 05	Y
15	WSL	DF	2014-07-12 14:40	2014-07-12 18:00	20.6	01, 02 , 03, 04, 05	Y
16	WSL	DF	2014-07-20 20:00	2014-07-21 03:00	12.7	01, 02 , 03, 04, 05	Y
17	WSL	DF	2014-07-23 23:00	2014-07-24 05:00	65.6	01, 02 , 04	Y
18	WSL	DF	2014-07-28 16:00	2014-07-28 20:50	54.3	01, 02 , 03, 04, 05	Y
19	WSL	DF	2014-07-28 20:55	2014-07-29 06:00	19.5	01, 02 , 03, 04, 05	Y

Table S2. Training Events from 2013 to 2014

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20	GFZ	DF	2014-08-02 17·30	2014-08-02 21·30	45.7	01, 02 , 03, 04, 05	Y
21	GFZ	DF	2014-08-08	2014-08-08	16.8	01, 02 , 04, 05	Y
22	GFZ	DF	2014-08-11 02:00	2014-08-11 18:00	7.7	01, 02 , 03, 04, 05	Y
23	GFZ	DF	2014-08-13 04:30	2014-08-13 15:00	0.4	01, 02 , 04, 05	N
24	WSL	DF	2014-09-08 19:30	2014-09-08 23:00	175.6	01, 02 , 04, 05	Y

N: Event number (N1-N4 were non-used events because data from IGB02 is not available). **Type**: DF is debris flow, F is flood. **Start time and End time** are labeled manually based on data from the IGB02 station. The format is *yyyy-mm-dd hh:mm* format. **SNR**, signal-to-noise ratio (N1-N4 none-used event by IGB10, 1-24 used event by IGB02). **Available station**, IGB01 abbreviation is 01. The total duration of the 24 used events is approximately 166 hours. The calculating window size is 60 seconds with 12000 data points. **BL**, from the **Start time** to the **End time** of the event, if the output of any time window from Chi-squared and Kolmogorov-Smirnov tests accepts the null hypothesis, that event will be marked as Y (Yes, follow BL); otherwise, N (No, does not follow BL).

N	Туре	CD1 Time (UTC)	Start Time (UTC)	End Time (UTC)	Volume (m ³)	Data Source	BL
25	DF	2017-05-19 11:41:00	2017-05-19 10:00	2017-05-19 13:00	No Data	No Data	No data
26	DF	2017-05-29 16:58:31	2017-05-29 15:00	2017-05-29 19:00	100000	ILL02	Y
27	DF	2017-06-03 20:23:07	2017-06-03 18:00	2017-06-03 22:30	No Data	ILL02	Y
28	DF	2017-06-03 23:27:38	2017-06-03 22:40	2017-06-04 02:00	25000	ILL02	Y
29	DF	2017-06-14 19:30:48	2017-06-14 18:00	2017-06-14 23:30	No Data	ILL02	Y
30	DF	2018-06-11 10:46:39	2018-06-11 09:00	2018-06-11 14:00	35000	ILL12	Y
31	DF	2018-06-12 18:29:16	2018-06-12 17:00	2018-06-12 22:00	No Data	ILL12	Y
32	DF	2018-07-25 16:56:40	2018-07-25 15:00	2018-07-25 20:00	2018-07-25 <50000		Y
33	DF	2018-08-08 17:49:25	2018-08-08 16:00	2018-08-08 20:00	-08-08):00 <100000		Y
34	DF	2019-06-10 17:02:51	2019-06-10 16:00	2019-06-10 20:00	3300	ILL12	Ν
35	DF	2019-06-10 22:01:17	2019-06-10 21:00	2019-06-11 03:00	6600	ILL12	Y
36	DF	2019-06-20 09:12:17	2019-06-20 07:00	2019-06-20 11:00	No Data	ILL12	Ν
37	DF	2019-06-21 19:34:42	2019-06-21 19:00	2019-06-21 22:00	83000	ILL12	Y
38	DF	2019-07-01 23:00:29	2019-07-01 22:00	2019-07-02 04:00	78000	ILL12	Y
39	DF	2019-07-02 22:09:28	2019-07-02 21:00	2019-07-02 23:30	39000	ILL12	Y
40	DF	2019-07-03 16:43:15	2019-07-03 15:00	2019-07-03 20:00	No Data	ILL12	N
41	DF	2019-07-15 03:40:21	2019-07-15 02:00	2019-07-15 06:00	16000	ILL12	N
42	DF	2019-07-26 17:33:12	2019-07-26 16:30	2019-07-26 19:30	64000	ILL12	Y
43	DF	2019-08-11 17:02:34	2019-08-11 16:00	2019-08-11 19:00	53000	ILL12	Y
44	DF	2019-08-20 16:40:59	2019-08-20 15:00	2019-08-20 18:00	13000	ILL12	Y
45	DF	2019-10-09 11:45:28	2019-10-09 10:30	2019-10-09 13:30	No Data	ILL12	Y

Table S3. Validation Events from 2017 to 2019

N: Event number. **CD1** time is the arrival time at CD1 and **Volume** is the integrated sum of discharge over the entire debris-flow wave (Chmiel et al., 2021). **The start time and End time** are labeled manually from ILL02 or ILL12 station. The format is *yyyy-mm-dd hh:mm* format. The calculating window size is 60 seconds with 6000 data points. **BL**, same as training events, Y (Yes, follow BL); N (No, does not follow BL).

N	Туре	Location	Date (UTC)	Station	Sensor	Data Logger	Sampling Frequenc y	Windo w Size	Ref	BL
47	Rockfall 1	lllgraben, Switzerland	2013-07-22 16:24	IGB01	Trillium Compact TC120s	Omnirecs Cube 3ext with breakout box	200 Hz	10 s	Burtin et al., 2016	N
48	Rockfall 2 Event 30 in ref	Lauterbrunne n, Switzerland	2015-04-06 13:23	Funny Rain- TC120s- Cube3e xtBOB	Trillium Compact TC120s	Omnirecs Cube 3ext with breakout box	200 Hz	10 s	Dietze, Mohadjer, et al., 2017; Dietze, Turowski, et al., 2017	N
49	Landslide	Askja, Iceland	2014-07-21 23:24	MOFO	Güralp CMG-6TD	Nanometrics Taurus	100 Hz	60 s	Schöpa et al., 2018	Y
50	Hurricane- induced Lahar	Volcán de Colima, Meixco	2015-10-24 01:00	SHK2	Sercel SG- 10	No info.	250 Hz	60 s	Capra et al., 2018	Y
51	Glacial-lake- outburst flood	Bhotekoshi, Nepal	2016-07-05 13:30	Hindi (NEP08)	Trillium Compact TC120s	Omnirecs Cube 3ext with breakout box	200 Hz	60 s	Cook et al., 2018	Y
52	Bedload transport	Liwu, Hualien	2021-10-12 01:05	TA64	Sensor Nederlan d PE-6/B	Omnirecs Cube 3ext	100 Hz	60 s		N

Table S4. Signals Generated by Different Mass Movements and Fluvial Processes

BL, from the **Start time** to the **End time** of the event, if the output of any time window from Chi-squared and Kolmogorov-Smirnov tests accepts the null hypothesis, that event will be marked as Y (Yes, follow BL); otherwise, N (No, does not follow BL).

N	$oldsymbol{arphi}$ optimal	Time (UTC)	Data Length	R ²	а	Ь	с	α	BL
1	36.62	2013-07-22 16:52:00	2	0.767	8.776E+02	2.885E-02	2.602E+04	1.10	Y
2	20.60	2013-07-29 07:35:00	5	0.899	2.600E+01	2.209E-02	1.061E+03	1.14	Y
3	77.97	2014-04-08 13:34:00	5	0.246	-2.902E+05	-1.319E-05	2.905E+05	1.00	Y
4	81.90	2014-04-26 23:58:00	1	0.771	4.071E+00	6.973E-02	7.531E+03	1.11	Y
5	81.76	2014-04-29 11:00:00	4	0.808	8.628E+01	1.649E-02	1.091E+03	1.00	Y
6	79.82	2014-04-30 12:44:00	5	0.052	7.613E-02	7.834E-02	4.425E+02	1.00	Y
7	-36.38	2014-05-02 08:00:00	4	0.771	2.639E-08	8.310E-02	1.980E+02	5.22	Ν
8	80.78	2014-05-07 15:53:00	5	0.940	1.690E+02	1.626E-02	-3.612E+02	1.12	Y
9	84.18	2014-05-23 04:42:00	5	0.920	7.927E+01	2.083E-02	1.217E+03	1.11	Y
10	84.98	2014-05-24 09:36:00	3	0.171	3.535E-54	5.886E-01	1.561E+04	1.11	Y
11	78.36	2014-05-27 13:01:00	2	0.747	9.090E-21	1.262E+01	4.595E+02	1.00	Y
12	70.16	2014-06-19 14:25:00	1	0.692	3.480E+02	5.768E-02	2.879E+04	1.00	Y
13	74.50	2014-06-23 18:12:00	3	0.736	7.486E+02	1.854E-02	1.370E+04	1.12	Y
14	80.25	2014-07-08 10:13:00	2	0.618	6.075E+07	3.890E-06	-6.072E+07	1.10	Y
15	87.97	2014-07-12 14:58:00	5	0.944	2.274E+00	3.252E-02	1.494E+03	1.10	Y
16	79.11	2014-07-20 20:45:00	5	0.905	3.962E+03	5.455E-03	2.568E+03	1.12	Y
17	93.10	2014-07-23 23:03:00	5	0.868	1.573E-01	3.858E-02	1.902E+03	1.10	Y
18	87.46	2014-07-28 17:09:00	1	0.838	1.775E+00	9.960E-02	6.508E+04	1.09	Y
19	80.08	2014-07-29 01:00:00	2	0.904	3.577E+02	3.034E-02	2.800E+04	1.10	Y
20	82.17	2014-08-02 19:17:00	5	0.882	4.056E+02	2.646E-02	6.460E+03	1.10	Y
21	79.50	2014-08-08 19:18:00	1	0.832	3.044E+00	8.648E-02	2.808E+04	1.10	Y
22	82.95	2014-08-11 17:02:00	2	0.930	3.404E-13	9.804E-01	7.724E+02	1.15	Y
23	-4.72	2014-08-13 10:30:00	3	0.199	4.305E-14	4.446E-01	2.564E+02	32.77	N
24	89.14	2014-09-08 19:58:00	5	0.842	9.514E-01	3.641E-02	3.316E+03	1.11	Y
25	No data	2017-05-19							
26	92.75	2017-05-29 17:00:00	5	0.894	1.125E-02	3.508E-02	1.322E+02	1.00	Y
27	80.60	2017-06-03 20:30:00	5	0.794	6.382E+01	5.378E-03	2.322E+01	1.00	Y
28	86.79	2017-06-03 23:30:00	5	0.814	3.505E-01	2.558E-02	1.594E+02	1.14	Y

Table S5. Coefficients of the Exponential Fitting

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29	88.78	2017-06-14 19:32:00	5	0.947	1.507E-01	2.829E-02	1.659E+02	1.14	Y
30	77.82	2018-06-11 11:00:00	5	0.924	1.257E+02	6.730E-03	-1.350E+01	1.00	Y
31	73.20	2018-06-12 18:46:00	5	0.439	4.804E+05	1.561E-06	-4.800E+05	1.00	Y
32	83.37	2018-07-25 16:59:00	5	0.930	6.260E+00	1.854E-02	8.504E+01	1.00	Y
33	87.17	2018-08-08 17:52:00	5	0.910	7.524E+00	2.069E-02	2.320E+02	1.00	Y
34	68.99	2019-06-10 17:30:00	3	0.290	2.464E+01	1.999E-02	1.725E+03	1.13	Ν
35	77.29	2019-06-10 22:54:00	5	0.918	6.911E+01	1.106E-02	6.430E+02	1.14	Y
36	72.41	2019-06-20 09:23:00	5	0.840	2.176E+03	3.061E-03	-2.148E+03	1.15	Ν
37	80.18	2019-06-21 19:39:00	5	0.878	6.205E+03	2.586E-03	-6.752E+03	1.13	Y
38	80.54	2019-07-01 23:35:00	4	0.527	9.053E+02	6.602E-03	2.048E+03	1.13	Y
39	75.99	2019-07-02 22:16:00	5	0.847	6.091E+06	1.954E-06	-6.091E+06	1.13	Y
40	48.59	2019-07-03 17:08:00	3	0.511	6.445E+00	2.704E-02	9.103E+02	1.00	Ν
41	67.80	2019-07-15 03:49:00	5	0.854	9.492E+05	9.374E-06	-9.491E+05	1.13	Ν
42	81.66	2019-07-26 17:38:00	5	0.905	1.367E+02	1.071E-02	-2.983E+01	1.13	Y
43	87.39	2019-08-11 17:04:00	5	0.899	8.506E+00	2.044E-02	1.775E+02	1.12	Y
44	79.34	2019-08-20 16:49:00	5	0.894	3.303E+02	9.328E-03	2.485E+02	1.18	Y
45	74.21	2019-10-09 11:57:00	5	0.915	1.345E+02	1.019E-02	5.004E+02	1.00	Y
46	-24.42	2019-10-15 16:19:00	5	0.888	1.275E+02	6.050E-03	1.586E+01	1.22	N

N: Event number. $\varphi_{optimal}$ is the optimal goodness of fit between manually labeled start and end time (unit by %). **Time** corresponds to the moment when φ is at its optimal. **Data length** is the length of the data used to fit the exponential function (before φ_{optima} , in minutes). **R**² is the coefficient of determination of exponential fitting. **a**, **b**, and **c** are the coefficients of the exponential fitting in $S(t) = a \times e^{bt} + c$ (Text S3). α is the power law exponent at optimal goodness of fit time.

Code S1.

All codes are available in

https://github.com/Nedasd/Benfords-law-in-environmental-seismology.git

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