Patterns of Alluviation in Mixed Bedrock-Alluvial Channels: 2. Controls on the Formation of Alluvial Patches

Jongseok Cho\textsuperscript{1} and Peter A. Nelson\textsuperscript{1}

\textsuperscript{1}Colorado State University

June 11, 2023

Abstract

Understanding the development and spatial distribution of alluvial patches in mixed bedrock-alluvial rivers is necessary to predict the mechanisms of the interactions between sediment transport, alluvial cover, and bedrock erosion. This study aims to analyze patterns of bedrock alluviation using a 2D morphodynamic model, and to use the model results to better understand the mechanisms responsible for alluvial patterns observed experimentally. A series of simulations are conducted to explore how alluvial patterns in mixed bedrock-alluvial channels form and evolve for different channel slopes and antecedent sediment layer thicknesses. In initially bare bedrock low-slope channels, the model predicts a linear relationship between sediment cover and sediment supply because areas of subcritical flow enable sediment deposition, while in steep-slope channels the flow remains fully supercritical and the model predicts so-called runaway alluviation. For channels initially covered with sediment, the model predicts a slope-dependent sediment supply threshold above which a linear relationship between bedrock exposure and sediment supply develops, and below which the bedrock becomes fully exposed. For a given sediment supply, the fraction of bedrock exposure and average alluvial thickness converge toward the equilibrium value regardless of the initial cover thickness so long as it exceeds a minimum threshold. Steep channels are able to maintain a continuous strip of sediment under sub-capacity sediment supply conditions by achieving a balance between increased form drag as bedforms develop and reduced surface roughness as the portion of alluvial cover decreases. In lower-slope channels, alluvial patches are distributed sporadically in regions of the subcritical flow.
Patterns of Alluviation in Mixed Bedrock-Alluvial Channels: 2. Controls on the Formation of Alluvial Patches

Jongseok Cho$^1$ and Peter A. Nelson$^1$

$^1$Department of Civil and Environmental Engineering, Colorado State University, Fort Collins, Colorado, USA.

Corresponding author: Peter Nelson (peter.nelson@colostate.edu)

Key Points:

- Numerical simulations are conducted in mixed bedrock-alluvial channels to analyze controls on alluvial patterns.

- Distinct alluviation patterns are observed depending on the channel slopes and initial alluviation conditions.

- The interactions between flow, bed topography, and roughness control the mechanisms of bed evolution.
Abstract

Understanding the development and spatial distribution of alluvial patches in mixed bedrock-alluvial rivers is necessary to predict the mechanisms of the interactions between sediment transport, alluvial cover, and bedrock erosion. This study aims to analyze patterns of bedrock alluviation using a 2D morphodynamic model, and to use the model results to better understand the mechanisms responsible for alluvial patterns observed experimentally. A series of simulations are conducted to explore how alluvial patterns in mixed bedrock-alluvial channels form and evolve for different channel slopes and antecedent sediment layer thicknesses. In initially bare bedrock low-slope channels, the model predicts a linear relationship between sediment cover and sediment supply because areas of subcritical flow enable sediment deposition, while in steep-slope channels the flow remains fully supercritical and the model predicts so-called runaway alluviation. For channels initially covered with sediment, the model predicts a slope-dependent sediment supply threshold above which a linear relationship between bedrock exposure and sediment supply develops, and below which the bedrock becomes fully exposed. For a given sediment supply, the fraction of bedrock exposure and average alluvial thickness converge toward the equilibrium value regardless of the initial cover thickness so long as it exceeds a minimum threshold. Steep channels are able to maintain a continuous strip of sediment under sub-capacity sediment supply conditions by achieving a balance between increased form drag as bedforms develop and reduced surface roughness as the portion of alluvial cover decreases. In lower-slope channels, alluvial patches are distributed sporadically in regions of the subcritical flow.

Plain Language Summary
Bedrock rivers may have patches of alluvial sediment that covers some or all of the underlying bedrock. The amount of this sediment cover can change dynamically over time depending on the flow, upstream sediment supply, channel morphology, and antecedent sediment conditions. Here, we use a numerical model to simulate flow and sediment transport so that we may better understand what controls sediment cover in mixed bedrock-alluvial rivers. We use the model to simulate channels of varying slopes, sediment supplies, and initial sediment cover, and we analyze the model’s output to gain insight on how alluvial patches form and what controls their extent and dynamics. Our numerical model results produce phenomena that have been observed in physical experiments, and they show that the channel slope and initial sediment thickness plays an important role in determining whether and how much sediment can be deposited. Our results also show that flow transitions provide critical locations where sediment deposits can start to form. Persistent sediment cover in bedrock channels can develop when a delicate equilibrium is reached between sediment roughness and the flow field.

1 Introduction

Bedrock channels are characterized by occasional or continuous exposures of nonalluviated bedrock, which is a consequence of these channels receiving a sediment supply that is less than their transport capacity. A wide variety of models for drainage network evolution distinguish between bedrock and alluvial reaches (Howard, 1980; Howard et al., 1994; Howard & Kerby, 1983; Montgomery et al., 1996) by using channel slope and discharge to express channel transport capacity. Field investigations (Lamb et al., 2008; Massong & Montgomery, 2000; Montgomery et al., 1996; Montgomery & Buffington, 1997) demonstrate that bedrock channels occur at slopes greater than a critical value \( S > S_c \), and alluvial channels form at a slope less than the critical value \( S \leq S_c \).
The spatial distribution of alluvial cover in mixed bedrock-alluvial channels has
importance for determining rates and patterns of bedrock erosion, hydrodynamics, and aquatic
habitat. Mechanistic models of bedrock erosion incorporate the erosional mechanism of saltating
bedload particles impacting and eroding bedrock (e.g., Demeter et al., 2005; Hartshorn et al.,
dependence on sediment supply and bedrock exposure into the calculation of erosion rates.

Competition between the tools and cover effects controls the spatial distribution of the bedrock
channel erosion, resulting in lateral and vertical channel erosion and meandering (Finnegan et al.,
2007; Lamb et al., 2015; Turowski et al., 2007; Turowski, Hovius, Meng-Long, et al., 2008;
Turowski, Hovius, Wilson, et al., 2008). The cover effect is typically demonstrated by linear or
exponential relations of fractional bedrock exposure as a function of sediment supply to transport
capacity ratio (Sklar & Dietrich, 1998, 2004; Turowski et al., 2007). Additionally, sediment
supply and alluvial cover impact the maintenance and distribution of aquatic habitat and attached
micro-organisms (Buffington et al., 2004; Detert & Parker, 2010; Huston & Fox, 2015, 2016;

Alluvial patterns in bedrock channels are controlled by spatial and temporal variations in
sediment flux, transport capacity, and bed topography. These channels can exhibit alluvial
patterns ranging from continuous and concentrated longitudinal strips of sediment to spatially
discontinuous patches of sediment. Experiments in mixed bedrock-alluvial channels have
observed spatially concentrated sediment cover and storage in low parts of the underlying
bedrock topography (Finnegan et al., 2007; Johnson & Whipple, 2007), indicating that
topographic roughness induces changes in local flow properties and threshold of sediment
motion. Inoue et al. (2014) observed inconsistent development of alluvial cover in the inner
channel through their field experiments. Hodge and Hoey’s (2016a, 2016b) experiments show that velocity is an important control on sediment deposition, as they did not observe sediment cover at low areas of the bed where the flow velocity remained high. The results from several experimental studies (Chatanantavet & Parker, 2008; Johnson & Whipple, 2010; Sklar & Dietrich, 2004; Turowski et al., 2007) suggest a few major factors control grain entrainment and bedrock exposure, such as sediment supply rate, channel slope, material size, and bed roughness.

A set of flume experiments (Chatanantavet & Parker, 2008) using different channel bed slopes has shown that for lower slopes ($S < 0.0115$) bedrock exposure decreased more or less linearly with increasing the ratio of sediment supply rate to capacity transport rate. However, for sufficiently higher slopes ($S \geq 0.0115$), the bedrock remained fully exposed when the ratio of sediment supply to transport capacity is less than a critical value, while a linear relationship between the degree of bedrock exposure and sediment supply rate to transport capacity ratio prevailed when the sediment supply exceeds transport capacity. These experiments also documented a slope-dependent “runaway alluviation,” where for initially bare-bedrock conditions, low-slope channels develop linearly increasing sediment cover with increasing sediment supply, while high-slope channels remain completely exposed until sediment supply exceeds the transport capacity, beyond which the channel becomes fully alluviated.

Chatanantavet and Parker (2008) suggested this may be a result of slope-dependent grain interactions; later experiments by Mishra and Inoue (2020) indicated that runaway alluviation occurred when the bedrock roughness was lower than the sediment roughness, but the gradual alluvial cover could develop when the bedrock roughness was higher than that of the sediment. A full explanation of the slope-dependent behavior of sediment dynamics in mixed bedrock-alluvial channels is still needed.
Morphodynamic models have struggled to replicate the types of observations made in mixed bedrock-alluvial experiments. Promising results have been presented by Hodge and Hoey (2012), who developed a cellular automaton (CA) model where the probability of entrainment of individual grains was specified for bedrock or alluvial areas. Their model found relationships between bedrock exposure and the ratio of sediment supply to transport capacity ($q_s/q_c$) similar to those observed by Chatanantavet and Parker (2008), but in contrast to the experimental observations, the majority of their CA model runs predicted that the presence or absence of sediment cover on the bed at the beginning of the run did not affect the steady state sediment cover. Additionally, the entrainment probabilities needed to change from run to run for their results to achieve the diversity of findings Chatanantavet and Parker (2008) reported. While CA models like this provide interesting insight on the potential importance of grain dynamics for alluvial patterns in bedrock-alluvial channels, the absence of a flow model makes connecting probabilities of sediment entrainment and deposition to physical mechanisms of sediment transport and alluviation challenging.

Because of the complexity inherent in the roughness relationship incorporated to the flow resistance and sediment transport, evaluating the relative influence of different roughness mechanisms for channel evolution is currently a challenging problem. The patterns of sediment cover over the bedrock bed affect the spatial distribution of local roughness, flow rate, and sediment transport. Here we use a new morphodynamic model to reproduce many of the phenomena that have been observed in mixed bedrock-alluvial channels, and we use the model predictions to untangle the mechanisms responsible for the range of dynamic sediment behavior in these environments. In particular, we use the model to investigate the following questions: 1) what explains the apparent slope dependence for runaway alluviation? 2) How can mixed
bedrock-alluvial channels maintain alluvial cover when the sediment supply is less than the transport capacity? 3) How do initial conditions of sediment cover affect the temporal development of alluvial cover, and the overall relationship between bedrock exposure and sediment supply? Our results reveal the important interactions between dynamic channel roughness, flow patterns, sediment transport rates, bedform development, and alluvial cover.

2 Methods

2.1 Morphodynamic model

We have developed a two-dimensional numerical morphodynamic model for mixed bedrock-alluvial channels. The model is fully described in Cho and Nelson (submitted), and we summarize the key aspects of the model here. The model consists of three major components: (1) the 2D shallow water equations (SWE) in the depth-averaged form are applied to solve the hydrodynamical component, (2) a sediment transport model calculates bedload transport rates associated with the hydrodynamic variables and topographic variation, and (3) the modified Exner equation, which computes changes in alluvial concentration or thickness due to sediment transport divergence. The novel aspects of this model, compared to previous models applied to mixed bedrock-alluvial morphodynamics, are 1) the Exner equation of sediment continuity accounts for the volume of bedload in transport and the fraction of bed covered in sediment, 2) a composite alluvial and bedrock roughness is used in the flow calculation, 3) the friction for sediment transport is modified to account for the effects of bedforms, and 4) the numerical scheme is robust and capable of handling Froude transitions and capturing shocks. We summarize the key components of the model below.
A key assumption in most numerical models combining water flow, sediment transport, and morphological evolution is that the response time of bed evolution is relatively long compared to the timescales of relevance to the flow of water (McLean et al., 1994; Nelson et al., 2003; Tubino et al., 1999). This allows a decoupling between the water flow computation and the sediment equation by assuming a quasi-steady approximation of morphodynamic process that the bed level does not change rapidly during an infinitesimal time interval while the flow field adapts instantaneously. Thus, the decoupled model practically solves for the flow field and topographic evolution using an iterative procedure.

Mixed bedrock-alluvial channels exhibit complex roughness feedbacks due to differential roughness of alluvium and bedrock surfaces, the development of bedforms and associated form drag, as well as bed shear stress taken up by sediment transport itself. To account for this, our model uses a composite roughness partitioned into surface roughness of alluvial and bedrock bed, sediment transport roughness, and form drag. The ripple factor is applied to the shear stress to remove the form drag of the bedforms assuming the remaining part is responsible for sediment transport.

Water depth and flux are calculated using the Harten-Lax-van Leer-Contract (HLLC) scheme with the weighted average flux (WAF) method (Toro, 1992a, 1992b) and an explicit application of the central difference of the viscous and friction terms at each computational cell center in a 2D domain. Because of the local and global change in flow resistance associated with different substrate roughness between the bedrock and bed material and bedform evolution, a treatment of transcritical flow is necessary. The flow resistance consists of skin friction, form drag, and bedload transport roughness. A linear fractional cover model representing the relation between the alluvial bed and bare bedrock bed, the volume of local bed material per volume of a
monolayer of sediment grains, is used to calculate local skin friction. The form drag effect is determined using the local bed slope and topographic variation averaged over the area of interest. Additionally, the bedload layer thickness is added to the total roughness where the sediment transport occurs. The explicit calculation of each of these components of roughness is critical to be able to replicate observations of persistent sediment cover in mixed bedrock-alluvial channels (Cho and Nelson, submitted).

The bed morphology is updated using the modified Exner equation for sediment continuity (Inoue et al., 2014, 2016; Luu et al., 2004). The sediment transport capacity on the alluvial bed is estimated from Wong et al. (2007). A correction of the bedload transport rate on the pure bedrock bed is necessary considering a relatively small volume of bedload transport:

\[
q_b = \begin{cases} 
\frac{V_b}{V_{bc}}q_{bc} & \text{for } 0 \leq V_b < V_{bc} \\
q_{bc} & \text{for } V_{bc} \leq V_b 
\end{cases}
\]  \hspace{1cm} (1)

where \(q_b\) is the bedload transport rate per unit width, \(q_{bc}\) is the bedload transport capacity per unit width, \(V_b\) is the volume of sediment per unit area in the bedload layer, and \(V_{bc}\) is the saturation volume per unit area in the bedload layer. When the bed is completely bare, only the volume of sediment in the bedload layer is brought into bedload transport without resting on the bed, referred to as the throughput bedload. When the volume of the bedload layer exceeds the saturation value, sediment starts to deposit on the bed and the linear cover fraction model is utilized to determine whether the bed is in the state of partial or complete cover:

\[
\eta_a = \begin{cases} 
\frac{V_{ba} - V_{bc}}{1 - \lambda} & \text{for } V_{bc} \leq V_{ba} \\
0 & \text{for } 0 \leq V_{ba} < V_{bc}
\end{cases}
\]  \hspace{1cm} (2)
where $\eta_a$ is the alluvial layer thickness, $V_{ba}$ is the total volume of sediment per unit area in alluvial bedload layers, $\lambda$ is the porosity, and

$$V_b = \begin{cases} V_{bc} & \text{for } V_{bc} \leq V_{ba} \\ V_{ba} & \text{for } 0 \leq V_{ba} < V_{bc} \end{cases}$$

(3)

where $V_{bc} = q_{bc}/u_s$ and $u_s$ is the saltation velocity.

The skin friction is used for the calculation of dimensionless shear stress to account for the fact that only the near-bed grain roughness is responsible for the sediment transport, using a correction referred as to the ripple factor (Ribberink, 1987; Vermeer, 1986). The dimensionless shear stress (Struiksma, 1985; Talmon et al., 1995) and critical Shields number (Calantoni, 2002; Duan & Julien, 2005; Soulsby, 1997) are corrected for spatially varying bed topography in the direction of flow.

The critical Shields parameter calculated as a function of the ratio of the bedrock hydraulic roughness to the grain size (Inoue et al., 2014; Johnson, 2014; Mishra & Inoue, 2020) is adopted instead of constant value for bedrock and alluvial surfaces. We use a modified dimensionless critical shear stress model to simulate bed evolution in different channel slopes:

$$\tau_c^* = \alpha_c (k_0/d)^{0.6}$$

(4)

where $\alpha_c$ is the correction factor for different channel slopes, $k_0$ is the hydraulic roughness height, and $d$ is the grain size.

2.2 Simulation conditions

We conducted a set of numerical experiments to explore how channel slope, initial sediment cover thickness, and sediment supply impact the evolution of patterns of alluvial cover
in mixed bedrock-alluvial channels. The simulations were designed to complement the
experiments of Chatanantavet and Parker (2008) and provide mechanistic insight into controls on
alluvial patterns in bedrock channels. Table 1 summarizes the initial flow and sediment condition
used in each experiment based on the flume experiments conducted by Chatanantavet and Parker
(2008). The computational channel is straight and longer than the experimental channel to avoid
possible problems regarding bedform development sensitive to boundary disturbances. All
simulations impose constant water and sediment supply, uniform sediment size, and non-erodible
bedrock to exclude potential disturbances to bed topography created by unsteady conditions. The
computations are stopped when near equilibrium conditions of bed topography are achieved, in
which the average sediment cover thickness and the fraction of bedrock cover vary around stable
values.

All simulations are performed in a rectangular bedrock channel having a length of 20 m
and a width of 0.9 m. Small topographic variations with a standard deviation of 2.2 mm and
peak-to-peak bed elevation of 9 mm are applied to the bedrock bed. First, two sets of simulations
are performed: (1) with some initial cover thickness with different channel slopes (Runs 2-A5
and 2-B5) and (2) without antecedent sediment cover (Runs 2-Ax, 2-Bx, and 2-Dx) to explore
the effect of channel slopes. Second, a set of Run 2-B is conducted with the \( q_s/q_c = 0.6 \) and
various initial sediment cover thicknesses of 1, 4, and 6 cm. Two types of uniform grains, fine 2
mm and coarse 7 mm gravels, are employed in the experiments with a mild slope channel (\( S \leq 0.0115 \)) and with a steep slope channel (\( S = 0.02 \)), respectively.
The bedrock and grain roughness heights are back-calculated from the logarithmic law depending on the channel slope and corresponding hydraulic conditions. The spatially varying friction coefficient is then calculated as a function of the flow depth and composite roughness, which depends upon the local status of fraction cover, bedform dimension, and bedload transport.

<table>
<thead>
<tr>
<th>RUN</th>
<th>$k_b$</th>
<th>$\alpha_c$</th>
<th>$S$</th>
<th>$Z_{bi}$</th>
<th>$Q_w$</th>
<th>$Q_c$</th>
<th>$Q_s$</th>
<th>$Q_s/Q_c$</th>
<th>$d$</th>
<th>$H$</th>
<th>$U$</th>
<th>$F_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-A1</td>
<td>0.4</td>
<td>0.03</td>
<td>1.15</td>
<td>1.5</td>
<td>24</td>
<td>25</td>
<td>5</td>
<td>0.2</td>
<td>2</td>
<td>4.3</td>
<td>62</td>
<td>0.78</td>
</tr>
<tr>
<td>2-A2</td>
<td>0.4</td>
<td>0.03</td>
<td>1.15</td>
<td>1.5</td>
<td>24</td>
<td>25</td>
<td>10</td>
<td>0.4</td>
<td>2</td>
<td>4.3</td>
<td>62</td>
<td>0.6</td>
</tr>
<tr>
<td>2-A3</td>
<td>0.4</td>
<td>0.03</td>
<td>1.15</td>
<td>1.5</td>
<td>24</td>
<td>25</td>
<td>15</td>
<td>0.6</td>
<td>2</td>
<td>4.3</td>
<td>62</td>
<td>0.41</td>
</tr>
<tr>
<td>2-A4</td>
<td>0.4</td>
<td>0.03</td>
<td>1.15</td>
<td>1.5</td>
<td>24</td>
<td>25</td>
<td>20</td>
<td>0.8</td>
<td>2</td>
<td>4.3</td>
<td>62</td>
<td>0.27</td>
</tr>
<tr>
<td>2-A5</td>
<td>0.4</td>
<td>0.03</td>
<td>1.15</td>
<td>1.5</td>
<td>24</td>
<td>25</td>
<td>25</td>
<td>1.0</td>
<td>2</td>
<td>4.5</td>
<td>59</td>
<td>0</td>
</tr>
<tr>
<td>2-B1</td>
<td>4</td>
<td>0.05</td>
<td>2</td>
<td>2</td>
<td>55</td>
<td>110</td>
<td>44</td>
<td>0.4</td>
<td>7</td>
<td>5</td>
<td>122</td>
<td>1</td>
</tr>
<tr>
<td>2-B2</td>
<td>4</td>
<td>0.05</td>
<td>2</td>
<td>2</td>
<td>55</td>
<td>110</td>
<td>66</td>
<td>0.6</td>
<td>7</td>
<td>5.5</td>
<td>111</td>
<td>0.4</td>
</tr>
<tr>
<td>2-B3</td>
<td>4</td>
<td>0.05</td>
<td>2</td>
<td>2</td>
<td>55</td>
<td>110</td>
<td>88</td>
<td>0.8</td>
<td>7</td>
<td>5.5</td>
<td>111</td>
<td>0.25</td>
</tr>
<tr>
<td>2-B4</td>
<td>4</td>
<td>0.05</td>
<td>2</td>
<td>2</td>
<td>55</td>
<td>110</td>
<td>110</td>
<td>1.0</td>
<td>7</td>
<td>6</td>
<td>102</td>
<td>0</td>
</tr>
<tr>
<td>2-B2-a</td>
<td>4</td>
<td>0.05</td>
<td>2</td>
<td>1</td>
<td>55</td>
<td>110</td>
<td>66</td>
<td>0.6</td>
<td>7</td>
<td>5</td>
<td>122</td>
<td>1</td>
</tr>
<tr>
<td>2-B2-b</td>
<td>4</td>
<td>0.05</td>
<td>2</td>
<td>4</td>
<td>55</td>
<td>110</td>
<td>66</td>
<td>0.6</td>
<td>7</td>
<td>6</td>
<td>102</td>
<td>0.4</td>
</tr>
<tr>
<td>2-B2-c</td>
<td>4</td>
<td>0.05</td>
<td>2</td>
<td>6</td>
<td>55</td>
<td>110</td>
<td>66</td>
<td>0.6</td>
<td>7</td>
<td>6</td>
<td>102</td>
<td>0.4</td>
</tr>
<tr>
<td>2-Ax</td>
<td>0.4</td>
<td>0.03</td>
<td>1.15</td>
<td>0</td>
<td>24</td>
<td>150</td>
<td>150</td>
<td>1.0</td>
<td>2</td>
<td>2.9</td>
<td>92</td>
<td>0</td>
</tr>
<tr>
<td>2-Bx</td>
<td>4</td>
<td>0.05</td>
<td>2</td>
<td>0</td>
<td>55</td>
<td>350</td>
<td>350</td>
<td>1.0</td>
<td>7</td>
<td>5</td>
<td>122</td>
<td>0</td>
</tr>
<tr>
<td>2-Dx-a</td>
<td>0.2</td>
<td>0.08</td>
<td>0.3</td>
<td>0</td>
<td>55</td>
<td>8</td>
<td>2</td>
<td>0.25</td>
<td>2</td>
<td>7.5</td>
<td>81</td>
<td>0.7</td>
</tr>
<tr>
<td>2-Dx-b</td>
<td>0.2</td>
<td>0.08</td>
<td>0.3</td>
<td>0</td>
<td>55</td>
<td>8</td>
<td>4</td>
<td>0.5</td>
<td>2</td>
<td>7.5</td>
<td>81</td>
<td>0.6</td>
</tr>
<tr>
<td>2-Dx-c</td>
<td>0.2</td>
<td>0.08</td>
<td>0.3</td>
<td>0</td>
<td>55</td>
<td>8</td>
<td>6</td>
<td>0.75</td>
<td>2</td>
<td>7.5</td>
<td>81</td>
<td>0.52</td>
</tr>
<tr>
<td>2-Dx-d</td>
<td>0.2</td>
<td>0.08</td>
<td>0.3</td>
<td>0</td>
<td>55</td>
<td>8</td>
<td>8</td>
<td>1.0</td>
<td>2</td>
<td>7.5</td>
<td>81</td>
<td>0</td>
</tr>
</tbody>
</table>

The run names correspond to Chatanantavet and Parker’s (2008) experimental conditions. $k_b$ denotes bedrock roughness height, $\alpha_c$ denotes Shields number correction coefficient, $S$ denotes channel slope, $Z_{bi}$ denotes initial sediment cover thickness, $Q_w$ denotes water discharge, $Q_c$ denotes sediment transport capacity, $Q_s$ denotes sediment supply rate, $q_s/q_c$ denotes the ratio of sediment supply to transport capacity, $d$ denotes grain size, $H$ denotes flow depth, $U$ denotes flow velocity, and $F_e$ denotes the fraction of bedrock exposure, respectively.

Table 1. Summary of Flow, Sediment Transport, and Topographic Conditions
rate. The critical Shields parameters for each channel slope are back-calculated by equating the sediment transport capacity from our numerical simulations and experimental runs of Chatanantavet and Parker (2008) conducted in bedrock channels, and adjustment to the correction coefficient $\alpha_c$ is employed.

In each simulation, the sediment supply $q_s$ was specified as a target fraction of the transport capacity $q_c$. To determine $q_c$, we conducted simulations for each run, assuming that the initial bed is nearly flat for either with initial sediment cover (Runs 2-A and 2-B) or bare bedrock surface (Runs 2-Ax, 2-Bx, and 2-Dx) without the presence of bedforms. The bedrock transport capacity is the lowest rate of sediment input per unit width fed into the initially bare bedrock bed that results in equilibrium conditions where the bed is completely covered ($F_c = 0$). The alluvial transport capacity is the channel averaged sediment transport rate in the alluvial channel.

3 Results

3.1 Simulations with no initial alluvial cover

Runs 2-Ax, 2-Bx, and 2-Dx all commenced with a bare bedrock bed, and are used to investigate how channel slope affects sudden vs. gradual alluviation as sediment supply increases. The lowest-slope simulations ($S = 0.003$) of Run 2-Dx exhibited a gradual decrease in bedrock exposure at increasing sediment supplies. An example of bed evolution for these runs is shown in Figure 1 for Run 2-Dx-a, for which bedrock surface roughness was 0.2 mm and the ratio of sediment supply to transport capacity was $q_s/q_c = 0.25$. The time evolution of alluvial cover illustrates that the sediment grains initially deposit in the lower topography areas, and sediment patches gradually grow thicker and expand in the upstream direction.
Figure 2. Simulated bed evolution of Run 2-Dx-a, which began with a bare bedrock bed and sediment supplied at $q_s/q_c = 0.25$. Colorbar shows the thickness of the sediment cover, and white areas correspond to the exposed bedrock surface.

Figure 2 shows the final state of alluviation for all of the low-slope 2-Dx runs. As sediment supply (i.e., $q_s/q_c$) increases, the alluviated patches grow in size and the overall fraction of exposed bedrock ($F_e$) decreases.

Figure 1. Plan view of bedrock topography (top). Colorbar scale indicates the detrended bed elevation. Simulated bed evolution of Run 2-Bx with $q_s/q_c = 1$ (bottom). Colorbar shows the thickness of the sediment cover, and white areas correspond to the exposed bedrock surface.
Figure 3 shows the time evolution of alluvial patch formation for simulation Run 2-Bx, which commenced from the bare bedrock bed in a channel with a higher slope of 0.02. For this run, bedrock roughness was 4 mm and sediment was supplied at the channel’s transport capacity ($q_s/q_c = 1$). The sediment tends to deposit on the stoss side of the bedrock mounds where low flow velocity results in low Froude number, shear stress, and sediment transport capacity. The sediment forms small patches that grow in size as they move downstream. The migrating alluvial patches bypass the high bedrock areas by moving through the topographic lows. A sediment strip develops and shifts from one side of the channel to the other, and the entire channel bed is eventually covered with sediment. Complete alluviation of the bedrock bed begins from the upstream end of the channel, and alternate-bar-like patterns form as sediment moves across the channel.

Unlike the lower-slope runs of 2-Dx, simulations for 2-Ax ($S = 0.0115$) and 2-Bx ($S = 0.02$) were not able to achieve persistent alluvial cover for sediment supplies less than the transport capacity. Figure 4a demonstrates how the fraction of exposed bedrock surface varies with the sediment supply to transport capacity ratio in the initially bare bedrock channel. In steep-slope channels ($S \geq 0.0115$) without preexisting sediment, the bedrock bed remains fully

**Figure 3.** Plan view of Runs 2-Dx at equilibrium. The sediment is supplied at $q_s/q_c = 0.25$, 0.5, 0.75, and 1.0, from top to bottom. Colorbar shows the thickness of the sediment cover, and white areas correspond to the exposed bedrock surface.
exposed while the sediment supply rate is less than the channel sediment transport capacity. When the sediment supply momentarily exceeds the threshold value, the entire bedrock bed is completely covered by sediment, so-called “runaway alluviation”. In contrast, in the lower slope channel (\( S = 0.003 \), Run 2-Dx), the fractional bedrock exposure declines with increasing sediment supply in a more or less linear fashion at below-capacity sediment supply. These results are qualitatively and quantitatively similar to the observations of Chatanantavet and Parker (2008), whose experimental results are plotted alongside ours on the same figure (Figure 4).

3.2 Simulations with initial alluvial cover and varying slope

Figure 4b compares the model-predicted and experimentally observed relationships between bedrock exposure fraction and sediment supply in channels with initial sediment cover. In a steeper slope channel (\( S = 0.02 \), Run 2-B), the bedrock is completely exposed when the sediment supply to transport capacity rate is less than 0.4. However, a linear relationship between the extent of sediment cover and sediment supply is exhibited when the sediment supply exceeds the threshold value (\( q_s/q_c > 0.6 \)). This threshold sediment supply rate beyond which there is a linear relationship between sediment cover fraction and sediment supply ratio.

![Figure 4](image_url)

**Figure 4.** Results from the numerical simulations and flume experiments (Chatanantavet & Parker, 2008) of the fraction of bedrock exposure with varying sediment supply to transport capacity ratio for the simulations commenced from (a) bare bedrock channel and (b) alluvial channel.
decreases as the channel slope decreases (Run 2-A). In a channel slope of 0.0115, the linear relationship is observed throughout the entire range of $q_s/q_c$ that the fraction of bedrock exposure increases gradually as the sediment supply to capacity ratio decreases.

3.3 Simulations with various initial sediment layer thickness

The simulations of Runs 2-B2 and 2-B2-a to 2-B2-c are conducted to explore the effect of antecedent sediment layer thickness on alluvial patterns. Figure 5 shows the time evolution of bedrock exposure and averaged alluvial thickness in the 2% slope channel for both the simulations and Chatanantavet and Parker (2008) experiments. The initial sediment cover thickness was varied between 1 and 6 cm while the sediment flux of 66 g/s is constantly supplied. In the simulation starting with 1 cm sediment cover thickness, the sediment quickly erodes and washes out from the channel. The simulations with 2 cm or higher initial alluvial thickness present that the fraction of bedrock exposure increases gradually and converges approximately at 0.4. For those runs, the mean alluvial layer thickness over the sediment-covered area evolves toward 2 cm when the initial cover thickness equals or exceeds 2 cm.

Figure 5. The time evolution of the (a) fraction of bedrock exposure from numerical simulations, (b) flume experiments (Chatanantavet & Parker, 2008), and (c) alluvial cover thickness averaged over the covered area only. The Runs of 2-B2-a, 2-B2, 2-B2-b, and 2-B2-c correspond to the initial cover thickness of 1, 2, 4, and 6 cm, respectively.
4 Discussion

Our results replicated several phenomena observed in the Chatanantavet & Parker (2008) experiments: 1) runaway alluviation for high-slope channels with no initial sediment cover, and gradual alluviation for low-slope channels with no initial sediment cover (Figure 4a), 2) a threshold sediment supply for higher-slope channels with the initial cover below which sediment washes out, but above which a linear relationship between sediment supply and fraction exposure develops (Figure 4b), and 3) a threshold initial sediment cover thickness above which quasi-steady sediment cover develops, but below which full bedrock exposure and sediment washout occurs. The model’s complete description of the hydrodynamic and sediment transport conditions throughout the experiments help explain why these phenomena occurred.

4.1 Runaway alluviation vs. gradual alluviation

The gradual decrease in the fraction of bedrock exposed with increasing sediment supply at the low-slope (2-Dx) condition, and the runaway alluviation at higher slopes where persistent (total) sediment cover occurred only at sediment supply greater than the transport capacity, with complete bedrock exposure at sediment supplies below that value, was observed in both our numerical simulations and in the experiments of Chatanantavet and Parker (2008). Chatanantavet and Parker (2008) suggested that this phenomenon may be in part due to sediment grain interactions, such that at low slopes, frequent grain collisions lead to the development of alluvial patches, which then increase local roughness and grow in size, whereas at higher slopes those collisions may be less frequent and less likely to develop incipient patches. Hodge and Hoey’s (2012) cellular automata model runs with varying grain entrainment probability on bedrock and alluvial surfaces suggest that the much higher grain entrainment probability on the bedrock surface than the alluvial surface is a key factor of runaway alluviation.
Our model treats sediment transport as a continuum and is therefore not able to model grain interactions directly, but it still exhibited the same sort of slope-dependent relationship between gradual vs. runaway alluviation. Our model results suggest this is a result of the development of transcritical flows over bare low-slope beds, but not in higher-slope channels.

Figure 6 shows the Froude numbers calculated in the bare bedrock channel before feeding sediment at the inlet for the Runs 2-Ax, 2-Bx, and 2-Dx. The flow is supercritical in steep slope channels (2-Ax and 2-Bx), while locally subcritical flow is observed in lower slope channels (2-Dx). The flow field variables and shear stress result in a low sediment transport capacity where the flow is subcritical in the low-slope channel (Figure 7). These subcritical zones become areas of deposition, inducing a cascade of local changes in roughness, velocity, shear stress, and critical dimensionless shear stress which permit the growth of alluvial patches. However, in the steep slope channels, the large sediment transport capacity produced by high flow velocity and shear stress results in the particles passing through the channel reach without ever residing on the bed when the sediment supply is less than the transport capacity (Figure 4a).

Figure 6. Initially calculated Froude number for Runs (a) 2-Dx, (b) 2-Ax, and (c) 2-Bx in bare bedrock channel. Colorbars indicate the scale of computed values, respectively.
Figure 7. Initially calculated variables in bare bedrock channel for the Run 2-Dx: (a) flow depth, (b) flow velocity, (c) flow discharge, (d) dimensionless shear stress, (e) critical Shields parameter, and (f) dimensionless bedload transport capacity. Colorbars indicate the scale of computed values, respectively.

Thus, while grain interactions and roughness feedbacks may play a role in developing persistent alluvial sediment cover at below-capacity sediment supply in low-slope channels, the flow field – especially the presence of regions of transcritical flow – may be a necessary and important condition leading to this behavior.

4.2 Sediment supply threshold for persistent alluvial patches

The 2% slope channel simulations with 2 cm initial cover thickness (Runs 2-B1, 2-B2, 2-B3, and 2-B4) show sediment washing out when the sediment supply rate is less than the
threshold capacity of about . The sediment forms alternate bars that produce additional
resistance to the flow. The increased form drag roughness competes with decreased bed surface
roughness due to exposed bedrock beds. The channel experiences complete bedrock exposure
when the sediment supply does not meet the threshold amount to maintain the bedform. In other
words, the sediment supplied to the system is redistributed to create bedforms which increase
overall roughness and encourage sediment deposition and persistence of the alluvial cover; if the
supply is insufficient to build the bedforms, the overall roughness decreases as bedrock becomes
exposed and all of the sediment washes out (Figure 8).

Similar mechanisms explain the relationship between the initial thickness of sediment
cover and the development of persistent alluvial cover as opposed to sediment washout at below-
threshold initial thicknesses (Figure 5). The 1.15 % slope channel simulations with 1.5 cm initial
cover thickness (Runs 2-A1 through 2-A5) show the linear relationship between the fraction
cover and sediment supply to transport capacity. Transcritical flow is observed in this simulation
(Figure 9). The flow is subcritical in the alluvial area and supercritical over the bedrock surface.
The sediment forms a series of discrete sediment patches with a thin layer rather than a

Figure 8. Simulated bed evolution of Run 2-B1 with \( q_s/q_c = 0.4 \). Colorbar shows the thickness of
the sediment cover, and white areas correspond to the exposed bedrock surface.
continuous strip of sediment. The subcritical flow promotes the thin layer of alluvial patches to form even in low sediment supply channels.

Figure 10 shows how roughness changes from the initial value of grain and sediment transport roughness, form drag, and total hydraulic roughness through time. The non-bedform

**Figure 9.** Simulation results commenced from alluvial channel for the Run 2-A2 at $t = 5$ h: (a) alluvial thickness, (b) flow depth, (c) flow velocity, (d) Froude number, (e) dimensionless shear stress, (f) critical Shields parameter, and (g) dimensionless bedload transport capacity. Colorbars indicate the scale of computed values, respectively. White areas correspond to (a) the exposed bedrock surface and (g) no bedload transport.
Figure 10. The time evolution of roughness difference from the initial roughness value: (a) bed surface roughness, (b) bedform roughness, and (c) total hydraulic roughness. The Runs of 2-B2-a, 2-B2, 2-B2-b, and 2-B2-c correspond to the initial cover thickness of 1, 2, 4, and 6 cm, respectively.

Conclusions

We used a two-dimensional morphodynamic model to conduct a series of numerical experiments in the mixed bedrock-alluvial channel. This study explores the interaction between bedrock alluviation and morphological evolution. Simulations with varying sediment supply are conducted in different slope and antecedent channel conditions. The model replicated observations from a mixed bedrock-alluvial experiment (Chatantanavet & Parker, 2008), including a) the relationship between channel slope and gradual vs. runaway alluviation, b) the slope-dependent sediment supply threshold for development of persistent alluvial cover, c) the relationship between decreasing bedrock exposure and increasing sediment supply, and d) the
development of constant alluvial cover thickness regardless of initial sediment thickness, provided the initial thickness exceeds a minimum value necessary to maintain bedform dimensions.

The model results provide physical insight on the mechanisms responsible for these phenomena. Transcritical flow plays an important role in initiating sediment deposition over initially bare bedrock, and the development of transcritical zones in low-slope simulations but not high-slope simulations may explain the apparent slope dependence of runaway alluviation. Persistent sediment cover in high-slope channels is possible when rough alluvial surfaces balance the extent of lower-roughness bedrock surfaces, and steeper channels require higher sediment supply to exceed an apparent threshold where that balance can occur.

Open Research

The output data from numerical simulations used in this analysis can be found at https://github.com/rcemorpho/morph2d.

References


Patterns of Alluviation in Mixed Bedrock-Alluvial Channels: 2. Controls on the Formation of Alluvial Patches

Jongseok Cho$^1$ and Peter A. Nelson$^1$

$^1$Department of Civil and Environmental Engineering, Colorado State University, Fort Collins, Colorado, USA.

Corresponding author: Peter Nelson (peter.nelson@colostate.edu)

Key Points:

- Numerical simulations are conducted in mixed bedrock-alluvial channels to analyze controls on alluvial patterns.
- Distinct alluviation patterns are observed depending on the channel slopes and initial alluviation conditions.
- The interactions between flow, bed topography, and roughness control the mechanisms of bed evolution.
Abstract

Understanding the development and spatial distribution of alluvial patches in mixed bedrock-alluvial rivers is necessary to predict the mechanisms of the interactions between sediment transport, alluvial cover, and bedrock erosion. This study aims to analyze patterns of bedrock alluviation using a 2D morphodynamic model, and to use the model results to better understand the mechanisms responsible for alluvial patterns observed experimentally. A series of simulations are conducted to explore how alluvial patterns in mixed bedrock-alluvial channels form and evolve for different channel slopes and antecedent sediment layer thicknesses. In initially bare bedrock low-slope channels, the model predicts a linear relationship between sediment cover and sediment supply because areas of subcritical flow enable sediment deposition, while in steep-slope channels the flow remains fully supercritical and the model predicts so-called runaway alluviation. For channels initially covered with sediment, the model predicts a slope-dependent sediment supply threshold above which a linear relationship between bedrock exposure and sediment supply develops, and below which the bedrock becomes fully exposed. For a given sediment supply, the fraction of bedrock exposure and average alluvial thickness converge toward the equilibrium value regardless of the initial cover thickness so long as it exceeds a minimum threshold. Steep channels are able to maintain a continuous strip of sediment under sub-capacity sediment supply conditions by achieving a balance between increased form drag as bedforms develop and reduced surface roughness as the portion of alluvial cover decreases. In lower-slope channels, alluvial patches are distributed sporadically in regions of the subcritical flow.

Plain Language Summary
Bedrock rivers may have patches of alluvial sediment that covers some or all of the underlying bedrock. The amount of this sediment cover can change dynamically over time depending on the flow, upstream sediment supply, channel morphology, and antecedent sediment conditions. Here, we use a numerical model to simulate flow and sediment transport so that we may better understand what controls sediment cover in mixed bedrock-alluvial rivers. We use the model to simulate channels of varying slopes, sediment supplies, and initial sediment cover, and we analyze the model’s output to gain insight on how alluvial patches form and what controls their extent and dynamics. Our numerical model results produce phenomena that have been observed in physical experiments, and they show that the channel slope and initial sediment thickness plays an important role in determining whether and how much sediment can be deposited. Our results also show that flow transitions provide critical locations where sediment deposits can start to form. Persistent sediment cover in bedrock channels can develop when a delicate equilibrium is reached between sediment roughness and the flow field.

1 Introduction

Bedrock channels are characterized by occasional or continuous exposures of nonalluviated bedrock, which is a consequence of these channels receiving a sediment supply that is less than their transport capacity. A wide variety of models for drainage network evolution distinguish between bedrock and alluvial reaches (Howard, 1980; Howard et al., 1994; Howard & Kerby, 1983; Montgomery et al., 1996) by using channel slope and discharge to express channel transport capacity. Field investigations (Lamb et al., 2008; Massong & Montgomery, 2000; Montgomery et al., 1996; Montgomery & Buffington, 1997) demonstrate that bedrock channels occur at slopes greater than a critical value ($S > S_c$), and alluvial channels form at a slope less than the critical value ($S \leq S_c$).
The spatial distribution of alluvial cover in mixed bedrock-alluvial channels has importance for determining rates and patterns of bedrock erosion, hydrodynamics, and aquatic habitat. Mechanistic models of bedrock erosion incorporate the erosional mechanism of saltating bedload particles impacting and eroding bedrock (e.g., Demeter et al., 2005; Hartshorn et al., 2002; Sklar & Dietrich, 1998, 2001, 2004; Zhang et al., 2015), implicitly introducing dependence on sediment supply and bedrock exposure into the calculation of erosion rates.

Competition between the tools and cover effects controls the spatial distribution of the bedrock channel erosion, resulting in lateral and vertical channel erosion and meandering (Finnegan et al., 2007; Lamb et al., 2015; Turowski et al., 2007; Turowski, Hovius, Meng-Long, et al., 2008; Turowski, Hovius, Wilson, et al., 2008). The cover effect is typically demonstrated by linear or exponential relations of fractional bedrock exposure as a function of sediment supply to transport capacity ratio (Sklar & Dietrich, 1998, 2004; Turowski et al., 2007). Additionally, sediment supply and alluvial cover impact the maintenance and distribution of aquatic habitat and attached micro-organisms (Buffington et al., 2004; Detert & Parker, 2010; Huston & Fox, 2015, 2016; Kuhnle et al., 2013; Lisle & Hilton, 1992; Lisle & Lewis, 1992; Madej, 2001).

Alluvial patterns in bedrock channels are controlled by spatial and temporal variations in sediment flux, transport capacity, and bed topography. These channels can exhibit alluvial patterns ranging from continuous and concentrated longitudinal strips of sediment to spatially discontinuous patches of sediment. Experiments in mixed bedrock-alluvial channels have observed spatially concentrated sediment cover and storage in low parts of the underlying bedrock topography (Finnegan et al., 2007; Johnson & Whipple, 2007), indicating that topographic roughness induces changes in local flow properties and threshold of sediment motion. Inoue et al. (2014) observed inconsistent development of alluvial cover in the inner
channel through their field experiments. Hodge and Hoey’s (2016a, 2016b) experiments show that velocity is an important control on sediment deposition, as they did not observe sediment cover at low areas of the bed where the flow velocity remained high. The results from several experimental studies (Chatanantavet & Parker, 2008; Johnson & Whipple, 2010; Sklar & Dietrich, 2004; Turowski et al., 2007) suggest a few major factors control grain entrainment and bedrock exposure, such as sediment supply rate, channel slope, material size, and bed roughness.

A set of flume experiments (Chatanantavet & Parker, 2008) using different channel bed slopes has shown that for lower slopes ($S < 0.0115$) bedrock exposure decreased more or less linearly with increasing the ratio of sediment supply rate to capacity transport rate. However, for sufficiently higher slopes ($S \geq 0.0115$), the bedrock remained fully exposed when the ratio of sediment supply to transport capacity is less than a critical value, while a linear relationship between the degree of bedrock exposure and sediment supply rate to transport capacity ratio prevailed when the sediment supply exceeds transport capacity. These experiments also documented a slope-dependent “runaway alluviation,” where for initially bare-bedrock conditions, low-slope channels develop linearly increasing sediment cover with increasing sediment supply, while high-slope channels remain completely exposed until sediment supply exceeds the transport capacity, beyond which the channel becomes fully alluviated. Chatanantavet and Parker (2008) suggested this may be a result of slope-dependent grain interactions; later experiments by Mishra and Inoue (2020) indicated that runaway alluviation occurred when the bedrock roughness was lower than the sediment roughness, but the gradual alluvial cover could develop when the bedrock roughness was higher than that of the sediment. A full explanation of the slope-dependent behavior of sediment dynamics in mixed bedrock-alluvial channels is still needed.
Morphodynamic models have struggled to replicate the types of observations made in mixed bedrock-alluvial experiments. Promising results have been presented by Hodge and Hoey (2012), who developed a cellular automaton (CA) model where the probability of entrainment of individual grains was specified for bedrock or alluvial areas. Their model found relationships between bedrock exposure and the ratio of sediment supply to transport capacity ($q_s/q_c$) similar to those observed by Chatanantavet and Parker (2008), but in contrast to the experimental observations, the majority of their CA model runs predicted that the presence or absence of sediment cover on the bed at the beginning of the run did not affect the steady state sediment cover. Additionally, the entrainment probabilities needed to change from run to run for their results to achieve the diversity of findings Chatanantavet and Parker (2008) reported. While CA models like this provide interesting insight on the potential importance of grain dynamics for alluvial patterns in bedrock-alluvial channels, the absence of a flow model makes connecting probabilities of sediment entrainment and deposition to physical mechanisms of sediment transport and alluviation challenging.

Because of the complexity inherent in the roughness relationship incorporated to the flow resistance and sediment transport, evaluating the relative influence of different roughness mechanisms for channel evolution is currently a challenging problem. The patterns of sediment cover over the bedrock bed affect the spatial distribution of local roughness, flow rate, and sediment transport. Here we use a new morphodynamic model to reproduce many of the phenomena that have been observed in mixed bedrock-alluvial channels, and we use the model predictions to untangle the mechanisms responsible for the range of dynamic sediment behavior in these environments. In particular, we use the model to investigate the following questions: 1) what explains the apparent slope dependence for runaway alluviation? 2) How can mixed
bedrock-alluvial channels maintain alluvial cover when the sediment supply is less than the
transport capacity? 3) How do initial conditions of sediment cover affect the temporal
development of alluvial cover, and the overall relationship between bedrock exposure and
sediment supply? Our results reveal the important interactions between dynamic channel
roughness, flow patterns, sediment transport rates, bedform development, and alluvial cover.

2 Methods

2.1 Morphodynamic model

We have developed a two-dimensional numerical morphodynamic model for mixed
bedrock-alluvial channels. The model is fully described in Cho and Nelson (submitted), and we
summarize the key aspects of the model here. The model consists of three major components: (1)
the 2D shallow water equations (SWE) in the depth-averaged form are applied to solve the
hydrodynamical component, (2) a sediment transport model calculates bedload transport rates
associated with the hydrodynamic variables and topographic variation, and (3) the modified
Exner equation, which computes changes in alluvial concentration or thickness due to sediment
transport divergence. The novel aspects of this model, compared to previous models applied to
mixed bedrock-alluvial morphodynamics, are 1) the Exner equation of sediment continuity
accounts for the volume of bedload in transport and the fraction of bed covered in sediment, 2) a
composite alluvial and bedrock roughness is used in the flow calculation, 3) the friction for
sediment transport is modified to account for the effects of bedforms, and 4) the numerical
scheme is robust and capable of handling Froude transitions and capturing shocks. We
summarize the key components of the model below.
A key assumption in most numerical models combining water flow, sediment transport, and morphological evolution is that the response time of bed evolution is relatively long compared to the timescales of relevance to the flow of water (McLean et al., 1994; Nelson et al., 2003; Tubino et al., 1999). This allows a decoupling between the water flow computation and the sediment equation by assuming a quasi-steady approximation of morphodynamic process that the bed level does not change rapidly during an infinitesimal time interval while the flow field adapts instantaneously. Thus, the decoupled model practically solves for the flow field and topographic evolution using an iterative procedure.

Mixed bedrock-alluvial channels exhibit complex roughness feedbacks due to differential roughness of alluvium and bedrock surfaces, the development of bedforms and associated form drag, as well as bed shear stress taken up by sediment transport itself. To account for this, our model uses a composite roughness partitioned into surface roughness of alluvial and bedrock bed, sediment transport roughness, and form drag. The ripple factor is applied to the shear stress to remove the form drag of the bedforms assuming the remaining part is responsible for sediment transport.

Water depth and flux are calculated using the Harten-Lax-van Leer-Contract (HLLC) scheme with the weighted average flux (WAF) method (Toro, 1992a, 1992b) and an explicit application of the central difference of the viscous and friction terms at each computational cell center in a 2D domain. Because of the local and global change in flow resistance associated with different substrate roughness between the bedrock and bed material and bedform evolution, a treatment of transcritical flow is necessary. The flow resistance consists of skin friction, form drag, and bedload transport roughness. A linear fractional cover model representing the relation between the alluvial bed and bare bedrock bed, the volume of local bed material per volume of a
monolayer of sediment grains, is used to calculate local skin friction. The form drag effect is determined using the local bed slope and topographic variation averaged over the area of interest. Additionally, the bedload layer thickness is added to the total roughness where the sediment transport occurs. The explicit calculation of each of these components of roughness is critical to be able to replicate observations of persistent sediment cover in mixed bedrock-alluvial channels (Cho and Nelson, submitted).

The bed morphology is updated using the modified Exner equation for sediment continuity (Inoue et al., 2014, 2016; Luu et al., 2004). The sediment transport capacity on the alluvial bed is estimated from Wong et al. (2007). A correction of the bedload transport rate on the pure bedrock bed is necessary considering a relatively small volume of bedload transport:

\[ q_b = \begin{cases} 
\frac{V_b}{V_{bc}} q_{bc} & \text{for } 0 \leq V_b < V_{bc} \\
q_{bc} & \text{for } V_{bc} \leq V_b 
\end{cases} \]  

(1)

where \( q_b \) is the bedload transport rate per unit width, \( q_{bc} \) is the bedload transport capacity per unit width, \( V_b \) is the volume of sediment per unit area in the bedload layer, and \( V_{bc} \) is the saturation volume per unit area in the bedload layer. When the bed is completely bare, only the volume of sediment in the bedload layer is brought into bedload transport without resting on the bed, referred to as the throughput bedload. When the volume of the bedload layer exceeds the saturation value, sediment starts to deposit on the bed and the linear cover fraction model is utilized to determine whether the bed is in the state of partial or complete cover:

\[ \eta_a = \begin{cases} 
\frac{V_{ba} - V_{bc}}{1 - \lambda} & \text{for } V_{bc} \leq V_{ba} \\
0 & \text{for } 0 \leq V_{ba} < V_{bc} 
\end{cases} \]  

(2)
where $\eta_a$ is the alluvial layer thickness, $V_{ba}$ is the total volume of sediment per unit area in alluvial bedload layers, $\lambda$ is the porosity, and

$$V_b = \begin{cases} V_{bc} & \text{for } V_{bc} \leq V_{ba} \\ V_{ba} & \text{for } 0 \leq V_{ba} < V_{bc} \end{cases}$$

(3)

where $V_{bc} = q_{bc}/u_s$ and $u_s$ is the saltation velocity.

The skin friction is used for the calculation of dimensionless shear stress to account for the fact that only the near-bed grain roughness is responsible for the sediment transport, using a correction referred as to the ripple factor (Ribberink, 1987; Vermeer, 1986). The dimensionless shear stress (Struiksma, 1985; Talmon et al., 1995) and critical Shields number (Calantoni, 2002; Duan & Julien, 2005; Soulsby, 1997) are corrected for spatially varying bed topography in the direction of flow.

The critical Shields parameter calculated as a function of the ratio of the bedrock hydraulic roughness to the grain size (Inoue et al., 2014; Johnson, 2014; Mishra & Inoue, 2020) is adopted instead of constant value for bedrock and alluvial surfaces. We use a modified dimensionless critical shear stress model to simulate bed evolution in different channel slopes:

$$\tau_c' = \alpha_c (k_0/d)^{0.6}$$

(4)

where $\alpha_c$ is the correction factor for different channel slopes, $k_0$ is the hydraulic roughness height, and $d$ is the grain size.

2.2 Simulation conditions

We conducted a set of numerical experiments to explore how channel slope, initial sediment cover thickness, and sediment supply impact the evolution of patterns of alluvial cover
in mixed bedrock-alluvial channels. The simulations were designed to complement the
experiments of Chatanantavet and Parker (2008) and provide mechanistic insight into controls on
alluvial patterns in bedrock channels. Table 1 summarizes the initial flow and sediment condition
used in each experiment based on the flume experiments conducted by Chatanantavet and Parker
(2008). The computational channel is straight and longer than the experimental channel to avoid
possible problems regarding bedform development sensitive to boundary disturbances. All
simulations impose constant water and sediment supply, uniform sediment size, and non-erodible
bedrock to exclude potential disturbances to bed topography created by unsteady conditions. The
computations are stopped when near equilibrium conditions of bed topography are achieved, in
which the average sediment cover thickness and the fraction of bedrock cover vary around stable
values.

All simulations are performed in a rectangular bedrock channel having a length of 20 m
and a width of 0.9 m. Small topographic variations with a standard deviation of 2.2 mm and
peak-to-peak bed elevation of 9 mm are applied to the bedrock bed. First, two sets of simulations
are performed: (1) with some initial cover thickness with different channel slopes (Runs 2-A5
and 2-B5) and (2) without antecedent sediment cover (Runs 2-Ax, 2-Bx, and 2-Dx) to explore
the effect of channel slopes. Second, a set of Run 2-B is conducted with the \( q_s/q_c = 0.6 \) and
various initial sediment cover thicknesses of 1, 4, and 6 cm. Two types of uniform grains, fine 2
mm and coarse 7 mm gravels, are employed in the experiments with a mild slope channel (\( S \leq 0.0115 \)) and with a steep slope channel (\( S = 0.02 \)), respectively.
The bedrock and grain roughness heights are back-calculated from the logarithmic law depending on the channel slope and corresponding hydraulic conditions. The spatially varying friction coefficient is then calculated as a function of the flow depth and composite roughness, which depends upon the local status of fraction cover, bedform dimension, and bedload transport.
rate. The critical Shields parameters for each channel slope are back-calculated by equating the sediment transport capacity from our numerical simulations and experimental runs of Chatanantavet and Parker (2008) conducted in bedrock channels, and adjustment to the correction coefficient $\alpha_c$ is employed.

In each simulation, the sediment supply $q_s$ was specified as a target fraction of the transport capacity $q_c$. To determine $q_c$, we conducted simulations for each run, assuming that the initial bed is nearly flat for either with initial sediment cover (Runs 2-A and 2-B) or bare bedrock surface (Runs 2-Ax, 2-Bx, and 2-Dx) without the presence of bedforms. The bedrock transport capacity is the lowest rate of sediment input per unit width fed into the initially bare bedrock bed that results in equilibrium conditions where the bed is completely covered ($F_e = 0$). The alluvial transport capacity is the channel averaged sediment transport rate in the alluvial channel.

3 Results

3.1 Simulations with no initial alluvial cover

Runs 2-Ax, 2-Bx, and 2-Dx all commenced with a bare bedrock bed, and are used to investigate how channel slope affects sudden vs. gradual alluviation as sediment supply increases. The lowest-slope simulations ($S = 0.003$) of Run 2-Dx exhibited a gradual decrease in bedrock exposure at increasing sediment supplies. An example of bed evolution for these runs is shown in Figure 1 for Run 2-Dx-a, for which bedrock surface roughness was 0.2 mm and the ratio of sediment supply to transport capacity was $q_s/q_c = 0.25$. The time evolution of alluvial cover illustrates that the sediment grains initially deposit in the lower topography areas, and sediment patches gradually grow thicker and expand in the upstream direction.
Figure 2. Simulated bed evolution of Run 2-Dx-a, which began with a bare bedrock bed and sediment supplied at $q_s/q_c = 0.25$. Colorbar shows the thickness of the sediment cover, and white areas correspond to the exposed bedrock surface.

Figure 2 shows the final state of alluviation for all of the low-slope 2-Dx runs. As sediment supply (i.e., $q_s/q_c$) increases, the alluviated patches grow in size and the overall fraction of exposed bedrock ($F_e$) decreases.

Figure 1. Plan view of bedrock topography (top). Colorbar scale indicates the detrended bed elevation. Simulated bed evolution of Run 2-Bx with $q_s/q_c = 1$ (bottom). Colorbar shows the thickness of the sediment cover, and white areas correspond to the exposed bedrock surface.
Figure 3 shows the time evolution of alluvial patch formation for simulation Run 2-Bx, which commenced from the bare bedrock bed in a channel with a higher slope of 0.02. For this run, bedrock roughness was 4 mm and sediment was supplied at the channel’s transport capacity ($q_s/q_c = 1$). The sediment tends to deposit on the stoss side of the bedrock mounds where low flow velocity results in low Froude number, shear stress, and sediment transport capacity. The sediment forms small patches that grow in size as they move downstream. The migrating alluvial patches bypass the high bedrock areas by moving through the topographic lows. A sediment strip develops and shifts from one side of the channel to the other, and the entire channel bed is eventually covered with sediment. Complete alluviation of the bedrock bed begins from the upstream end of the channel, and alternate-bar-like patterns form as sediment moves across the channel.

Unlike the lower-slope runs of 2-Dx, simulations for 2-Ax ($S = 0.0115$) and 2-Bx ($S = 0.02$) were not able to achieve persistent alluvial cover for sediment supplies less than the transport capacity. Figure 4a demonstrates how the fraction of exposed bedrock surface varies with the sediment supply to transport capacity ratio in the initially bare bedrock channel. In steep-slope channels ($S \geq 0.0115$) without preexisting sediment, the bedrock bed remains fully

**Figure 3.** Plan view of Runs 2-Dx at equilibrium. The sediment is supplied at $q_s/q_c = 0.25, 0.5, 0.75,$ and 1.0, from top to bottom. Colorbar shows the thickness of the sediment cover, and white areas correspond to the exposed bedrock surface.
exposed while the sediment supply rate is less than the channel sediment transport capacity.

When the sediment supply momentarily exceeds the threshold value, the entire bedrock bed is completely covered by sediment, so-called “runaway alluviation”. In contrast, in the lower slope channel (\( S = 0.003 \), Run 2-Dx), the fractional bedrock exposure declines with increasing sediment supply in a more or less linear fashion at below-capacity sediment supply. These results are qualitatively and quantitatively similar to the observations of Chatanantavet and Parker (2008), whose experimental results are plotted alongside ours on the same figure (Figure 4).

### 3.2 Simulations with initial alluvial cover and varying slope

Figure 4b compares the model-predicted and experimentally observed relationships between bedrock exposure fraction and sediment supply in channels with initial sediment cover. In a steeper slope channel (\( S = 0.02 \), Run 2-B), the bedrock is completely exposed when the sediment supply to transport capacity rate is less than 0.4. However, a linear relationship between the extent of sediment cover and sediment supply is exhibited when the sediment supply exceeds the threshold value (\( q_s/q_c > 0.6 \)). This threshold sediment supply rate beyond which there is a linear relationship between sediment cover fraction and sediment supply ratio

![Figure 4](image)

**Figure 4.** Results from the numerical simulations and flume experiments (Chatanantavet & Parker, 2008) of the fraction of bedrock exposure with varying sediment supply to transport capacity ratio for the simulations commenced from (a) bare bedrock channel and (b) alluvial channel.
decreases as the channel slope decreases (Run 2-A). In a channel slope of 0.0115, the linear relationship is observed throughout the entire range of $q_s/q_c$ that the fraction of bedrock exposure increases gradually as the sediment supply to capacity ratio decreases.

3.3 Simulations with various initial sediment layer thickness

The simulations of Runs 2-B2 and 2-B2-a to 2-B2-c are conducted to explore the effect of antecedent sediment layer thickness on alluvial patterns. Figure 5 shows the time evolution of bedrock exposure and averaged alluvial thickness in the 2 % slope channel for both the simulations and Chatanantavet and Parker (2008) experiments. The initial sediment cover thickness was varied between 1 and 6 cm while the sediment flux of 66 g/s is constantly supplied. In the simulation starting with 1 cm sediment cover thickness, the sediment quickly erodes and washes out from the channel. The simulations with 2 cm or higher initial alluvial thickness present that the fraction of bedrock exposure increases gradually and converges approximately at 0.4. For those runs, the mean alluvial layer thickness over the sediment-covered area evolves toward 2 cm when the initial cover thickness equals or exceeds 2 cm.

Figure 5. The time evolution of the (a) fraction of bedrock exposure from numerical simulations, (b) flume experiments (Chatanantavet & Parker, 2008), and (c) alluvial cover thickness averaged over the covered area only. The Runs of 2-B2-a, 2-B2, 2-B2-b, and 2-B2-c correspond to the initial cover thickness of 1, 2, 4, and 6 cm, respectively.
4 Discussion

Our results replicated several phenomena observed in the Chatanantavet & Parker (2008) experiments: 1) runaway alluviation for high-slope channels with no initial sediment cover, and gradual alluviation for low-slope channels with no initial sediment cover (Figure 4a), 2) a threshold sediment supply for higher-slope channels with the initial cover below which sediment washes out, but above which a linear relationship between sediment supply and fraction exposure develops (Figure 4b), and 3) a threshold initial sediment cover thickness above which quasi-steady sediment cover develops, but below which full bedrock exposure and sediment washout occurs. The model’s complete description of the hydrodynamic and sediment transport conditions throughout the experiments help explain why these phenomena occurred.

4.1 Runaway alluviation vs. gradual alluviation

The gradual decrease in the fraction of bedrock exposed with increasing sediment supply at the low-slope (2-Dx) condition, and the runaway alluviation at higher slopes where persistent (total) sediment cover occurred only at sediment supply greater than the transport capacity, with complete bedrock exposure at sediment supplies below that value, was observed in both our numerical simulations and in the experiments of Chatanantavet and Parker (2008). Chatanantavet and Parker (2008) suggested that this phenomenon may be in part due to sediment grain interactions, such that at low slopes, frequent grain collisions lead to the development of alluvial patches, which then increase local roughness and grow in size, whereas at higher slopes those collisions may be less frequent and less likely to develop incipient patches. Hodge and Hoey’s (2012) cellular automata model runs with varying grain entrainment probability on bedrock and alluvial surfaces suggest that the much higher grain entrainment probability on the bedrock surface than the alluvial surface is a key factor of runaway alluviation.
Our model treats sediment transport as a continuum and is therefore not able to model grain interactions directly, but it still exhibited the same sort of slope-dependent relationship between gradual vs. runaway alluviation. Our model results suggest this is a result of the development of transcritical flows over bare low-slope beds, but not in higher-slope channels.

Figure 6 shows the Froude numbers calculated in the bare bedrock channel before feeding sediment at the inlet for the Runs 2-Ax, 2-Bx, and 2-Dx. The flow is supercritical in steep slope channels (2-Ax and 2-Bx), while locally subcritical flow is observed in lower slope channels (2-Dx). The flow field variables and shear stress result in a low sediment transport capacity where the flow is subcritical in the low-slope channel (Figure 7). These subcritical zones become areas of deposition, inducing a cascade of local changes in roughness, velocity, shear stress, and critical dimensionless shear stress which permit the growth of alluvial patches. However, in the steep slope channels, the large sediment transport capacity produced by high flow velocity and shear stress results in the particles passing through the channel reach without ever residing on the bed when the sediment supply is less than the transport capacity (Figure 4a).

Figure 6. Initially calculated Froude number for Runs (a) 2-Dx, (b) 2-Ax, and (c) 2-Bx in bare bedrock channel. Colorbars indicate the scale of computed values, respectively.
Thus, while grain interactions and roughness feedbacks may play a role in developing persistent alluvial sediment cover at below-capacity sediment supply in low-slope channels, the flow field – especially the presence of regions of transcritical flow – may be a necessary and important condition leading to this behavior.

4.2 Sediment supply threshold for persistent alluvial patches

The 2% slope channel simulations with 2 cm initial cover thickness (Runs 2-B1, 2-B2, 2-B3, and 2-B4) show sediment washing out when the sediment supply rate is less than the

Figure 7. Initially calculated variables in bare bedrock channel for the Run 2-Dx: (a) flow depth, (b) flow velocity, (c) flow discharge, (d) dimensionless shear stress, (e) critical Shields parameter, and (f) dimensionless bedload transport capacity. Colorbars indicate the scale of computed values, respectively.
threshold capacity of about . The sediment forms alternate bars that produce additional
resistance to the flow. The increased form drag roughness competes with decreased bed surface
roughness due to exposed bedrock beds. The channel experiences complete bedrock exposure
when the sediment supply does not meet the threshold amount to maintain the bedform. In other
words, the sediment supplied to the system is redistributed to create bedforms which increase
overall roughness and encourage sediment deposition and persistence of the alluvial cover; if the
supply is insufficient to build the bedforms, the overall roughness decreases as bedrock becomes
exposed and all of the sediment washes out (Figure 8).

Similar mechanisms explain the relationship between the initial thickness of sediment
cover and the development of persistent alluvial cover as opposed to sediment washout at below-
threshold initial thicknesses (Figure 5). The 1.15 % slope channel simulations with 1.5 cm initial
cover thickness (Runs 2-A1 through 2-A5) show the linear relationship between the fraction
cover and sediment supply to transport capacity. Transcritical flow is observed in this simulation
(Figure 9). The flow is subcritical in the alluvial area and supercritical over the bedrock surface.
The sediment forms a series of discrete sediment patches with a thin layer rather than a

![Figure 8](image_url)

**Figure 8.** Simulated bed evolution of Run 2-B1 with $q_s/q_c = 0.4$. Colorbar shows the thickness of
the sediment cover, and white areas correspond to the exposed bedrock surface.
continuous strip of sediment. The subcritical flow promotes the thin layer of alluvial patches to form even in low sediment supply channels.

Figure 10 shows how roughness changes from the initial value of grain and sediment transport roughness, form drag, and total hydraulic roughness through time. The non-bedform

![Figure 9](image-url)

**Figure 9.** Simulation results commenced from alluvial channel for the Run 2-A2 at $t = 5$ h: (a) alluvial thickness, (b) flow depth, (c) flow velocity, (d) Froude number, (e) dimensionless shear stress, (f) critical Shields parameter, and (g) dimensionless bedload transport capacity. Colorbars indicate the scale of computed values, respectively. White areas correspond to (a) the exposed bedrock surface and (g) no bedload transport.
roughness gradually decreases as bedrock exposure increases for all Runs of 2-B2. However, the
bedform roughness of Runs 2-B2, 2-B2-b, and 2-B2-c increases to 5 mm at the equilibrium state,
whereas the bedform roughness of Run 2-B2-a increases at the early stage and decreases back to
0. The changes in total roughness height indicate that the channel initially requires a thicker
sediment cover layer than the threshold value to develop bars and maintain alluvial strips, or
sediment washing out occurs.

5 Conclusions

We used a two-dimensional morphodynamic model to conduct a series of numerical
experiments in the mixed bedrock-alluvial channel. This study explores the interaction between
bedrock alluviation and morphological evolution. Simulations with varying sediment supply are
conducted in different slope and antecedent channel conditions. The model replicated
observations from a mixed bedrock-alluvial experiment (Chatanantavet & Parker, 2008),
including a) the relationship between channel slope and gradual vs. runaway alluviation, b) the
slope-dependent sediment supply threshold for development of persistent alluvial cover, c) the
relationship between decreasing bedrock exposure and increasing sediment supply, and d) the
development of constant alluvial cover thickness regardless of initial sediment thickness, 
provided the initial thickness exceeds a minimum value necessary to maintain bedform dimensions.

The model results provide physical insight on the mechanisms responsible for these phenomena. Transcritical flow plays an important role in initiating sediment deposition over initially bare bedrock, and the development of transcritical zones in low-slope simulations but not high-slope simulations may explain the apparent slope dependence of runaway alluviation. Persistent sediment cover in high-slope channels is possible when rough alluvial surfaces balance the extent of lower-roughness bedrock surfaces, and steeper channels require higher sediment supply to exceed an apparent threshold where that balance can occur.

Open Research

The output data from numerical simulations used in this analysis can be found at https://github.com/rcemorpho/morph2d.

References


z.search.serialssolution.com?ctx_ver=Z39.88-2004&ctx_enc=info:ofi/enc:UTF-


