Check Your Work: The Value of Analyzing Units of Measure and Dimensional Homogeneity, as Demonstrated in a Fluid Mechanics Exam

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Abstract

Fluid mechanics is a field rife with diverse units of measure and, consequently, potential for errors. Unlike their experience in mathematics, undergraduate students must learn to manipulate both numbers and units in order to satisfy dimensional homogeneity and succeed in engineering. Using written exam responses from students in an undergraduate fluid mechanics class, this paper 1) compares scores according to whether students carefully handled units or not and 2) provides examples of mistakes that may have been avoided by analyzing units. In the exams, students who consistently tracked units scored higher and with less variance than those who did not; both results are statistically significant. The selected examples highlight errors in algebra, fluid properties, and geometry that may have been detected had students properly handled units. The findings suggest that the habit of analyzing units is desirable for students to develop, helping them both understand concepts and check their work as they become engineers.

Keywords

Unit conversion; fluid mechanics; hydraulics; engineering education; dimensional homogeneity

1. Introduction

Perhaps in no field of engineering are units of measure so diverse or so essential as in fluid mechanics. Problems frequently involve forces, volumes, lengths, flow rates, areas, and pressures in various units and must be dimensionally correct. Even a single measurable quantity like flow rate, for example, is commonly expressed in gallons per minute for piped systems, cubic feet per second for open channels, and million gallons per day for wastewater treatment works. The Système International d’Unités (SI) can be just as perplexing with seemingly arbitrary expressions, like 1 kg·m/(s²m²) = 1 Pa, and a plethora of prefixes for powers of 10 that continually befuddle students (Sokolowski 2015) and even teachers (Dincer and Osmanoglu 2018). To further complicate matters, fluid mechanics employs dimensionless numbers, like the Froude number and Reynolds number, as well as empirical relationships with non-integer dimensional exponents, like the Manning equation and Hazen–Williams equation, that are meaningful only with their prescribed units of input.

To succeed in fluid mechanics and engineering in general, students must learn to properly handle units. Equations must be dimensionally homogeneous, meaning the left and right sides, and all additive terms, must have the same dimensions—e.g., combinations of length, mass, time, and/or temperature (Bridgman 1922; Gruber and Olsen 1992). Undergraduate students arriving in fluid mechanics, though otherwise well-equipped from their mathematics courses, are not necessarily accustomed to handling dimensions and units—a skill developed primarily in science and engineering courses dealing with specific physical quantities rather than general numbers and variables (Butterfield et al. 2011; Raje 2019). Moving on to engineering courses, students must learn to set up and carry out calculations not just with the numbers but with the units.
Fluid mechanics is an ideal place to do so because students can practice this new skill frequently given the variety of problems, dimensions, and units involved. Many fluid mechanics textbooks—such as those by Chin (2017), Mott and Untener (2015), Potter et al. (2012), Fox et al. (2009), Young et al. (2007), and Çengel and Cimbala (2006), just to name a few—rightly dedicate some early sections to the treatment of units, dimensions, and physical quantities. Students should learn that carefully handling units in each problem is a powerful technique for understanding concepts and checking their work: getting the right units can confirm the answer; getting the wrong units almost always indicates an error, whether in units, logic, algebra, or something else. A missing unit often indicates a missing step.

To illustrate the necessity—and benefit—of analyzing units in fluid mechanics, the author 1) compares exam scores for students who did and did not consistently track units and 2) gives three examples from exams where errors went undetected due to students’ poor handling of units. Implications for pedagogy are discussed alongside the results.

2. Methods

Data for the analysis came from 47 students’ written solutions to midterm exams in a junior-level fluid mechanics course which the author taught in 2021. This is an ideal situation because of the relatively large class, the visibility of the students’ approach in the written work (as opposed to a multiple-choice test), and the variety of dimensions and units involved in fluid mechanics problems. Further, unlike homework problems, the exams were solo performances where students could not consult one another, the teaching assistant, or the instructor and instead had to rely on their own skills. Students had been taught how to analyze units in class and were encouraged, but not required, to track units in their exam problems. Accordingly, students’ exam responses clearly showed the habits—good and bad—they had developed in this regard.

Each of the 47 exam responses, prior to grading, was classified as either “consistently tracking units” (i.e., in every term of every calculation) or “not consistently tracking units” (i.e., ignoring units, omitting units, using wrong units, informally treating units, and/or failing to follow units through the entire problem). Thereafter, only the final answers were graded; students’ work leading up to the final answer, including handling of units, was not part of the score.

Exam scores were divided into two groups depending on whether the students consistently handled units or not. The author employed a two-tailed t-test to compare the means of the two groups and a two-tailed F-test to compare their variances. The theory and procedure of the tests are generally known and the reader is referred to Kennedy (2008) for details. A 95% confidence level was chosen for each test, meaning that differences in the groups’ means and variances would be declared statistically significant only if there was less than a 5% probability (p < 0.05) that the differences would occur randomly, i.e., without invoking the students’ handling of units.

3. Results and discussion

3.1 Exam scores

Of the 47 students in the sample, 27 consistently tracked units in their exam work and 20 did not. The score distributions of each group appear in Fig. 1.

Students who consistently tracked units scored 8 points higher on average than those who did not (94/100 and 86/100, respectively). According to the t-test, the difference was statistically significant with p = 0.00016. Students who consistently tracked units performed better than their counterparts on the same exam. Further, and perhaps more interestingly, while perfect scores were observed in both groups, the
spread of scores for students who consistently tracked units was noticeably tighter (standard deviation $[SD] = 4.6$) than for those who did not ($SD = 7.7$). According to the $F$-test, the difference was statistically significant with $p = 0.015$. Students who consistently tracked units, therefore, also scored more predictably.

In short, students who paid attention to units, though they may have made other mistakes, were more likely to score higher and score more predictably (with less variance) than those who did not. Students who missed units missed opportunities to diagnose and correct mistakes, as the next section will show. The distributions of scores in each group are fundamentally different, indicating a strong link between unit handling and test performance. By extension, one may infer a strong link between unit handling and mastery of fluid mechanics concepts.

Accordingly, instructors should recognize the need for students to write, analyze, transform, and cancel units as part of their engineering education. Learning objectives and learning activities to help them develop the skills should be designed for this purpose, and instructors should model the skills regularly when teaching.

3.2 Example problems

From the exams, three examples were selected where a student arrived at an incorrect answer and at the same time ignored units. The examples illustrate how students seemed to struggle with the new experience of tracking units and demonstrate why students should develop the habit of tracking units in all their work if for no other reason than as a technique to detect errors. Thibault et al. (2010) showed a similar example involving gas–liquid mass transfer. Interestingly, none of the problems presented below involved unit conversion. Rather, the errors were with algebra (Example 1), fluid properties (Example 2), and geometry (Example 3). The point is that many types of errors will ultimately appear as unit errors in written work; analyzing the units can help students recognize and resolve such errors.

3.2.1 Example 1

One exam problem required the computation of head loss ($h_L$) using Darcy’s equation, which depends on pipe length ($L$), pipe diameter ($D$), velocity head ($v^2/(2g)$), and a dimensionless friction factor ($f$) from the Moody diagram. The student correctly determined the inputs and then assumed an answer in meters. But the student failed to square the velocity, leading to an incorrect answer:

$$h_L = f \frac{L}{D} \frac{v^2}{2g} = (0.019) \left( \frac{435}{0.5} \frac{m}{m} \right) \left( \frac{1.8}{2} \frac{m}{s} \right) \left( \frac{9.81}{m^2/s^2} \right) = 1.52 \text{ m (incorrect)} \quad (1)$$

Checking the units as written would have revealed that the resulting unit was not actually meters, violating the requirement of dimensional homogeneity:

$$\frac{m}{m} \left( \frac{m}{s} \right) \left( \frac{m}{s^2} \right) = \frac{m}{s} \left( \frac{1}{s} \right) = \frac{m}{s^2}$$

Seeing that the expected units were missing, the student could have corrected the error. The correct expression is:

$$h_L = f \frac{L}{D} \frac{v^2}{2g} = (0.019) \left( \frac{435}{0.5} \frac{m}{m} \right) \left( \frac{1.8}{2} \frac{m^2}{s^2} \right) = 2.73 \text{ m} \quad (3)$$

which could be confirmed by checking the units:
\[ \frac{m}{m} \left( \frac{m^2}{s^2} \right) = \frac{m \cdot m^2 \cdot s^2}{m \cdot s^2 \cdot m} = m \]  

(4)

3.2.2 Example 2

Another exam problem required determining the Reynolds number \((Re)\), which is the ratio of inertial forces (velocity, \(v\); diameter, \(D\); and density, \(\rho\)) to viscous forces (dynamic viscosity, \(\eta\)). The student seemed to know that the Reynolds number should be dimensionless. But the student computed:

\[ Re = \frac{vD\rho}{\eta} = \frac{(4.8 \frac{m}{s})(0.05 \text{ m})(9.81)}{0.0013} = 1,811 \text{ (incorrect)} \]

(5)

The student’s blunder was to insert the wrong fluid property—the specific weight of water \((9.81 \text{ kN/m}^3)\), instead of its density \((1,000 \text{ kg/m}^3)\)—in the numerator, without its corresponding units. The student also omitted the dynamic viscosity units of Pascal-seconds in the denominator. Had the student attempted to cancel out the units as written, the student would have noticed that the result was not dimensionless as it should have been:

\[ \frac{m}{s} \cdot \frac{m^2}{s} = \frac{m}{s} \]

(6)

and may have gone back to check the setup. The correct expression is:

\[ Re = \frac{vD\rho}{\eta} = \frac{(4.8 \frac{m}{s})(0.05 \text{ m})(1,000 \frac{\text{kg}}{\text{m}^3})}{(0.0013 \text{ Pa} \cdot \text{s})} = 185,000 \]

(7)

which could be confirmed by checking the units, recalling that 1 Pa·s is the same as 1 N·s/m\(^2\) or 1 kg/(m·s):

\[ \frac{m}{s} \cdot \frac{\text{kg}}{\text{m}^2} \cdot \frac{m \cdot s}{\text{kg}} = 1 \]

(8)

leaving a dimensionless answer as expected.

3.2.3 Example 3

In this problem, the student was asked to recommend a Schedule 40 steel pipe diameter (in millimeters) to convey a flow, \(Q\), of 9,400 L/min at a maximum velocity, \(v\), of 2.0 m/s. The student began with the continuity equation relating the flow and velocity to the cross-sectional area, \(A\):

\[ A = \frac{Q}{V} = \left( \frac{9,400 \text{ L/min}}{2.0 \frac{m}{s}} \right) \left( \frac{1 \text{ m}^3}{1,000 \text{ L}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 0.078 \text{ m} = 78 \text{ mm} \text{ (incorrect)} \]

(9)

The student then incorrectly recommended a nominal diameter \((D)\) of 80 mm (DN 80 pipe). The calculation was, of course, incomplete: the student had only computed the area, not the diameter. The missing unit should have alerted the student to the missing step. The number so far was correct, but cancelling the units would have shown it was not what the student was looking for:

\[ \frac{\text{L}}{\text{s}} \cdot \frac{\text{m}^2}{\text{L}} \cdot \frac{\text{min}}{\text{min}} = \text{m}^2 \]

(10)

Recognizing this as an area, the final step would have been to determine the diameter:
\[ D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(0.078 \text{ m}^2)}{\pi}} = 0.315 \text{ m} = 315 \text{ mm} \] (11)

There is no standard pipe size with a 315 mm diameter, so the appropriate choice is the next one up, DN 350. The units of the foregoing equation are consistent; the square root of a squared unit leaves the single unit, confirming that the answer is correct.

3.3 Limitations and Further Work

The relationship between proper unit analysis and good exam performance is not necessarily causal. Higher scores may reflect better understanding of the subject matter overall, not just better handling of units. Conversely, lower scores may have come from students who would have performed poorly regardless of their ability to handle units. The sample is admittedly just one sample—some might even say anecdotal—and the results may reflect only the learning experience of the specific class and exam in question. The results are nonetheless logical and informative, and author invites further work to strengthen the case that analyzing units, as a preparatory skill, improves student performance. Such data should be collected across multiple students and semesters (preferably with the same instructor and similar exams), or even across various schools with a standardized exam deliberately designed to capture the effect of interest.

4. Conclusion

To succeed in fluid mechanics and other engineering courses, students must learn to properly handle units. This is usually a new skill for them, differing from mathematics in that it requires proper manipulation of numbers and units.

This study analyzed fluid mechanics exam scores for 47 students according to the students’ handling of units. Students who consistently paid attention to units in their written solutions were more likely to score higher and with less variance than those who did not. Both results are statistically significant. Three examples were given to illustrate how students may have avoided mistakes in algebra, fluid properties, and geometry by analyzing units when they had supposedly finished the problem. Checking units is an opportunity to catch errors.

The analysis presented evidence that analyzing units (not just converting units) is a desirable habit associated with better exam performance, which leads to better confidence in the solutions. Instructors should emphasize unit analysis and students should strive to develop this important skill. Fluid mechanics courses are a logical place to do both, given the variety of problems, dimensions, and units of measure involved. Attention to units should occur early in the course and should not be rushed. Throughout the semester—and throughout their careers—students should check their work by analyzing the units of the answer.

Acknowledgment

The author thanks Dan Jones of Hansen, Allen & Luce, Inc., for his feedback on an early draft of this article.

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