Patterns of Alluviation in Mixed Bedrock-Alluvial Channels: 1. Numerical Model

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Abstract

Mixed bedrock-alluvial rivers can exhibit partial alluvial cover, which may play an important role in controlling bedrock erosion rates and landscape evolution. However, numerical morphodynamic models generally are unable to predict the pattern of alluviation in these channels. Hence we present a new two-dimensional depth-averaged morphodynamic model that can be applied to both fully alluvial and mixed bedrock-alluvial channels, and we use the model to gain insight into the mechanisms responsible for the development of sediment patches and patterns of bedrock alluviation. The model computes hydrodynamics, sediment transport, and bed evolution, using a roughness partitioning that accounts for differential roughness of sediment and bedrock, roughness due to sediment transport, and form drag. The model successfully replicates observations of bar development and migration from a fully alluvial flume experiment, and it models persistent sediment patches observed in a mixed bedrock-alluvial flume experiment. Numerical experiments in which the form drag, sediment transport roughness, and ripple factor correction were neglected did not successfully reproduce the observed persistent sediment cover in the mixed bedrock-alluvial case, suggesting that accounting for these different roughness components is critical to successfully model sediment dynamics in bedrock channels.
Patterns of Alluviation in Mixed Bedrock-Alluvial Channels: 1. Numerical Model

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Key Points:

- A new model is developed to predict the dynamics of alluvial patterns in mixed bedrock-alluvial channels.

- The model accurately predicts bedform evolution in an alluvial channel and alluvial patch formation in a mixed bedrock-alluvial channel.

- Accounting for differential roughness and shear stress correction are important for model prediction of the degree of exposed bedrock.
Abstract

Mixed bedrock-alluvial rivers can exhibit partial alluvial cover, which may play an important role in controlling bedrock erosion rates and landscape evolution. However, numerical morphodynamic models generally are unable to predict the pattern of alluviation in these channels. Hence we present a new two-dimensional depth-averaged morphodynamic model that can be applied to both fully alluvial and mixed bedrock-alluvial channels, and we use the model to gain insight into the mechanisms responsible for the development of sediment patches and patterns of bedrock alluviation. The model computes hydrodynamics, sediment transport, and bed evolution, using a roughness partitioning that accounts for differential roughness of sediment and bedrock, roughness due to sediment transport, and form drag. The model successfully replicates observations of bar development and migration from a fully alluvial flume experiment, and it models persistent sediment patches observed in a mixed bedrock-alluvial flume experiment. Numerical experiments in which the form drag, sediment transport roughness, and ripple factor correction were neglected did not successfully reproduce the observed persistent sediment cover in the mixed bedrock-alluvial case, suggesting that accounting for these different roughness components is critical to successfully model sediment dynamics in bedrock channels.

Plain Language Summary

The spatial distribution of sediment patches in bedrock rivers can play an important role in the local hydraulic conditions as well as long-term channel and landscape evolution, because sediment in these channels is an important mechanism of bedrock erosion. Flume experiments and field studies have shown that the amount of sediment cover and the pattern of sediment patches in these streams can sometimes reach a sort of equilibrium, but numerical models have been largely unable to replicate those observations. Here, we present a new numerical model for
mixed bedrock-alluvial rivers that explicitly accounts for different roughness feedbacks between
the bedrock, sediment, and flow field. We show that the model can successfully simulate
observations from flume experiments conducted with and without exposed bedrock, and it is able
to simulate persistent partial sediment cover. These conditions develop because of the
complicated roughness feedbacks in these channels, which we demonstrate through simulations
where different roughness components are “turned off,” which result in completely exposed
bedrock without sediment cover.

1 Introduction

River channels can be classified as either alluvial, bedrock, or mixed bedrock-alluvial
are entirely covered by sediment of sufficient thickness so that the underlying bedrock is not
exposed (Tinkler & Wohl, 1998; Whipple, 2004). In contrast, bedrock channels are characterized
by frequently exposed bedrock and a lack of continuous alluvial cover in the channel bed and
banks. Mixed bedrock-alluvial channels tend to have sediment supply that is less than their
sediment transport capacity, and they feature the exposed bedrock interspersed with patches of
alluvial cover, potentially taking the form of alternate bars (e.g., Chatanantavet & Parker, 2008)
or point bars at meander bends (P. A. Nelson et al., 2014; Nittrouer et al., 2011).

The pattern of alluvial cover in mixed bedrock-alluvial channels likely plays a role in the
morphological evolution of those channels, and consequently on landscape evolution (Gasparini
et al., 2007; Hodge & Hoey, 2012; Howard et al., 1994; Seidl & Dietrich, 1992; Tinkler & Wohl,
1998; Whipple & Tucker, 2002; Wohl, 1993). Bedrock channel erosion sets the lower boundary
condition for landscape evolution and bedrock channels convey climatic and tectonic
perturbation through the landscape (Whipple, 2001; Whipple & Tucker, 1999). Alluvial cover is

an important component of mechanistic models of bedrock erosion; for example, the saltation-abrasion model incorporates the erosional mechanism of saltating bedload particles impacting and eroding bedrock (Demeter et al., 2005; Hartshorn et al., 2002; Sklar & Dietrich, 1998, 2001, 2004; Zhang et al., 2015). Competition between the tools and cover effects controls the spatial distribution of the bedrock channel erosion that results in lateral and vertical channel erosion and meandering (Finnegan et al., 2007; Lamb et al., 2015; Turowski et al., 2007; Turowski, Hovius, Meng-Long, et al., 2008; Turowski, Hovius, Wilson, et al., 2008).

Observations from flume experiments documenting the development of alluvial cover patterns on bedrock beds have shown that alluvial cover in these channels depends on channel slope, the initial thickness of alluvial sediment, sediment supply, bedrock roughness, and channel topography. Chatanantavet and Parker (2008) conducted a series of experiments in a straight flume where they varied the rate of sediment supply, the initial cover of sediment on the bed, the slope, and the grain size. Their experiments illuminated several exciting phenomena in mixed bedrock-alluvial channels. First, the pattern and trajectory of the alluvial cover appear to be slope-dependent, wherein at low slopes ($S = 0.0115$) the exposed bed fraction linearly decreases with increasing sediment supply, akin to the linear relationship hypothesized by Sklar and Dietrich (2004). However, at higher slopes ($S = 0.02$), experiments starting with a bare bed did not develop persistent alluvial cover at any supplied rate until that rate exceeded the overall transport capacity of the channel. At that point, “runaway alluviation” occurred, and the entire channel became covered in sediment. Second, the thickness of the initial alluvial cover affected the dynamics of alluviation on the bed, wherein at low initial alluvial thickness the bed was stripped clean, whereas thicker initial sediment covers eventually reached a non-zero fraction of
bedrock exposure. Third, some of their experiments developed continuous strips of sediment moving from one side of the channel to the other, akin to alternate bars.

Other experiments have suggested that bedrock topography and the relative hydraulic roughness of bedrock relative to grain size plays a role in alluvial dynamics of mixed bedrock-alluvial channels. Mishra and Inoue (2020) performed flume experiments with varying bedrock roughness. The observed the extent of alluvial cover increases with increasing sediment supply when the hydraulic roughness of the bedrock bed is larger than that of the alluvial surface. However, a sudden transition from bare bedrock bed to full alluviation was observed as sediment supply momentarily exceeded the channel's transport capacity when the ratio of hydraulic roughness height of bedrock to grain size ($k_{sb}/d$) is 1.9 or lower. They also proposed an approximation of dimensionless critical shear stress for incipient particle motion over bedrock beds as a function of relative roughness height. Hodge and Hoey (2016a, 2016b) performed experiments in a 3D printed scale model of a jointed limestone bedrock river in which the patterns of sediment deposits at different sediment supply rates were documented. Their experiments pointed out the importance of bedrock topography on depositional patterns, as patches of sediment tended to form in the lowest portions of the bed, and at higher discharge and sediment supply, the bed topography played a less important role than sediment-sediment and sediment-flow interactions in stabilizing patches of alluvium.

Numerical models of sediment transport and bed evolution in mixed bedrock-alluvial channels have struggled to capture the dynamics of alluvial cover observed in experiments or in the field. Morphodynamic models simulate river channel evolution by iteratively using hydraulic flow field calculations to estimate sediment transport rates, which are then used in the conservation of sediment mass (i.e., the Exner equation) to calculate bed erosion and deposition
patterns. These types of models have been used for decades to understand the dynamics of alluvial rivers, such as the development and migration of alternate bars (e.g., Bernini et al., 2006; Defina, 2003; Qian et al., 2017), sediment sorting (e.g., P. A. Nelson et al., 2015a, 2015b), braiding (e.g., Murray & Paola, 1997; Schuurman et al., 2013), meandering (e.g., Smith & Mclean, 1984; J. M. Nelson et al., 2003), and armoring (e.g., Parker & Klingeman, 1982; Parker & Toro-Escobar, 2002). However, alluvial morphodynamic models generally assume that the sediment supply equals or exceeds the sediment transport capacity, which is not the case for mixed bedrock-alluvial rivers.

This assumption has only recently begun to be relaxed in attempts to use morphodynamic models to better understand mixed bedrock-alluvial rivers (e.g., P. A. Nelson & Seminara, 2012). Using a cellular automaton model governed by probabilities of individual grain movement, Hodge and Hoey (2012) studied the relationship between the fraction of bedrock exposure and the ratio of sediment supply to capacity on noneroding bedrock beds. Zhang et al. (2015) expressed bedrock cover fraction as a ratio of vertical length scale between alluvial thickness and macro-roughness of bedrock topography representing the statistical characteristics of bedrock surface fluctuations. This MRSAA (Macro-Roughness-based Saltation-Abrasion-Alluviation) model was later implemented to investigate the knickpoint migration (Zhang et al., 2018, 2019) and formation of the cyclic steps (Izumi et al., 2017) to spatiotemporal variation of sediment supply. Inoue et al. (2016) provided an early attempt to investigate alternate bar formation and bedrock incision in mixed bedrock-alluvial channels in response to the different ratio of sediment supply to channel’s transport capacity.

Larger-than-grain-scale bedrock topography, sometimes called “macrotopography”, has recently been incorporated into one-dimensional morphodynamic models (Zhang et al., 2015).
The model solutions for alluvial response to sediment supply show that stripping of antecedent sediment from upstream to downstream occurs with the termination of sediment supply, and the development of an alluvial layer with a thickness corresponding to the ratio of sediment supply to transport capacity \( (q_s/q_c) \), which propagates downstream over bare bedrock when the sediment supply is increased. Application of nonlinear wave speed to the alluvial layer predicted that alluvium over a bedrock surface with a small cover fraction migrated downstream much faster than those consisting of a higher fraction cover.

Despite these recent advances in modeling the morphodynamics of mixed bedrock-alluvial rivers, there remain uncertainties about how differences in sediment and bedrock roughness influence in mixed bedrock-alluvial rivers, and how hydraulics and macroroughness features may interact to control alluvial patterns. Existing models are not able to resolve these issues because either they are one-dimensional and unable to predict lateral variation in alluvial cover, or they do not fully account for roughness effects of local sediment cover or transport. For example, current models of mixed bedrock-alluvial morphodynamics have not been able to predict persistent longitudinal alluvial strips, or to fully replicate the relationship between overall sediment cover and the sediment supply to capacity ratio as reported in Chatanantavet and Parker (2008).

This study intends to resolve the limitations models encounter when attempting to simulate sediment dynamics in a mixed bedrock-alluvial channel by developing a new model that uses: (1) a two-dimensional approach to account for the influence of potential bedforms (e.g., alternate bars, antidunes, and dunes) in a straight channel; (2) the effect of flow resistance partitioned into skin friction due to stationary particles and bedrock surface roughness, form drag associated with dimensions of alluvial cover and irregular bedrock topography, and bedload
roughness produced by saltating sediment; (3) application of a ripple factor to take into account for an intermediate state between the planar bed and small-scale bedforms.

We hypothesize that accounting for differential bedrock and sediment roughness, roughness from sediment transport, form drag, and a near-bed shear stress correction are necessary to capture sediment dynamics in mixed bedrock-alluvial channels. We use two experiments to demonstrate that our model can reproduce observations made in fully alluvial and mixed bedrock-alluvial channels, and we explore the importance of the different roughness terms we include by repeating the simulation with those terms neglected. We simulate an experiment of Lanzoni (2000) for alternate bar formation in an alluvial channel and an experiment of Chatanantavet and Parker (2008) for gravel patch development in a mixed bedrock-alluvial channel to assess model performance. Our model successfully replicates sediment dynamics in both experiments. The numerical experiments that neglect form drag and transport components of the roughness do not reproduce observations from the Chatanantavet and Parker (2008) experiment, indicating that those components play a critical role in mixed bedrock-alluvial sediment dynamics.

2 Morphodynamic model

We have developed a new morphodynamic model that simulates sediment transport and deposition in mixed bedrock-alluvial channels. This model consists of three components: a hydrodynamic model describing the depth-averaged flow field, a sediment transport model describing bedload sediment transport rate, and a bed evolution model updating bed elevation and the areal fraction of bedrock cover. Unlike previous models of mixed bedrock-alluvial morphodynamics, this new model accounts for form drag and sediment transport roughness, in addition to surface (grain and bedrock) roughness and a roughness-dependent critical Shields
stress. In addition, the total shear stress is corrected with a ripple factor in the sediment transport model, as the bedform drag does not contribute to the bedload transport.

2.1 Flow model

The governing equations for calculating flow depth and velocity are composed of the depth-averaged form of mass continuity and momentum balance in a 2D Cartesian coordinate system:

\[
\frac{\partial Q}{\partial t} + \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + S_b + S_f
\]  

where \( t \) is time, \( x \) and \( y \) are Cartesian coordinates, \( Q \) are the conservative variables, \( F_x \) and \( F_y \) are the convective fluxes, \( D_x \) and \( D_y \) are the diffusive fluxes, \( S_b \) are the bed slope terms, and \( S_f \) are the friction slope.
where \( h \) is the flow depth, \( u \) and \( v \) are the velocities in \( x \)- and \( y \)-directions, respectively, \( g \) is the gravitational acceleration, \( \nu \) and \( \nu_t \) are the kinematic viscosity of water and the turbulent eddy viscosity, respectively, \( \eta \) is the water surface elevation, \( z_b \) is the bed elevation, and \( \rho \) is the density of water. The bed shear stresses \( \tau_{bx} \) and \( \tau_{by} \) in \( x \)- and \( y \)-direction are given by

\[
(\tau_{bx}, \tau_{by}) = \rho C_f \sqrt{u^2 + v^2} (u, v)
\]

where \( C_f \) is a friction coefficient estimated using the law of the wall as a function of the flow depth \( h \) and total roughness height \( k_0 \):

\[
C_f = \left[ \frac{1}{\kappa} \ln \left( \frac{11h}{k_0} \right) \right]^2.
\]
2.2 Depth-averaged mixing-length model

The calculation of the turbulent viscosity term is based on a mixing-length model with depth-averaged terms developed by Stansby (2003):

\[
\nu_t = \sqrt{l_h^4 \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 \right] + \left( \gamma u_* h \right)^2 }
\]  

(5)

where \( l_h \) is a horizontal mixing length scale (\( l_h \approx 0.267\kappa h \)), \( \kappa \) is the von Karman constant (\( \kappa \approx 0.408 \)), and \( \gamma \) is a constant that accounts for vertical mixing (\( \gamma \approx 0.067 \)). The local shear velocity, \( u_* \), is defined as;

\[
u u_* = \tau_b / \rho
\]

(6)

where \( \tau_b = \sqrt{\tau_{bx}^2 + \tau_{by}^2} \) is the local bed shear stress vector, with components \( \tau_{bx} \) and \( \tau_{by} \) in the \( x \)- and \( y \)-directions, respectively.

2.3 Bed roughness

The total roughness height \( k_0 \) is partitioned into three fractional roughness components, including skin friction \( k_s \), form drag \( k_f \), and bedload transport \( k_i \):

\[
k_0 = k_s + k_f + k_i.
\]

(7)

The skin friction is induced by the viscous shear stress and pressure force acting on the individual grains on the bed, and it relates to the size of the bed material. The local skin friction roughness height varies with surface particle size in the completely alluvial channel and the degree of irregularity of the bed surface in the bedrock channel. The calculation of skin friction
in the mixed bedrock-alluvial channel is based on the assumption that the skin friction is linearly associated with the changes in the fraction of bedrock covered by alluvium:

\[ k_s = P_c k_{sa} + (1 - P_c) k_{sb} \]  

(8)

where \( k_{sa} \) and \( k_{sb} \) are the hydraulic roughness height of the alluvial bed and bedrock bed, respectively. The local areal fraction of alluvial cover \( P_c = \frac{\eta_a}{C_m} \leq 1 \), in which \( C_m = \pi d / 6 \) is the maximum volume of a monolayer of spherical sediment grains of constant diameter \( d \) uniformly distributed over the bed surface (P. A. Nelson & Seminara, 2012), and \( \eta_a \) is the thickness of the alluvial layer.

The form drag component of roughness results from the pressure force acting over entire bedforms and is not responsible for the bedload motion of sediment particles (Maddux, McLean, et al., 2003; Maddux, Nelson, et al., 2003). We calculate the form drag component of roughness as a function of bed morphology through the empirical relation of Grant and Madsen (1982):

\[ k_f = 30 a_r \frac{\eta_f^2}{\lambda_f} \]  

(9)

where \( \eta_f \) and \( \lambda_f \) are bed form height and wavelength, and \( a_r \) is a coefficient in the range from 0.3 to 3. Grant and Madsen (1982) suggested \( a_r = 0.923 \), which we use here.

For the sediment transport component of roughness, Wiberg and Rubin (1989) proposed for the flat bed condition:

\[ k_i = 30a_{ws} d \frac{a_i T_s}{1 + a_i T_s} \]  

(10)
where $T = \tau^*/\tau^*_c$ is the transport stage, $\tau^*$ is the dimensionless shear stress, $\tau^*_c$ is the critical dimensionless Shields stress, $a_1 = 0.68$, $a_2 = 0.0204 (\ln 100d)^{1.5} + 0.0220 (\ln 100d) + 0.0709$, and $\alpha_{ws} = 0.056$. Here, the effect of local variation of bed topography is applied to the critical bed shear stress to account for the gravity effect, as described in Section 2.5.

2.4 Bed deformation model

The local volumetric concentration of sediment per unit area is calculated using the sediment conservation model for mixed bedrock-alluvial channel beds proposed by Luu, Egashira, and Takebayashi (2004):

$$\frac{\partial V_{ba}}{\partial t} + \frac{\partial q_{bx}}{\partial x} + \frac{\partial q_{by}}{\partial y} = 0$$

(11)

where $V_{ba}$ is the total volume of sediment per unit area and $q_{bx}$ and $q_{by}$ are vectors of the bedload transport rate per unit width in the $x$- and $y$-directions, respectively. The thickness of the alluvial layer $\eta_a$ and the volume of sediment in the bedload layer $V_{bc}$ are separately updated considering the saturation volume of the bedload layer $V_{bc}$. This saturation volume is a threshold value that determines whether the particles deposit on the bed or rapidly saltate over the surface without resting on the bed. When $V_{ba}$ exceeds $V_{bc}$, a volume of sediment equal to the difference between $V_{ba}$ and $V_{bc}$ deposits on the bed as an alluvial layer. When $V_{ba}$ is less than $V_{bc}$, the particles pass over the bedrock surface as throughput load without deposition:

$$\eta_a = \begin{cases} \frac{V_{ba} - V_{bc}}{1 - \lambda} & \text{for } V_{bc} \leq V_{ba} \\ 0 & \text{for } 0 \leq V_{ba} < V_{bc} \end{cases}$$

(12)
where $\lambda$ represents the porosity of the bed and $V_{bc}$ is the saturation volume of the bedload layer per unit area. When the bedrock is wholly exposed ($\eta = 0$), the volume of the throughput bedload layer is lower than the saturation volume. When the bed is partially or fully covered with sediment ($\eta > 0$), $V_b$ equates to $V_{bc}$ because sediment particles exchange occurs between alluvial and bedload layers:

$$V_b = \begin{cases} V_{bc} & \text{for } V_{bc} \leq V_{ba} \\ V_{ba} & \text{for } 0 \leq V_{ba} < V_{bc} \end{cases} \quad (13)$$

The saturation volume of bedload per unit area $V_{bc}$ is defined by

$$V_{bc} = \frac{q_{bc}}{u_s}, \quad (14)$$

where $q_{bc}$ is the bedload transport capacity per unit width and $u_s$ is the saltation velocity, calculated here with the empirical excess shear stress relation presented in (Sklar & Dietrich, 2004):

$$\frac{u_s}{\sqrt{R_b gd}} = 1.56 \left( \frac{\mu \tau^{*}}{\tau_c} - 1 \right)^{0.56} \quad (15)$$

where $R_b = (\rho_s - \rho) / \rho$ denotes submerged specific gravity of sediment and $\rho_s$ is the density of the sediment. The bedload transport capacity per unit width $q_{bc}$ is estimated using the relation based on Ashida and Michiue (1972):

$$\frac{q_{bc}}{\sqrt{R_b gd^2}} = 17 \left( \sqrt{\mu \tau^{*}} - \sqrt{\tau_c^*} \right) \left( \mu \tau^{*} - \tau_c^* \right) \quad (16)$$

where dimensionless shear stress $\tau^*$ is defined as
and \( \mu \leq 1 \) is the ripple factor, the ratio of the grain roughness to bed roughness (Ribberink, 1987) (discussed further in Section 2.5).

The volumetric sediment transport rate per unit width in the \( x \)- and \( y \)-directions is denoted

\[
\left( q_{bx}, q_{by} \right) = q_b \left( \cos \alpha, \sin \alpha \right)
\]

(18)

where \( \alpha \) is the angle of bedload transport and the sediment transport intensity, \( q_b \), depends on the ratio of the volume of sediment and its saturation value. The bedload transport rate in bedrock with a sufficient local volume of sediment equals the sediment transport capacity. However, in bedrock rivers without sediment cover, the bedload transport rate is less than the bedload transport capacity:

\[
q_b = \begin{cases} 
V_b & \text{for } 0 \leq V_b < V_{bc} \\
q_{bc} & \text{for } V_{bc} \leq V_b 
\end{cases}
\]

(19)

When considering the effect of gravity acting on particles for gradually varying bed elevation, the sediment transport direction deviates from the direction of the boundary shear stress. Here we adopt the well-known relationship (Struiksma, 1985):

\[
\tan \alpha = \frac{\sin \delta - \frac{1}{f(t^*)} \frac{\partial z_b}{\partial y}}{\cos \delta - \frac{1}{f(t^*)} \frac{\partial z_b}{\partial x}}
\]

(20)
where $\delta$ is the near-bed flow direction estimated to account for the influence of spiral water motion induced by bed topography as

$$\delta = \tan^{-1}\left(\frac{v}{u}\right) - \tan^{-1}\left(A\frac{h}{r_s}\right), \quad (21)$$

where the local radius of depth-averaged stream curvature is

$$r_s = \frac{U^3}{u^2 \frac{\partial v}{\partial x} + u v \left(\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x}\right) - v^2 \frac{\partial u}{\partial y}}, \quad (22)$$

where $U = \sqrt{u^2 + v^2}$ is the local flow velocity and the coefficient weighting the intensity of helical flow is

$$A = \frac{2}{\kappa^2} \left(1 - \frac{\sqrt{C_f}}{\kappa}\right), \quad (23)$$

and $f(\tau^*)$ is a function weighting the influence of the bed slope, following the form proposed by Talmon, Struiksma, and Van Mierlo et al. (1995):

$$f(\tau^*) = 9 \left(\frac{d}{h}\right)^{0.3} \sqrt{\tau^*}. \quad (24)$$

### 2.5 Shear stress correction

The bedload sediment transport rate is expressed as a function of shear stress acting over the channel bed. The presence of bedforms leads to a partial reduction of the total shear stress related to the form drag, and the remainder is available for sediment transport. Traditionally this has been implemented in models by multiplying the dimensionless shear stress by a so-called
“ripple factor.” Several models for estimating the ripple factor have been proposed as a function of the skin friction relative to the total friction (Ribberink, 1987; Vermeer, 1986):

\[
\mu = \left( \frac{C_{fs}}{C_f} \right)^{n/2}
\]  

(25)

where \( C_{fs} \) is the friction coefficient accounting for skin friction obtained by substituting \( k_0 \) with \( k_e + k_i \) in equation (4), and the exponent \( n \) is defined by

\[
n = 1.8 + 0.27 \log q^*,
\]

(26)

where \( q^* = q_{bc} \sqrt{R_g s d^3} \) is the dimensionless bedload transport rate. This relationship is valid for \( 0.001 \leq q^* \leq 1 \) (Vermeer, 1986).

2.6 Critical dimensionless Shields stress

Flume experiments conducted in bedrock channels (Inoue et al., 2014; Mishra & Inoue, 2020) have related the channel roughness and dimensionless critical shear stress of sediment movement. A power approximation proposed by Mishra and Inoue (2020) is applied to this model to take into account the effect of total hydraulic roughness on critical shear stress:

\[
\tau_{c0}^* = \tau_{c\alpha}^* \left( k_s/d \right)^{0.6}
\]

(27)

where \( \tau_{c\alpha}^* \) is the critical Shields stress back-calculated from sediment transport capacity measured from the experiment in the flat channel. In addition to the bed roughness, the local bed slope effect on the initiation of particle motion can be added because the local bed slope provides a gravitational component of the force exerted on the particle (Duan & Julien, 2005; Soulsby, 1997):
\[
\frac{\tau^*}{\tau_{c_0}^*} = \frac{\sin(\phi + \beta_s)}{\sin \phi} \frac{\cos \beta_n}{\sqrt{1 - \tan^2 \beta_n / \tan^2 \phi}}
\]  
(28)

where \( \phi \) is the grain angle of repose and \( \beta_s \) and \( \beta_n \) are the slope in the streamwise and cross-stream direction of the sediment transport, respectively. A simple approach was proposed by Wiberg and Smith (1987) for the treatment of heterogeneous bed conditions through the geometric relation of friction angle as a function of particle size to the roughness length scale of the bed:

\[
\phi = \cos^{-1}\left(\frac{d/k_s - 0.02}{d/k_s + 1}\right)
\]  
(29)

The bed slopes in the streamwise and cross-stream direction of sediment transport are

\[
\beta_s = \tan^{-1}\left(\frac{\partial z}{\partial x} \cos \alpha + \frac{\partial z}{\partial y} \sin \alpha\right), \quad \beta_n = \tan^{-1}\left(\frac{\partial z}{\partial y} \cos \alpha - \frac{\partial z}{\partial x} \sin \alpha\right).
\]  
(30)

2.7 Solution procedure

The model uses a decoupled approximation of the morphodynamic system by assuming that the response time of the bed evolution is relatively long compared to the time scale of the hydraulic processes (De Vries, 1965; Defina, 2003; J. M. Nelson & Smith, 1989). Therefore the hydrodynamic solver is followed by the calculation of the modified Exner equation.

First, local water depth and flow velocity are calculated using the hydrodynamic model (equation (1) - (2)). The friction term is estimated by taking into account the roughness of the bedrock \( k_{sb} \) and alluvial surfaces \( k_{sa} \), topographic variability \( k_f \), and the effect of saltating grains during sediment transport \( k_r \). Flow-conservative variables (\( h \), \( hu \), and \( hv \)) at each cell
interface are calculated using the total variation diminishing (TVD) scheme (Toro, 2009) for
cell-centered advection terms ($F_x$ and $F_y$).

Second, the local sediment transport rate $q_b$ (equation (19)) is estimated considering the
local circumstance of sediment deposition, which is determined by the fraction of bedrock cover
$P_c$ (equation (14) - (16)). The depth-averaged flow variables ($h$, $u$, $v$, and $k_s$) determine the
threshold of incipient sediment motion ($\tau^*$ and $\tau^*_c$), then the local bedload transport rate $q_b$,
grain saltation velocity $u_s$, and saturation volume of the bedload layer $V_{bc}$ are calculated.

Third, the sediment flux divergence updates the total sediment volume per unit area
(equation (11)). Finally, the new alluvial layer thickness $\eta_a$ and the volume of bedload transport
per unit area $V_b$ are determined by whether the bedrock is covered by alluvium or exposed
(equation (12) - (13)).

3 Model validation

We set up two simulations to demonstrate the model’s ability to replicate observations in
both alluvial and mixed bedrock-alluvial conditions. These benchmark laboratory flume
experiments documented: (1) the development of alternate bars in fully alluvial channels
(Lanzoni, 2000); and (2) flow and sediment transport in mixed bedrock-alluvial channels
(Chatanantavet & Parker, 2008). Table 1 summarizes the conditions for each experiment.

3.1 Alternate bar formation in an alluvial channel

To demonstrate alternate bars in alluvial conditions, we simulate Run P1505 described in
Lanzoni (2000). In the experiment, a straight, flow- and sediment-recirculating flume 1.5 m
wide, 1 m deep with a 55 m long test section was supplied with a steady water discharge of 30 l/s. The bed was composed of a uniform sand size of 0.48 mm and initially screeded flat. At the upstream end of the channel, a 16 m long, 5 m wide stilling basin was installed to ensure smooth and regular water and sediment discharge into the experimental channel. This experiment stopped as it reached equilibrium after 28 hours, when the water surface slope matched the bed slope. A summary of reach-averaged measurements is reported in Table 1. Alternate bars were formed with an average height of 7 cm, wavelength of 10 m, and celerity of 2.80 m/h.

Our simulation of Run P1505 is conducted with some modification of experimental conditions. First, a longer channel of 120 m was used to ensure the bar reached the equilibrium state before migrating out of the domain (Defina, 2003). Second, a small topographic bump 0.6 m long, 0.75 m wide, and 5 mm high was introduced near the upper boundary on the left side (looking downstream) to create an initial disturbance to induce free bar development because the arbitrarily distributed source of disturbances from the physical experiment was unknown. Numerical models generally require perturbations such as a topographic bump (Defina, 2003; Wu et al., 2011) or a bend upstream of the straight channel (Mendoza et al., 2017) to develop

<table>
<thead>
<tr>
<th>Table 1. Summary of Experimental Conditions.</th>
<th>m</th>
<th>m</th>
<th>%</th>
<th>l/s</th>
<th>g/s</th>
<th>mm</th>
<th>cm</th>
<th>m/s</th>
<th>hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P1505b</td>
<td>1.5</td>
<td>55</td>
<td>0.452</td>
<td>30</td>
<td>28</td>
<td>0.48</td>
<td>4.4</td>
<td>0.45</td>
<td>28</td>
</tr>
<tr>
<td>2-B2c</td>
<td>0.9</td>
<td>13</td>
<td>2</td>
<td>55</td>
<td>110</td>
<td>7</td>
<td>5.5</td>
<td>1.02</td>
<td>5</td>
</tr>
</tbody>
</table>

*B* is the channel width, *L* is the channel length, *S* is the channel slope, *Q_w* is the water discharge, *Q_c* is the sediment transport capacity, *D* is the grain size, *H* is the averaged flow depth, *U* is the averaged flow velocity, and *t* is the duration of the experiment.

b Run P1505 was performed by Lanzoni (2000).

c Run 2-B2 (*q_*/*q_c* = 0.56) was performed by Chatanantavet & Parker (2008).
bars in uniform flow over flat-bed conditions. Third, the constant and uniform water discharge and sediment feed rate were imposed at the inlet.

3.2 Patterns of bedrock alluviation with limited sediment supply

The second experiment Run 2-B2 ($q_s/q_c = 0.56$) (Chatanantavet & Parker, 2008), was carried out on a non-erodible bedrock surface in a 13 m long and 0.9 m wide straight, rectangular flume channel with a slope of 0.02. The bedrock bed was randomly abraded with a longitudinally averaged standard deviation of 2.4 mm, and the distance between the lowest and highest points of the profile is approximately 1 cm (Figure 1b and 3b in Chatanantavet & Parker, 2008).

The averaged values of initial experimental conditions are presented in Table 1. The representative hydraulic roughness of the bedrock surface was back-calculated from the Manning-Strickler relation under the flow conditions measured from the experimental results conducted in the channel slope of 0.0115. Initially, the bed was covered entirely with a 2 cm thick layer of uniform 7 mm sediment. The steady water discharge was 55 l/s, the constant sediment supply rate was 62 g/s, and the sediment transport capacity was estimated at 110 g/s for a wholly covered bed. The experiment stopped when the fraction of bedrock cover $P_c$ reached a steady state whose value was approximately 0.59.

The numerical model simulation of this experiment uses a 0.9 m wide, 20 m long bedrock bed with randomly generated topographic perturbations with a standard deviation of 2.2 mm and a peak-to-peak distance of bed elevation of 0.9 cm (Figure 1). The roughness height due to skin friction (grain equivalent roughness height) used in the numerical simulation was back calculated from equation (4) using experimentally measured average flow conditions for each case of bare bedrock bed $k_{sb}$ and alluvial bed $k_{sa}$. In the channel fully covered with sediment $H = 0.06$ m
and $U = 1.02$ m/s, and in case of flow over bare bedrock bed $H = 0.05$ m and $U = 1.22$ m/s, hence $k_{sa} = 7$ mm and $k_{sb} = 3$ mm. The sediment supply to transport capacity ratio $q_s/q_c$ is 0.6, which means the steady sediment supply rate is 66 g/s assuming the sediment transport capacity of the alluvial bed is 110 g/s. A small perturbation was given by changing sediment distribution patterns at the upstream end of the channel to prevent sediment and bedform from washing out from the upstream end of the channel (e.g., Figure 14 in Inoue et al., 2016).

3.3 Simulations investigating impacts of roughness components on mixed bedrock-alluvial sediment dynamics

To investigate the importance of different roughness components on the model’s performance of predicting alluvial patterns, we conducted numerical simulations without bedform roughness or ripple factor under the same conditions of Run 2-B2. The form drag effect on the flow is removed by setting the bedform roughness to zero; hence the ripple factor is no longer influential. The effect of bed-form solely on sediment transport is removed by setting the ripple factor to unity.

We also conducted a simulation where the sediment transport roughness was set to zero, to explore how that component influences morphodynamic predictions in mixed bedrock-alluvial channels. For this simulation, the total roughness consisted of bedform and grain roughness, the latter of which was increased relative to the base scenario so that overall flow depths and velocities still matched experimental observations.

**Figure 1.** Plan view of bedrock topography. Colorbar scale indicates the detrended bed elevation.
4 Results

4.1 Simulation of alluvial alternate bars (Run P1505)

Figure 2 shows the plan views of the evolution of detrended bed surface elevation in Run P1505. The initial upstream bump \((t = 0 \text{ h})\) deflects flow and sediment transport. The sediment eroded from the bump forms the first bar and triggers the development of smaller bars downstream \((t = 2 - 4 \text{ h})\). The bars get larger and taller as they migrate downstream. When they reach equilibrium, their downstream migration and growth rate vary slowly and become stable. The initially flat alluvial bed, during the bedform formation and migration process, develops into a clear pattern of alternate bars. The initial disturbance gradually spreads out by decreasing height and stretching in the flow direction, prompting new bars to develop downstream (e.g., Figure 3 in Defina, 2003).

The longitudinal bed profiles and the bed elevation difference between the right- and left-side walls from the numerical results (Figure 3) can be compared to the measured data at the equilibrium state from Lanzoni (2000, Figure 1g) (Figure 4). Bar amplitudes are calculated as the difference between the right and left side of the bed elevation, measured 20 cm from each side wall (Figure 3a, b). The amplitude and wavelength are calculated using half the vertical and twice the horizontal peak-to-peak distances of the detrended profile. The celerity of migrating

![Figure 2](image-url)

**Figure 2.** Detrended plan view of modeled bed evolution for the alluvial bar flume experiment (P1505) with the same longitudinal and transverse scale. Colorbar shows detrended bed elevation at the same scale for all plots.
bars is calculated by tracking topographic peaks over time. The longitudinal bed profiles from numerical experiments exhibit highly ordered wave patterns. The axis bed profiles in the numerical simulation and experiment indicate the transversally maximum value of bed elevation.

**Figure 3.** Modeled longitudinal bed profiles along the left- and right-side wall and axis at 6 h (a) and 12 h (b) and the difference between right- and left-side bed elevation at 6 h (c) and 12h (d).

**Figure 4.** Measured (a) longitudinal bed profiles along the left- and right-side wall and (b) the difference between right- and left-side bed elevation at equilibrium (Figure 1g, Lanzoni, 2000).
Figure 5. Computed time evolution of bar characteristics; celerity, wavelength, and amplitude (from top to bottom) on the left (a, c, and e) and right (b, d, and f) side of the channel.

Table 2. Comparison of Bar Characteristics from Measured Data and Computed Results.

<table>
<thead>
<tr>
<th>Run</th>
<th>$C_b$</th>
<th>$\lambda_b$</th>
<th>$H_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1505</td>
<td>2.8</td>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>Computed</td>
<td>3.2±0.5</td>
<td>14±2</td>
<td>7±1</td>
</tr>
</tbody>
</table>

As the calculated time changes in bar celerity (Figure 5a, b), wavelength (Figure 5c, d), and amplitude (Figure 5e, f) are small, and we consider the bed condition at near equilibrium.

The mean characteristics of the bar (bar height, wavelength, and celerity) near equilibrium are compared and reported in Table 2. The computed bar wavelength, height, and celerity are approximately 14 m, 7 cm, and 3.2 m/s, respectively. The computed bar height and celerity show reasonably good agreement with observations of Lanzoni (2000), but the wavelength is slightly
overestimated. However, the computed alternate bar wavelength is approximately nine times the channel width showing good agreement with Defina’s (2003) numerical result and laboratory data of alternate bar wavelength prediction (Ikeda, 1984).

4.2 Simulation of mixed bedrock-alluvial experiment (Run 2-B2)

Figure 6 shows a series of numerical results illustrating the time evolution of bed configuration for the mixed bedrock-alluvial experiment (Run 2-B2) commencing from a 2 cm thick alluvial cover \((t = 0 \text{ h})\). The non-uniform sediment distribution, feeding more sediment at

![Simulated bed evolution of Run 2-B2. Colorbar shows the thickness of the sediment cover, and white areas correspond to the exposed bedrock surface.](image)

**Figure 6.** Simulated bed evolution of Run 2-B2. Colorbar shows the thickness of the sediment cover, and white areas correspond to the exposed bedrock surface.

![Time evolution of bedrock exposure Run 2-B4 at times are 1, 4, and 7 hours from top to bottom: \(Q_s = 97 \text{ g/s}, q_s/q_c = 0.88, \text{ and } P_c = 0.78\) (Chatanantavet & Parker, 2008 (personal communication)). The channel is 0.9 m wide, 13 m long, water and sediment flow from left to right, and light and dark areas correspond to bedrock and sediment, respectively.](image)

**Figure 7.** Time evolution of bedrock exposure Run 2-B4 at times are 1, 4, and 7 hours from top to bottom: \(Q_s = 97 \text{ g/s}, q_s/q_c = 0.88, \text{ and } P_c = 0.78\) (Chatanantavet & Parker, 2008 (personal communication)). The channel is 0.9 m wide, 13 m long, water and sediment flow from left to right, and light and dark areas correspond to bedrock and sediment, respectively.
one side than the other, causes flow deflection induced by a laterally sloping bed at the upstream end \((t = 0.5 \text{ h})\). At an early stage, this disturbance quickly erodes sediment on the right side of the channel down to the bedrock bed and creates pools and bars downstream. Then the exposed bedrock bed area, indicated as a white area, increases as the pools deepen and bars grow taller \((t = 1 \text{ h})\). The sediment on the bed forms a strip of sediment shifting from one side to the other side of the channel through time while maintaining a consistent fraction of bedrock cover. Similar patterns were observed in the Chatanantavet and Parker (2008) experiments; for example, Figure 7 shows the time evolution of bedrock exposure from Run 2-B4 (Table 1 in Chatanantavet and Parker, 2008) with \(Q_c = 97 \text{ g/s, } q_s/q_c = 0.88\), and \(P_c = 0.78\) at equilibrium phase. Again, a continuous band of sediment forms comparable to Figure 6. However, the sediment cover is more expansive, and the bed sediment shifting from one to the other side is less sensitive because of the higher sediment supply rate than what was provided in Run 2-B2 modeled here.

Figure 8 shows the computed flow variables, bed topography, and sediment transport capacity at equilibrium \((t = 5\text{h})\). The flow depth and velocity over the exposed bedrock area are higher than on the alluvial bed. However, calculated shear stress is lower in exposed bedrock areas due to the smaller roughness and higher flow depth. The critical dimensionless shear stress is generally higher on the alluvial bed but smaller over the lee side of the bedforms. The sediment transport capacity tends to be higher over the exposed bedrock surface and stoss side of the bedform and lower on lee slopes.

Figure 9 shows the time evolution of channel averaged fraction of bedrock exposure \(F = 1 - P_c\), alluvial thickness, relative Shields parameter, and bedload transport rate. A ratio of alluvial coverage of bed surface in the area of interest determines the fraction of bedrock covered with sediment \(P_c = A_c/A_t\) (Johnson, 2014), where \(A_c\) is the area covered with sediment and \(A_t\)
is the total area of interest, or numerically \( P_e = \sum P_{ci} / N \), where \( P_{ci} \) is the local fraction of bedrock cover (equation 8) and \( N \) is the total number of cells in the domain. The fraction of bedrock exposure is initially zero when the bed is entirely covered by sediment and quickly converges toward a near-equilibrium state where \( F_e \) is 0.4 at \( t = 1 \) h (Figure 9a). The numerical result of the degree of bedrock exposure presents a similar value of \( F_e \) to the experimental observation. The averaged sediment cover thickness over an alluvial area varies little in the range

![Figure 8](image-url)

**Figure 8.** Plan view of (a) alluvial thickness, (b) flow depth, (c) flow velocity, (d) dimensionless shear stress, (e) critical Shields parameter, and (f) dimensionless bedload transport capacity at \( t = 5 \) h. Colorbars indicate the scale of computed values, respectively, and white areas correspond to (a) the exposed bedrock surface and (e) zero sediment transport rate.
of a single grain diameter (Figure 9b). The relative Shields parameter $\tau^*/\tau_c^*$ (Figure 9c) and bedload transport rate (Figure 9d) decrease from an initially higher value to approximate equilibrium and vary around 1.2 and 70 g/s, respectively.

The comparison between the computed and measured quantities available from experimental observations is presented in Table 3. The agreement is reasonably good in predicting the fraction of bedrock cover, but higher flow depth and lower velocity in the experiment result in a lower Froude number and lower dimensionless shear stress. The model predicted lower shear stress over the bare bedrock areas than the alluvial areas because the bedform developed during the simulation. The alluvial bars form where flow depth is low and velocity is high, and pools exist over the bare bedrock area with deep flow depth and low velocity.
The present study shows the results that bed topography comparable to experimental observations from (1) free bar formation in a mild slope alluvial channel with fine grains (Lanzoni, 2000) and; (2) alluvial pattern in the mixed bedrock-alluvial channel in a steep slope channel with coarse grains (Chatanantavet & Parker, 2008). These suggest that the model can simulate complex flow and sediment transport to gain insight into mechanisms of the bedform development and alluviation patterns in mixed bedrock-alluvial channels.

The discrepancies between the numerical and experimental results are mainly associated with the initial and boundary conditions, which are not successfully reflected in the numerical models, such as the size of the initial disturbances and patterns of flow and sediment distribution at the channel inlet.

### Table 3. Summary of Numerical Results

<table>
<thead>
<tr>
<th>Run</th>
<th>$P_c$</th>
<th>$H$</th>
<th>$U$</th>
<th>$Fr$</th>
<th>$\tau^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-B2 (C&amp;P)$^a$</td>
<td>0.59</td>
<td>5.5±1.5</td>
<td>1.11</td>
<td>1.51</td>
<td>0.11</td>
</tr>
<tr>
<td>2-B2$^b$</td>
<td>0.60</td>
<td>6.0</td>
<td>1.00</td>
<td>1.32</td>
<td>0.10</td>
</tr>
<tr>
<td>2-B2 (covered zones)$^c$</td>
<td></td>
<td>5.6</td>
<td>0.95</td>
<td>1.31</td>
<td>0.10</td>
</tr>
<tr>
<td>2-B2 (exposed zones)$^d$</td>
<td></td>
<td>7.5</td>
<td>1.07</td>
<td>1.26</td>
<td>0.08</td>
</tr>
</tbody>
</table>

$^a$ Experimental results from Chatanantavet and Parker (2008).

$^b$ Reach averaged numerical results.

$^c$ Reach averaged numerical results only over the alluviated zones.

$^d$ Reach averaged numerical results only over the exposed bedrock zones.
5.1 Discussion on ripple factor and form drag

Only a few studies have attempted to simulate sediment transport in a straight mixed bedrock-alluvial channel using a 2D numerical model (Inoue et al., 2016; P. A. Nelson & Seminara, 2012). These models estimated bed resistance to the flow using the grain roughness and bedrock surface irregularity which is applicable to the plane bed. Our model includes the effect of bed-form generation in roughness estimation since alternate bars often form in mixed bedrock-alluvial channels (Chatanantavet & Parker, 2008; Inoue et al., 2016; P. A. Nelson & Seminara, 2012). The hydraulic roughness height of alluvial bed changes according to the size of bedforms (i.e., dunes and ripples) (Engelund, 1977; Raudkivi, 1997; van Rijn, 1982, 1984; Vanoni & Hwang, 1967; Wiberg & Nelson, 1992).

Figure 10 shows the time evolution of bed topography with the same condition applied in Run 2-B2 without including the bedform effect in the roughness calculation, and hence the ineffective ripple factor. This simulation undergoes sediment washing out from the upstream end of the channel while forming a bar-like bedform downstream. This result is comparable with the results provided by Inoue et al. (2016).

**Figure 10.** The time evolution of bed exposure for run 2-B2 without form drag effect ($k_f = 0$). Colorbar means alluvial cover thickness, and white area is exposed bedrock surface.
The prediction of bedload transport is based on the corrected dimensionless shear stress for bedform roughness with a ripple factor (Meyer-Peter & Muller, 1948). The value of the ripple factor has not been reported explicitly but empirically estimated as a ratio of grain shear stress to the total bed shear stress (Ribberink, 1987; Vermeer, 1986). Nevertheless, the ripple factor is often applied to sediment transport models to generate bedforms in alluvial channels with a constant value of less than 1 (Defina, 2003; Van der Meer et al., 2011).

Figure 11 shows the plan view of the bed topography of Run 2-B2 without the dimensionless shear stress correction by using a ripple factor equal to 1 in equation (16). The upstream sediment erodes slower than the bed evolution in Figure 10, and the morphodynamic process has enough time to form a longitudinal sediment strip because the form drag generates higher flow resistance. However, larger dimensionless shear stress due to the absence of the ripple factor in the supply-limited channel causes a thin layer of sediment at the upstream end and a narrow strip of sediment. The strip of sediment is cut off at the narrowed section, and the sediment is eventually washed out.

**Figure 11.** The time evolution of bed exposure for Run 2-B2 without ripple factor ($\mu = 1$). Colorbar indicates alluvial cover thickness, and the white area is exposed to the bedrock surface.
Comparing Figures 6 and 10-11, it can be seen that introducing both form drag to the roughness and ripple factor of the shear stress in bedload transport is critical in developing bedforms and persistent alluviation in bedrock channels where the sediment supply is less than the capacity of the channel. Results from previous models attempting to replicate these experiments (Inoue et al., 2014) show alternate bar development over the alluvial surface, but sediment supply less than the transport capacity decreases the thickness of the alluvial layer leading to sediment washing out from upstream and exposure of the entire bedrock.

The roughness component for form drag effectively increases drag and reduces flow velocity, whereas the ripple factor reduces shear stress in bedform-developed areas. Unlike the form drag acting on entire bedform fields, skin friction is responsible for the local bed surface roughness only. The smaller surface roughness over the exposed bedrock area than over the alluvial area produces higher shear stress. Therefore the model absent of form drag and ripple factor predicts a higher sediment transport rate as the fraction of sediment cover decreases in limited sediment supply conditions, resulting in a total washout of the sediment from the channel.

5.2 The impact of sediment transport roughness

Our model explicitly accounts for both the grain roughness and roughness produced by moving particles (Dietrich, 1982; Grant & Madsen, 1982; Smith & McLean, 1977; Wiberg & Rubin, 1989). To explore the importance of accounting for each of these components individually, we conducted a simulation where we ignored sediment transport roughness and increased the grain roughness to compensate for reduced bed roughness caused by the absence of sediment transport roughness.
Figure 12 shows the time evolution of bed topography for this simulation of Run 2-B2, excluding sediment transport roughness. A strip of sediment forms shifting from one side to the other side of the channel momentarily ($t = 0.5\ h$), but then washes out from the upstream end of the channel ($t = 2\ h$), with a small residual alluvial patch persisting only in a topographic low area in the bedrock ($t = 4\ h$). The flow resistance decreases as bedrock gets exposed because of the underlying linear relationship between the fraction of bedrock cover and grain roughness. Unlike grain roughness, the sediment transport roughness is larger where grain movements actively take place. The sediment transport roughness tends to be higher over the bedrock surface partially covered with sediment because the sediment transport rate over the smoother bedrock surface, where the flow velocity is faster, is higher than over the rougher alluvial surface. The sediment transport roughness contributes to the formation of alluvial patches in the bedrock channel by reducing grain entrainment over the mixed bedrock-alluvial surface. These results indicate that separate accounting of grain and sediment transport roughness produces morphodynamic predictions that better match observations of persistent alluvial patterns in mixed bedrock-alluvial channels.

**Figure 12.** The time evolution of bed exposure for Run 2-B2 without sediment transport roughness from flow resistance ($k_t = 0$). Colorbar indicates alluvial cover thickness, and the white area is exposed to the bedrock surface.
6 Conclusions

In this study, we have developed a two-dimensional morphodynamic model to explore bar formation and migration in an alluvial channel and sediment transport mechanisms in a mixed bedrock-alluvial channel without sufficient sediment supply. Comparisons of model predictions with experimental observations from the free bar test show that the model predicts bed morphology and bedform development similar to the observations of the flume experiment. However, some discrepancies raise the need for proper tuning of model parameters and initial bottom perturbation.

The model predicts the flow field and sediment distribution patterns in mixed bedrock alluvial channels reasonably well. Numerical experiments show that the inclusion of bedform roughness and a shear stress correction for near-bed sediment transport is critical to be able to replicate the alluvial patterns over bare bedrock observed in flume experiments. The evolving interactions between the alluvial and bedrock bed surface, flow field, and sediment transport simultaneously modify the degree of sediment cover and bed topography. The numerical model presented in this study captures the behavior associated with the bedrock alluviation process and can be used to extend its applicability to various flow and sediment supply conditions with different channel slopes and antecedent topography.

Future work could explore the mechanisms of sediment pattern formation in mixed bedrock-alluvial rivers and characterize the effects of the channel slope, initial sediment cover thickness, difference between the grain and bedrock roughness, and bedrock configuration.

Acknowledgments
We thank Phairot Chatanantavet and Gary Parker for providing photographs of flume experiments.

**Open Research**

The code demonstrating the 2D morphodynamic model and numerical data of test cases shown herein are archived at https://github.com/rcemorpho/morph2d.

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Patterns of Alluviation in Mixed Bedrock-Alluvial Channels: 1. Numerical Model

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Key Points:

- A new model is developed to predict the dynamics of alluvial patterns in mixed bedrock-alluvial channels.

- The model accurately predicts bedform evolution in an alluvial channel and alluvial patch formation in a mixed bedrock-alluvial channel.

- Accounting for differential roughness and shear stress correction are important for model prediction of the degree of exposed bedrock.
Abstract

Mixed bedrock-alluvial rivers can exhibit partial alluvial cover, which may play an important role in controlling bedrock erosion rates and landscape evolution. However, numerical morphodynamic models generally are unable to predict the pattern of alluviation in these channels. Hence we present a new two-dimensional depth-averaged morphodynamic model that can be applied to both fully alluvial and mixed bedrock-alluvial channels, and we use the model to gain insight into the mechanisms responsible for the development of sediment patches and patterns of bedrock alluviation. The model computes hydrodynamics, sediment transport, and bed evolution, using a roughness partitioning that accounts for differential roughness of sediment and bedrock, roughness due to sediment transport, and form drag. The model successfully replicates observations of bar development and migration from a fully alluvial flume experiment, and it models persistent sediment patches observed in a mixed bedrock-alluvial flume experiment. Numerical experiments in which the form drag, sediment transport roughness, and ripple factor correction were neglected did not successfully reproduce the observed persistent sediment cover in the mixed bedrock-alluvial case, suggesting that accounting for these different roughness components is critical to successfully model sediment dynamics in bedrock channels.

Plain Language Summary

The spatial distribution of sediment patches in bedrock rivers can play an important role in the local hydraulic conditions as well as long-term channel and landscape evolution, because sediment in these channels is an important mechanism of bedrock erosion. Flume experiments and field studies have shown that the amount of sediment cover and the pattern of sediment patches in these streams can sometimes reach a sort of equilibrium, but numerical models have been largely unable to replicate those observations. Here, we present a new numerical model for
mixed bedrock-alluvial rivers that explicitly accounts for different roughness feedbacks between
the bedrock, sediment, and flow field. We show that the model can successfully simulate
observations from flume experiments conducted with and without exposed bedrock, and it is able
to simulate persistent partial sediment cover. These conditions develop because of the
complicated roughness feedbacks in these channels, which we demonstrate through simulations
where different roughness components are “turned off,” which result in completely exposed
bedrock without sediment cover.

1 Introduction

River channels can be classified as either alluvial, bedrock, or mixed bedrock-alluvial
are entirely covered by sediment of sufficient thickness so that the underlying bedrock is not
exposed (Tinkler & Wohl, 1998; Whipple, 2004). In contrast, bedrock channels are characterized
by frequently exposed bedrock and a lack of continuous alluvial cover in the channel bed and
banks. Mixed bedrock-alluvial channels tend to have sediment supply that is less than their
sediment transport capacity, and they feature the exposed bedrock interspersed with patches of
alluvial cover, potentially taking the form of alternate bars (e.g., Chatanantavet & Parker, 2008)
or point bars at meander bends (P. A. Nelson et al., 2014; Nittouer et al., 2011).

The pattern of alluvial cover in mixed bedrock-alluvial channels likely plays a role in the
morphological evolution of those channels, and consequently on landscape evolution (Gasparini
et al., 2007; Hodge & Hoey, 2012; Howard et al., 1994; Seidl & Dietrich, 1992; Tinkler & Wohl,
1998; Whipple & Tucker, 2002; Wohl, 1993). Bedrock channel erosion sets the lower boundary
condition for landscape evolution and bedrock channels convey climatic and tectonic
perturbation through the landscape (Whipple, 2001; Whipple & Tucker, 1999). Alluvial cover is
an important component of mechanistic models of bedrock erosion; for example, the saltation-
abrasion model incorporates the erosional mechanism of saltating bedload particles impacting
and eroding bedrock (Demeter et al., 2005; Hartshorn et al., 2002; Sklar & Dietrich, 1998, 2001,
2004; Zhang et al., 2015). Competition between the tools and cover effects controls the spatial
distribution of the bedrock channel erosion that results in lateral and vertical channel erosion and
meandering (Finnegan et al., 2007; Lamb et al., 2015; Turowski et al., 2007; Turowski, Hovius,

Observations from flume experiments documenting the development of alluvial cover
patterns on bedrock beds have shown that alluvial cover in these channels depends on channel
slope, the initial thickness of alluvial sediment, sediment supply, bedrock roughness, and channel
topography. Chatanantavet and Parker (2008) conducted a series of experiments in a straight
flume where they varied the rate of sediment supply, the initial cover of sediment on the bed, the
slope, and the grain size. Their experiments illuminated several exciting phenomena in mixed
bedrock-alluvial channels. First, the pattern and trajectory of the alluvial cover appear to be
slope-dependent, wherein at low slopes ($S = 0.0115$) the exposed bed fraction linearly decreases
with increasing sediment supply, akin to the linear relationship hypothesized by Sklar and
Dietrich (2004). However, at higher slopes ($S = 0.02$), experiments starting with a bare bed did
not develop persistent alluvial cover at any supplied rate until that rate exceeded the overall
transport capacity of the channel. At that point, “runaway alluviation” occurred, and the entire
channel became covered in sediment. Second, the thickness of the initial alluvial cover affected
the dynamics of alluviation on the bed, wherein at low initial alluvial thickness the bed was
stripped clean, whereas thicker initial sediment covers eventually reached a non-zero fraction of
bedrock exposure. Third, some of their experiments developed continuous strips of sediment moving from one side of the channel to the other, akin to alternate bars.

Other experiments have suggested that bedrock topography and the relative hydraulic roughness of bedrock relative to grain size plays a role in alluvial dynamics of mixed bedrock-alluvial channels. Mishra and Inoue (2020) performed flume experiments with varying bedrock roughness. The observed the extent of alluvial cover increases with increasing sediment supply when the hydraulic roughness of the bedrock bed is larger than that of the alluvial surface. However, a sudden transition from bare bedrock bed to full alluviation was observed as sediment supply momentarily exceeded the channel's transport capacity when the ratio of hydraulic roughness height of bedrock to grain size \( k_{bh}/d \) is 1.9 or lower. They also proposed an approximation of dimensionless critical shear stress for incipient particle motion over bedrock beds as a function of relative roughness height. Hodge and Hoey (2016a, 2016b) performed experiments in a 3D printed scale model of a jointed limestone bedrock river in which the patterns of sediment deposits at different sediment supply rates were documented. Their experiments pointed out the importance of bedrock topography on depositional patterns, as patches of sediment tended to form in the lowest portions of the bed, and at higher discharge and sediment supply, the bed topography played a less important role than sediment-sediment and sediment-flow interactions in stabilizing patches of alluvium.

Numerical models of sediment transport and bed evolution in mixed bedrock-alluvial channels have struggled to capture the dynamics of alluvial cover observed in experiments or in the field. Morphodynamic models simulate river channel evolution by iteratively using hydraulic flow field calculations to estimate sediment transport rates, which are then used in the conservation of sediment mass (i.e., the Exner equation) to calculate bed erosion and deposition.
patterns. These types of models have been used for decades to understand the dynamics of alluvial rivers, such as the development and migration of alternate bars (e.g., Bernini et al., 2006; Defina, 2003; Qian et al., 2017), sediment sorting (e.g., P. A. Nelson et al., 2015a, 2015b), braiding (e.g., Murray & Paola, 1997; Schuurman et al., 2013), meandering (e.g., Smith & Mclean, 1984; J. M. Nelson et al., 2003), and armoring (e.g., Parker & Klingeman, 1982; Parker & Toro-Escobar, 2002). However, alluvial morphodynamic models generally assume that the sediment supply equals or exceeds the sediment transport capacity, which is not the case for mixed bedrock-alluvial rivers.

This assumption has only recently begun to be relaxed in attempts to use morphodynamic models to better understand mixed bedrock-alluvial rivers (e.g., P. A. Nelson & Seminara, 2012). Using a cellular automaton model governed by probabilities of individual grain movement, Hodge and Hoey (2012) studied the relationship between the fraction of bedrock exposure and the ratio of sediment supply to capacity on noneroding bedrock beds. Zhang et al. (2015) expressed bedrock cover fraction as a ratio of vertical length scale between alluvial thickness and macro-roughness of bedrock topography representing the statistical characteristics of bedrock surface fluctuations. This MRSAA (Macro-Roughness-based Saltation-Abrasion-Alluviation) model was later implemented to investigate the knickpoint migration (Zhang et al., 2018, 2019) and formation of the cyclic steps (Izumi et al., 2017) to spatiotemporal variation of sediment supply. Inoue et al. (2016) provided an early attempt to investigate alternate bar formation and bedrock incision in mixed bedrock-alluvial channels in response to the different ratio of sediment supply to channel’s transport capacity.

Larger-than-grain-scale bedrock topography, sometimes called “macrotopography”, has recently been incorporated into one-dimensional morphodynamic models (Zhang et al., 2015).
The model solutions for alluvial response to sediment supply show that stripping of antecedent sediment from upstream to downstream occurs with the termination of sediment supply, and the development of an alluvial layer with a thickness corresponding to the ratio of sediment supply to transport capacity \( q_s/q_c \), which propagates downstream over bare bedrock when the sediment supply is increased. Application of nonlinear wave speed to the alluvial layer predicted that alluvium over a bedrock surface with a small cover fraction migrated downstream much faster than those consisting of a higher fraction cover.

Despite these recent advances in modeling the morphodynamics of mixed bedrock-alluvial rivers, there remain uncertainties about how differences in sediment and bedrock roughness influence in mixed bedrock-alluvial rivers, and how hydraulics and macroroughness features may interact to control alluvial patterns. Existing models are not able to resolve these issues because either they are one-dimensional and unable to predict lateral variation in alluvial cover, or they do not fully account for roughness effects of local sediment cover or transport. For example, current models of mixed bedrock-alluvial morphodynamics have not been able to predict persistent longitudinal alluvial strips, or to fully replicate the relationship between overall sediment cover and the sediment supply to capacity ratio as reported in Chatanantavet and Parker (2008).

This study intends to resolve the limitations models encounter when attempting to simulate sediment dynamics in a mixed bedrock-alluvial channel by developing a new model that uses: (1) a two-dimensional approach to account for the influence of potential bedforms (e.g., alternate bars, antidunes, and dunes) in a straight channel; (2) the effect of flow resistance partitioned into skin friction due to stationary particles and bedrock surface roughness, form drag associated with dimensions of alluvial cover and irregular bedrock topography, and bedload
roughness produced by saltating sediment; (3) application of a ripple factor to take into account for an intermediate state between the planar bed and small-scale bedforms.

We hypothesize that accounting for differential bedrock and sediment roughness, roughness from sediment transport, form drag, and a near-bed shear stress correction are necessary to capture sediment dynamics in mixed bedrock-alluvial channels. We use two experiments to demonstrate that our model can reproduce observations made in fully alluvial and mixed bedrock-alluvial channels, and we explore the importance of the different roughness terms we include by repeating the simulation with those terms neglected. We simulate an experiment of Lanzoni (2000) for alternate bar formation in an alluvial channel and an experiment of Chatanantavet and Parker (2008) for gravel patch development in a mixed bedrock-alluvial channel to assess model performance. Our model successfully replicates sediment dynamics in both experiments. The numerical experiments that neglect form drag and transport components of the roughness do not reproduce observations from the Chatanantavet and Parker (2008) experiment, indicating that those components play a critical role in mixed bedrock-alluvial sediment dynamics.

2 Morphodynamic model

We have developed a new morphodynamic model that simulates sediment transport and deposition in mixed bedrock-alluvial channels. This model consists of three components: a hydrodynamic model describing the depth-averaged flow field, a sediment transport model describing bedload sediment transport rate, and a bed evolution model updating bed elevation and the areal fraction of bedrock cover. Unlike previous models of mixed bedrock-alluvial morphodynamics, this new model accounts for form drag and sediment transport roughness, in addition to surface (grain and bedrock) roughness and a roughness-dependent critical Shields
stress. In addition, the total shear stress is corrected with a ripple factor in the sediment transport model, as the bedform drag does not contribute to the bedload transport.

2.1 Flow model

The governing equations for calculating flow depth and velocity are composed of the depth-averaged form of mass continuity and momentum balance in a 2D Cartesian coordinate system:

\[
\frac{\partial Q}{\partial t} + \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + S_b + S_f \tag{1}
\]

where \( t \) is time, \( x \) and \( y \) are Cartesian coordinates, \( Q \) are the conservative variables, \( F_x \) and \( F_y \) are the convective fluxes, \( D_x \) and \( D_y \) are the diffusive fluxes, \( S_b \) are the bed slope terms, and \( S_f \) are the friction slope:
where $h$ is the flow depth, $u$ and $v$ are the velocities in $x$- and $y$-directions, respectively, $g$ is the gravitational acceleration, $\nu$ and $\nu_t$ are the kinematic viscosity of water and the turbulent eddy viscosity, respectively, $\eta$ is the water surface elevation, $z_b$ is the bed elevation, and $\rho$ is the density of water. The bed shear stresses $\tau_{bx}$ and $\tau_{by}$ in $x$- and $y$-direction are given by

\[
\left(\tau_{bx}, \tau_{by}\right) = \rho C_f \sqrt{u^2 + v^2} (u, v)
\]

where $C_f$ is a friction coefficient estimated using the law of the wall as a function of the flow depth $h$ and total roughness height $k_0$:

\[
C_f = \left[ \frac{1}{\kappa} \ln \left( \frac{11h}{k_0} \right) \right]^2.
\]
2.2 Depth-averaged mixing-length model

The calculation of the turbulent viscosity term is based on a mixing-length model with depth-averaged terms developed by Stansby (2003):

\[

\nu_t = \sqrt{l_h^4 \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right)^2 \right] + (\gamma u_* h)^2}

\]

where \( l_h \) is a horizontal mixing length scale (\( l_h \approx 0.267 \kappa h \)), \( \kappa \) is the von Karman constant (\( \kappa \approx 0.408 \)), and \( \gamma \) is a constant that accounts for vertical mixing (\( \gamma \approx 0.067 \)). The local shear velocity, \( u_* \), is defined as;

\[

u_* = \sqrt{\tau_b / \rho}

\]

where \( \tau_b = \sqrt{\tau_{bx}^2 + \tau_{by}^2} \) is the local bed shear stress vector, with components \( \tau_{bx} \) and \( \tau_{by} \) in the \( x \)- and \( y \)-directions, respectively.

2.3 Bed roughness

The total roughness height \( k_0 \) is partitioned into three fractional roughness components, including skin friction \( k_s \), form drag \( k_f \), and bedload transport \( k_t \):

\[

k_0 = k_s + k_f + k_t.

\]
in the mixed bedrock-alluvial channel is based on the assumption that the skin friction is linearly associated with the changes in the fraction of bedrock covered by alluvium:

\[ k_s = P_c k_{sa} + (1 - P_c) k_{sb} \]  

(8)

where \( k_{sa} \) and \( k_{sb} \) are the hydraulic roughness height of the alluvial bed and bedrock bed, respectively. The local areal fraction of alluvial cover \( P_c = \eta_a/C_m \leq 1 \), in which \( C_m = \pi d / 6 \) is the maximum volume of a monolayer of spherical sediment grains of constant diameter \( d \) uniformly distributed over the bed surface (P. A. Nelson & Seminara, 2012), and \( \eta_a \) is the thickness of the alluvial layer.

The form drag component of roughness results from the pressure force acting over entire bedforms and is not responsible for the bedload motion of sediment particles (Maddux, McLean, et al., 2003; Maddux, Nelson, et al., 2003). We calculate the form drag component of roughness as a function of bed morphology through the empirical relation of Grant and Madsen (1982):

\[ k_f = 30 a_r \eta_r^2 / \lambda_r \]  

(9)

where \( \eta_r \) and \( \lambda_r \) are bed form height and wavelength, and \( a_r \) is a coefficient in the range from 0.3 to 3. Grant and Madsen (1982) suggested \( a_r = 0.923 \), which we use here.

For the sediment transport component of roughness, Wiberg and Rubin (1989) proposed for the flat bed condition:

\[ k_i = 30 a_w d \frac{a_t T_r}{1 + a_t T_r} \]  

(10)
where \( T^* = \frac{\tau^*}{\tau_c^*} \) is the transport stage, \( \tau^* \) is the dimensionless shear stress, \( \tau_c^* \) is the critical dimensionless Shields stress, \( a_1 = 0.68 \), \( a_2 = 0.0204 (\ln 100d)^2 + 0.0220 (\ln 100d) + 0.0709 \), and \( \alpha_{ns} = 0.056 \). Here, the effect of local variation of bed topography is applied to the critical bed shear stress to account for the gravity effect, as described in Section 2.5.

### 2.4 Bed deformation model

The local volumetric concentration of sediment per unit area is calculated using the sediment conservation model for mixed bedrock-alluvial channel beds proposed by Luu, Egashira, and Takebayashi (2004):

\[
\frac{\partial V_{ba}}{\partial t} + \frac{\partial q_{bx}}{\partial x} + \frac{\partial q_{by}}{\partial y} = 0
\]  

(11)

where \( V_{ba} \) is the total volume of sediment per unit area and \( q_{bx} \) and \( q_{by} \) are vectors of the bedload transport rate per unit width in the \( x \) - and \( y \) -directions, respectively. The thickness of the alluvial layer \( \eta_a \) and the volume of sediment in the bedload layer \( V_b \) are separately updated considering the saturation volume of the bedload layer \( V_{bc} \). This saturation volume is a threshold value that determines whether the particles deposit on the bed or rapidly saltate over the surface without resting on the bed. When \( V_{ba} \) exceeds \( V_{bc} \), a volume of sediment equal to the difference between \( V_{ba} \) and \( V_{bc} \) deposits on the bed as an alluvial layer. When \( V_{ba} \) is less than \( V_{bc} \), the particles pass over the bedrock surface as throughput load without deposition:

\[
\eta_a = \begin{cases} 
\frac{V_{ba} - V_{bc}}{1 - \lambda} & \text{for } V_{bc} \leq V_{ba} \\
0 & \text{for } 0 \leq V_{ba} < V_{bc}
\end{cases}
\]  

(12)
where $\lambda$ represents the porosity of the bed and $V_{bc}$ is the saturation volume of the bedload layer per unit area. When the bedrock is wholly exposed ($\eta_a = 0$), the volume of the throughput bedload layer is lower than the saturation volume. When the bed is partially or fully covered with sediment ($\eta_a > 0$), $V_b$ equates to $V_{bc}$ because sediment particles exchange occurs between alluvial and bedload layers:

$$V_b = \begin{cases} V_{bc} & \text{for } V_{bc} \leq V_{ba} \\ V_{ba} & \text{for } 0 \leq V_{ba} < V_{bc} \end{cases}$$

(13)

The saturation volume of bedload per unit area $V_{bc}$ is defined by

$$V_{bc} = \frac{q_{bc}}{u_s}$$

(14)

where $q_{bc}$ is the bedload transport capacity per unit width and $u_s$ is the saltation velocity, calculated here with the empirical excess shear stress relation presented in (Sklar & Dietrich, 2004):

$$\frac{u_s}{\sqrt{R_s gd}} = 1.56 \left(\frac{\mu \tau^*}{\tau_c^*} - 1\right)^{0.56}$$

(15)

where $R_s = (\rho_s - \rho)/\rho$ denotes submerged specific gravity of sediment and $\rho_s$ is the density of the sediment. The bedload transport capacity per unit width $q_{bc}$ is estimated using the relation based on Ashida and Michiue (1972):

$$\frac{q_{bc}}{\sqrt{R_s gd^3}} = 17 \left(\sqrt{\mu \tau^*} - \sqrt{\tau_c^*}\right)\left(\mu \tau^* - \tau_c^*\right)$$

(16)

where dimensionless shear stress $\tau^*$ is defined as
\[
\tau^* = \frac{\tau_b}{\rho R_g gd}
\]  

(17)

and \( \mu \leq 1 \) is the ripple factor, the ratio of the grain roughness to bed roughness (Ribberink, 1987) (discussed further in Section 2.5).

The volumetric sediment transport rate per unit width in the \( x \)- and \( y \)-directions is denoted

\[
(q_{tx}, q_{ty}) = q_b (\cos \alpha, \sin \alpha)
\]

(18)

where \( \alpha \) is the angle of bedload transport and the sediment transport intensity, \( q_b \), depends on the ratio of the volume of sediment and its saturation value. The bedload transport rate in bedrock with a sufficient local volume of sediment equals the sediment transport capacity.

However, in bedrock rivers without sediment cover, the bedload transport rate is less than the bedload transport capacity:

\[
q_b = \begin{cases} 
\frac{V_b}{V_{bc}}q_{bc} & \text{for } 0 \leq V_b < V_{bc} \\
q_{bc} & \text{for } V_{bc} \leq V_b
\end{cases}
\]

(19)

When considering the effect of gravity acting on particles for gradually varying bed elevation, the sediment transport direction deviates from the direction of the boundary shear stress. Here we adopt the well-known relationship (Struiksma, 1985):

\[
\tan \alpha = \frac{\sin \delta - \frac{1}{f(t^*)} \frac{\partial z_b}{\partial y}}{\cos \delta - \frac{1}{f(t^*)} \frac{\partial z_b}{\partial x}}
\]

(20)
where $\mathcal{S}$ is the near-bed flow direction estimated to account for the influence of spiral water motion induced by bed topography as

$$\mathcal{S} = \tan^{-1}\left( \frac{v}{u} \right) - \tan^{-1}\left( \frac{h}{r_s} A \right),$$  

(21)

where the local radius of depth-averaged stream curvature is

$$r_s = \frac{U^3}{u^2 \frac{\partial v}{\partial x} + \nu v \left( \frac{\partial v}{\partial y} - \frac{\partial u}{\partial x} \right) - v^2 \frac{\partial u}{\partial y}}$$  

(22)

where $U = \sqrt{u^2 + v^2}$ is the local flow velocity and the coefficient weighting the intensity of helical flow is

$$A = \frac{2}{k^2} \left( 1 - \sqrt{\frac{C_f}{k}} \right),$$  

(23)

and $f(\tau^*)$ is a function weighting the influence of the bed slope, following the form proposed by Talmon, Struiksma, and Van Mierlo et al. (1995):

$$f(\tau^*) = 9 \left( \frac{d}{h} \right)^{0.3} \sqrt{\tau^*}.$$  

(24)

2.5 Shear stress correction

The bedload sediment transport rate is expressed as a function of shear stress acting over the channel bed. The presence of bedforms leads to a partial reduction of the total shear stress related to the form drag, and the remainder is available for sediment transport. Traditionally this has been implemented in models by multiplying the dimensionless shear stress by a so-called
“ripple factor.” Several models for estimating the ripple factor have been proposed as a function of the skin friction relative to the total friction (Ribberink, 1987; Vermeer, 1986):

\[
\mu = \left( \frac{C_{fs}}{C_f} \right)^{n/2}
\]  

(25)

where \( C_{fs} \) is the friction coefficient accounting for skin friction obtained by substituting \( k_0 \) with \( k_s + k_i \) in equation (4), and the exponent \( n \) is defined by

\[
n = 1.8 + 0.27 \log q^*,
\]

(26)

where \( q^* = q_{bc}/\sqrt{R_s g d^3} \) is the dimensionless bedload transport rate. This relationship is valid for \( 0.001 \leq q^* \leq 1 \) (Vermeer, 1986).

2.6 Critical dimensionless Shields stress

Flume experiments conducted in bedrock channels (Inoue et al., 2014; Mishra & Inoue, 2020) have related the channel roughness and dimensionless critical shear stress of sediment movement. A power approximation proposed by Mishra and Inoue (2020) is applied to this model to take into account the effect of total hydraulic roughness on critical shear stress:

\[
\tau_{c0}^* = \tau_{cr}^* \left( k_s/d \right)^{0.6}
\]

(27)

where \( \tau_{cr}^* \) is the critical Shields stress back-calculated from sediment transport capacity measured from the experiment in the flat channel. In addition to the bed roughness, the local bed slope effect on the initiation of particle motion can be added because the local bed slope provides a gravitational component of the force exerted on the particle (Duan & Julien, 2005; Soulsby, 1997):
\[
\frac{\tau^*}{\tau_{c0}} = \frac{\sin(\phi + \beta_s)}{\sin \phi} \frac{\cos \beta_n}{\sqrt{1 - \tan^2 \beta_n / \tan^2 \phi}}
\]  \hspace{1cm} (28)

where \( \phi \) is the grain angle of repose and \( \beta_s \) and \( \beta_n \) are the slope in the streamwise and cross-stream direction of the sediment transport, respectively. A simple approach was proposed by Wiberg and Smith (1987) for the treatment of heterogeneous bed conditions through the geometric relation of friction angle as a function of particle size to the roughness length scale of the bed:

\[
\phi = \cos^{-1}\left(\frac{d/k_s - 0.02}{d/k_s + 1}\right)
\]  \hspace{1cm} (29)

The bed slopes in the streamwise and cross-stream direction of sediment transport are

\[
\beta_s = \tan^{-1}\left(\frac{\partial z}{\partial x} \cos \alpha + \frac{\partial z}{\partial y} \sin \alpha\right), \quad \beta_n = \tan^{-1}\left(\frac{\partial z}{\partial y} \cos \alpha - \frac{\partial z}{\partial x} \sin \alpha\right).
\]  \hspace{1cm} (30)

2.7 Solution procedure

The model uses a decoupled approximation of the morphodynamic system by assuming that the response time of the bed evolution is relatively long compared to the time scale of the hydraulic processes (De Vries, 1965; Defina, 2003; J. M. Nelson & Smith, 1989). Therefore the hydrodynamic solver is followed by the calculation of the modified Exner equation.

First, local water depth and flow velocity are calculated using the hydrodynamic model (equation (1) - (2)). The friction term is estimated by taking into account the roughness of the bedrock \( k_{sb} \) and alluvial surfaces \( k_{sa} \), topographic variability \( k_f \), and the effect of saltating grains during sediment transport \( k_r \). Flow-conservative variables (\( h \), \( hu \), and \( hv \)) at each cell
interface are calculated using the total variation diminishing (TVD) scheme (Toro, 2009) for cell-centered advection terms ($F_x$ and $F_y$).

Second, the local sediment transport rate $q_b$ (equation (19)) is estimated considering the local circumstance of sediment deposition, which is determined by the fraction of bedrock cover $P_c$ (equation (14) - (16)). The depth-averaged flow variables ($h$, $u$, $v$, and $k_s$) determine the threshold of incipient sediment motion ($\tau^*$ and $\tau_c^*$), then the local bedload transport rate $q_b$, grain saltation velocity $u_s$, and saturation volume of the bedload layer $V_{bc}$ are calculated.

Third, the sediment flux divergence updates the total sediment volume per unit area (equation (11)). Finally, the new alluvial layer thickness $\eta_a$ and the volume of bedload transport per unit area $V_b$ are determined by whether the bedrock is covered by alluvium or exposed (equation (12) - (13)).

3 Model validation

We set up two simulations to demonstrate the model’s ability to replicate observations in both alluvial and mixed bedrock-alluvial conditions. These benchmark laboratory flume experiments documented: (1) the development of alternate bars in fully alluvial channels (Lanzoni, 2000); and (2) flow and sediment transport in mixed bedrock-alluvial channels (Chatanantavet & Parker, 2008). Table 1 summarizes the conditions for each experiment.

3.1 Alternate bar formation in an alluvial channel

To demonstrate alternate bars in alluvial conditions, we simulate Run P1505 described in Lanzoni (2000). In the experiment, a straight, flow- and sediment-recirculating flume 1.5 m
wide, 1 m deep with a 55 m long test section was supplied with a steady water discharge of 30 l/s. The bed was composed of a uniform sand size of 0.48 mm and initially screeded flat. At the upstream end of the channel, a 16 m long, 5 m wide stilling basin was installed to ensure smooth and regular water and sediment discharge into the experimental channel. This experiment stopped as it reached equilibrium after 28 hours, when the water surface slope matched the bed slope. A summary of reach-averaged measurements is reported in Table 1. Alternate bars were formed with an average height of 7 cm, wavelength of 10 m, and celerity of 2.80 m/h.

Our simulation of Run P1505 is conducted with some modification of experimental conditions. First, a longer channel of 120 m was used to ensure the bar reached the equilibrium state before migrating out of the domain (Defina, 2003). Second, a small topographic bump 0.6 m long, 0.75 m wide, and 5 mm high was introduced near the upper boundary on the left side (looking downstream) to create an initial disturbance to induce free bar development because the arbitrarily distributed source of disturbances from the physical experiment was unknown. Numerical models generally require perturbations such as a topographic bump (Defina, 2003; Wu et al., 2011) or a bend upstream of the straight channel (Mendoza et al., 2017) to develop

<table>
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<th>Table 1. Summary of Experimental Conditions.</th>
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<td>Run</td>
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<td>P1505b</td>
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<td>2-B2c</td>
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</table>

a B is the channel width, L is the channel length, S is the channel slope, Qw is the water discharge, Qc is the sediment transport capacity, D is the grain size, H is the averaged flow depth, U is the averaged flow velocity, and t is the duration of the experiment.
b Run P1505 was performed by Lanzoni (2000).
c Run 2-B2 (q_w/q_c = 0.56) was performed by Chatanantavet & Parker (2008).
bars in uniform flow over flat-bed conditions. Third, the constant and uniform water discharge and sediment feed rate were imposed at the inlet.

3.2 Patterns of bedrock alluviation with limited sediment supply

The second experiment Run 2-B2 \( \left( \frac{q_s}{q_c} = 0.56 \right) \) (Chatanantavet & Parker, 2008), was carried out on a non-erodible bedrock surface in a 13 m long and 0.9 m wide straight, rectangular flume channel with a slope of 0.02. The bedrock bed was randomly abraded with a longitudinally averaged standard deviation of 2.4 mm, and the distance between the lowest and highest points of the profile is approximately 1 cm (Figure 1b and 3b in Chatanantavet & Parker, 2008).

The averaged values of initial experimental conditions are presented in Table 1. The representative hydraulic roughness of the bedrock surface was back-calculated from the Manning-Strickler relation under the flow conditions measured from the experimental results conducted in the channel slope of 0.0115. Initially, the bed was covered entirely with a 2 cm thick layer of uniform 7 mm sediment. The steady water discharge was 55 l/s, the constant sediment supply rate was 62 g/s, and the sediment transport capacity was estimated at 110 g/s for a wholly covered bed. The experiment stopped when the fraction of bedrock cover \( P_c \) reached a steady state whose value was approximately 0.59.

The numerical model simulation of this experiment uses a 0.9 m wide, 20 m long bedrock bed with randomly generated topographic perturbations with a standard deviation of 2.2 mm and a peak-to-peak distance of bed elevation of 0.9 cm (Figure 1). The roughness height due to skin friction (grain equivalent roughness height) used in the numerical simulation was back calculated from equation (4) using experimentally measured average flow conditions for each case of bare bedrock bed \( k_{sb} \) and alluvial bed \( k_{sa} \). In the channel fully covered with sediment \( H = 0.06 \) m
and $U = 1.02$ m/s, and in case of flow over bare bedrock bed $H = 0.05$ m and $U = 1.22$ m/s, hence $k_{sa} = 7$ mm and $k_{sb} = 3$ mm. The sediment supply to transport capacity ratio $q_s / q_c$ is 0.6, which means the steady sediment supply rate is 66 g/s assuming the sediment transport capacity of the alluvial bed is 110 g/s. A small perturbation was given by changing sediment distribution patterns at the upstream end of the channel to prevent sediment and bedform from washing out from the upstream end of the channel (e.g., Figure 14 in Inoue et al., 2016).

3.3 Simulations investigating impacts of roughness components on mixed bedrock-alluvial sediment dynamics

To investigate the importance of different roughness components on the model’s performance of predicting alluvial patterns, we conducted numerical simulations without bedform roughness or ripple factor under the same conditions of Run 2-B2. The form drag effect on the flow is removed by setting the bedform roughness to zero; hence the ripple factor is no longer influential. The effect of bed-form solely on sediment transport is removed by setting the ripple factor to unity.

We also conducted a simulation where the sediment transport roughness was set to zero, to explore how that component influences morphodynamic predictions in mixed bedrock-alluvial channels. For this simulation, the total roughness consisted of bedform and grain roughness, the latter of which was increased relative to the base scenario so that overall flow depths and velocities still matched experimental observations.

**Figure 1.** Plan view of bedrock topography. Colorbar scale indicates the detrended bed elevation.
4 Results

4.1 Simulation of alluvial alternate bars (Run P1505)

Figure 2 shows the plan views of the evolution of detrended bed surface elevation in Run P1505. The initial upstream bump \((t = 0 \text{ h})\) deflects flow and sediment transport. The sediment eroded from the bump forms the first bar and triggers the development of smaller bars downstream \((t = 2 - 4 \text{ h})\). The bars get larger and taller as they migrate downstream. When they reach equilibrium, their downstream migration and growth rate vary slowly and become stable. The initially flat alluvial bed, during the bedform formation and migration process, develops into a clear pattern of alternate bars. The initial disturbance gradually spreads out by decreasing height and stretching in the flow direction, prompting new bars to develop downstream (e.g., Figure 3 in Defina, 2003).

The longitudinal bed profiles and the bed elevation difference between the right- and left-side walls from the numerical results (Figure 3) can be compared to the measured data at the equilibrium state from Lanzoni (2000, Figure 1g) (Figure 4). Bar amplitudes are calculated as the difference between the right and left side of the bed elevation, measured 20 cm from each side wall (Figure 3a, b). The amplitude and wavelength are calculated using half the vertical and twice the horizontal peak-to-peak distances of the detrended profile. The celerity of migrating...
bars is calculated by tracking topographic peaks over time. The longitudinal bed profiles from numerical experiments exhibit highly ordered wave patterns. The axis bed profiles in the numerical simulation and experiment indicate the transversally maximum value of bed elevation.

**Figure 3.** Modeled longitudinal bed profiles along the left- and right-side wall and axis at 6 h (a) and 12 h (b) and the difference between right- and left-side bed elevation at 6 h (c) and 12h (d).

**Figure 4.** Measured (a) longitudinal bed profiles along the left- and right-side wall and (b) the difference between right- and left-side bed elevation at equilibrium (Figure 1g, Lanzoni, 2000).
As the calculated time changes in bar celerity (Figure 5a, b), wavelength (Figure 5c, d), and amplitude (Figure 5e, f) are small, and we consider the bed condition at near equilibrium. The mean characteristics of the bar (bar height, wavelength, and celerity) near equilibrium are compared and reported in Table 2. The computed bar wavelength, height, and celerity are approximately 14 m, 7 cm, and 3.2 m/s, respectively. The computed bar height and celerity show reasonably good agreement with observations of Lanzoni (2000), but the wavelength is slightly

**Figure 5.** Computed time evolution of bar characteristics; celerity, wavelength, and amplitude (from top to bottom) on the left (a, c, and e) and right (b, d, and f) side of the channel.

**Table 2.** Comparison of Bar Characteristics from Measured Data and Computed Results.

<table>
<thead>
<tr>
<th>Run</th>
<th>( C_b )</th>
<th>( \lambda_b )</th>
<th>( H_b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1505</td>
<td>2.8</td>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>Computed</td>
<td>3.2±0.5</td>
<td>14±2</td>
<td>7±1</td>
</tr>
</tbody>
</table>
overestimated. However, the computed alternate bar wavelength is approximately nine times the channel width showing good agreement with Defina’s (2003) numerical result and laboratory data of alternate bar wavelength prediction (Ikeda, 1984).

4.2 Simulation of mixed bedrock-alluvial experiment (Run 2-B2)

Figure 6 shows a series of numerical results illustrating the time evolution of bed configuration for the mixed bedrock-alluvial experiment (Run 2-B2) commencing from a 2 cm thick alluvial cover ($t = 0$ h). The non-uniform sediment distribution, feeding more sediment at

\[ Q_s = 97 \text{ g/s}, \quad q_s/q_c = 0.88, \quad \text{and} \quad P_c = 0.78 \] (Chatanantavet & Parker, 2008 (personal communication)). The channel is 0.9 m wide, 13 m long, water and sediment flow from left to right, and light and dark areas correspond to bedrock and sediment, respectively.

**Figure 6.** Simulated bed evolution of Run 2-B2. Colorbar shows the thickness of the sediment cover, and white areas correspond to the exposed bedrock surface.

**Figure 7.** Time evolution of bedrock exposure Run 2-B4 at times are 1, 4, and 7 hour from top to bottom: $Q_s = 97 \text{ g/s}$, $q_s/q_c = 0.88$, and $P_c = 0.78$ (Chatanantavet & Parker, 2008 (personal communication)). The channel is 0.9 m wide, 13 m long, water and sediment flow from left to right, and light and dark areas correspond to bedrock and sediment, respectively.
one side than the other, causes flow deflection induced by a laterally sloping bed at the upstream
end ($t = 0.5$ h). At an early stage, this disturbance quickly erodes sediment on the right side of the
channel down to the bedrock bed and creates pools and bars downstream. Then the exposed
bedrock bed area, indicated as a white area, increases as the pools deepen and bars grow taller ($t
= 1$ h). The sediment on the bed forms a strip of sediment shifting from one side to the other side
of the channel through time while maintaining a consistent fraction of bedrock cover. Similar
patterns were observed in the Chatanantavet and Parker (2008) experiments; for example, Figure
7 shows the time evolution of bedrock exposure from Run 2-B4 (Table 1 in Chatanantavet and
Parker, 2008) with $Q_s = 97$ g/s, $q_s/q_c = 0.88$, and $P_c = 0.78$ at equilibrium phase. Again, a
continuous band of sediment forms comparable to Figure 6. However, the sediment cover is
more expansive, and the bed sediment shifting from one to the other side is less sensitive because
of the higher sediment supply rate than what was provided in Run 2-B2 modeled here.

Figure 8 shows the computed flow variables, bed topography, and sediment transport
capacity at equilibrium ($t = 5$ h). The flow depth and velocity over the exposed bedrock area are
higher than on the alluvial bed. However, calculated shear stress is lower in exposed bedrock
areas due to the smaller roughness and higher flow depth. The critical dimensionless shear stress
is generally higher on the alluvial bed but smaller over the lee side of the bedforms. The
sediment transport capacity tends to be higher over the exposed bedrock surface and stoss side of
the bedform and lower on lee slopes.

Figure 9 shows the time evolution of channel averaged fraction of bedrock exposure
$F_c = 1 - P_c$, alluvial thickness, relative Shields parameter, and bedload transport rate. A ratio of
alluvial coverage of bed surface in the area of interest determines the fraction of bedrock covered
with sediment $P_c = A_c/A_i$ (Johnson, 2014), where $A_c$ is the area covered with sediment and $A_i$
is the total area of interest, or numerically $P_c = \sum P_{ci} / N$, where $P_{ci}$ is the local fraction of bedrock cover (equation 8) and $N$ is the total number of cells in the domain. The fraction of bedrock exposure is initially zero when the bed is entirely covered by sediment and quickly converges toward a near-equilibrium state where $F_e$ is 0.4 at $t = 1$ h (Figure 9a). The numerical result of the degree of bedrock exposure presents a similar value of $F_e$ to the experimental observation. The averaged sediment cover thickness over an alluvial area varies little in the range

![Figure 8](image)

**Figure 8.** Plan view of (a) alluvial thickness, (b) flow depth, (c) flow velocity, (d) dimensionless shear stress, (e) critical Shields parameter, and (f) dimensionless bedload transport capacity at $t = 5$ h. Colorbars indicate the scale of computed values, respectively, and white areas correspond to (a) the exposed bedrock surface and (e) zero sediment transport rate.
of a single grain diameter (Figure 9b). The relative Shields parameter $\tau^*/\tau^*_c$ (Figure 9c) and bedload transport rate (Figure 9d) decrease from an initially higher value to approximate equilibrium and vary around 1.2 and 70 g/s, respectively.

The comparison between the computed and measured quantities available from experimental observations is presented in Table 3. The agreement is reasonably good in predicting the fraction of bedrock cover, but higher flow depth and lower velocity in the experiment result in a lower Froude number and lower dimensionless shear stress. The model predicted lower shear stress over the bare bedrock areas than the alluvial areas because the bedform developed during the simulation. The alluvial bars form where flow depth is low and velocity is high, and pools exist over the bare bedrock area with deep flow depth and low velocity.
Table 3. Summary of Numerical Results

<table>
<thead>
<tr>
<th>Run</th>
<th>$P_c$</th>
<th>$H$ (cm)</th>
<th>$U$ (m/s)</th>
<th>Fr</th>
<th>$\tau^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-B2 (C&amp;P)&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.59</td>
<td>5.5±1.5</td>
<td>1.11</td>
<td>1.51</td>
<td>0.11</td>
</tr>
<tr>
<td>2-B2&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.60</td>
<td>6.0</td>
<td>1.00</td>
<td>1.32</td>
<td>0.10</td>
</tr>
<tr>
<td>2-B2 (covered zones)&lt;sup&gt;c&lt;/sup&gt;</td>
<td></td>
<td>5.6</td>
<td>0.95</td>
<td>1.31</td>
<td>0.10</td>
</tr>
<tr>
<td>2-B2 (exposed zones)&lt;sup&gt;d&lt;/sup&gt;</td>
<td></td>
<td>7.5</td>
<td>1.07</td>
<td>1.26</td>
<td>0.08</td>
</tr>
</tbody>
</table>

<sup>a</sup> Experimental results from Chatanantavet and Parker (2008).
<sup>b</sup> Reach averaged numerical results.
<sup>c</sup> Reach averaged numerical results only over the alluviated zones.
<sup>d</sup> Reach averaged numerical results only over the exposed bedrock zones.

5 Discussion

The present study shows the results that bed topography comparable to experimental observations from (1) free bar formation in a mild slope alluvial channel with fine grains (Lanzoni, 2000) and; (2) alluvial pattern in the mixed bedrock-alluvial channel in a steep slope channel with coarse grains (Chatanantavet & Parker, 2008). These suggest that the model can simulate complex flow and sediment transport to gain insight into mechanisms of the bedform development and alluviation patterns in mixed bedrock-alluvial channels.

The discrepancies between the numerical and experimental results are mainly associated with the initial and boundary conditions, which are not successfully reflected in the numerical models, such as the size of the initial disturbances and patterns of flow and sediment distribution at the channel inlet.
5.1 Discussion on ripple factor and form drag

Only a few studies have attempted to simulate sediment transport in a straight mixed bedrock-alluvial channel using a 2D numerical model (Inoue et al., 2016; P. A. Nelson & Seminara, 2012). These models estimated bed resistance to the flow using the grain roughness and bedrock surface irregularity which is applicable to the plane bed. Our model includes the effect of bed-form generation in roughness estimation since alternate bars often form in mixed bedrock-alluvial channels (Chatanantavet & Parker, 2008; Inoue et al., 2016; P. A. Nelson & Seminara, 2012). The hydraulic roughness height of alluvial bed changes according to the size of bedforms (i.e., dunes and ripples) (Engelund, 1977; Raudkivi, 1997; van Rijn, 1982, 1984; Vanoni & Hwang, 1967; Wiberg & Nelson, 1992).

Figure 10 shows the time evolution of bed topography with the same condition applied in Run 2-B2 without including the bedform effect in the roughness calculation, and hence the ineffective ripple factor. This simulation undergoes sediment washing out from the upstream end of the channel while forming a bar-like bedform downstream. This result is comparable with the results provided by Inoue et al. (2016).

Figure 10. The time evolution of bed exposure for run 2-B2 without form drag effect ($k_f = 0$). Colorbar means alluvial cover thickness, and white area is exposed bedrock surface.
The prediction of bedload transport is based on the corrected dimensionless shear stress for bedform roughness with a ripple factor (Meyer-Peter & Muller, 1948). The value of the ripple factor has not been reported explicitly but empirically estimated as a ratio of grain shear stress to the total bed shear stress (Ribberink, 1987; Vermeer, 1986). Nevertheless, the ripple factor is often applied to sediment transport models to generate bedforms in alluvial channels with a constant value of less than 1 (Defina, 2003; Van der Meer et al., 2011).

Figure 11 shows the plan view of the bed topography of Run 2-B2 without the dimensionless shear stress correction by using a ripple factor equal to 1 in equation (16). The upstream sediment erodes slower than the bed evolution in Figure 10, and the morphodynamic process has enough time to form a longitudinal sediment strip because the form drag generates higher flow resistance. However, larger dimensionless shear stress due to the absence of the ripple factor in the supply-limited channel causes a thin layer of sediment at the upstream end and a narrow strip of sediment. The strip of sediment is cut off at the narrowed section, and the sediment is eventually washed out.

Figure 11. The time evolution of bed exposure for Run 2-B2 without ripple factor ($\mu = 1$). Colorbar indicates alluvial cover thickness, and the white area is exposed to the bedrock surface.
Comparing Figures 6 and 10-11, it can be seen that introducing both form drag to the roughness and ripple factor of the shear stress in bedload transport is critical in developing bedforms and persistent alluviation in bedrock channels where the sediment supply is less than the capacity of the channel. Results from previous models attempting to replicate these experiments (Inoue et al., 2014) show alternate bar development over the alluvial surface, but sediment supply less than the transport capacity decreases the thickness of the alluvial layer leading to sediment washing out from upstream and exposure of the entire bedrock.

The roughness component for form drag effectively increases drag and reduces flow velocity, whereas the ripple factor reduces shear stress in bedform-developed areas. Unlike the form drag acting on entire bedform fields, skin friction is responsible for the local bed surface roughness only. The smaller surface roughness over the exposed bedrock area than over the alluvial area produces higher shear stress. Therefore the model absent of form drag and ripple factor predicts a higher sediment transport rate as the fraction of sediment cover decreases in limited sediment supply conditions, resulting in a total washout of the sediment from the channel.

5.2 The impact of sediment transport roughness

Our model explicitly accounts for both the grain roughness and roughness produced by moving particles (Dietrich, 1982; Grant & Madsen, 1982; Smith & McLean, 1977; Wiberg & Rubin, 1989). To explore the importance of accounting for each of these components individually, we conducted a simulation where we ignored sediment transport roughness and increased the grain roughness to compensate for reduced bed roughness caused by the absence of sediment transport roughness.
Figure 12 shows the time evolution of bed topography for this simulation of Run 2-B2, excluding sediment transport roughness. A strip of sediment forms shifting from one side to the other side of the channel momentarily ($t = 0.5 \, \text{h}$), but then washes out from the upstream end of the channel ($t = 2 \, \text{h}$), with a small residual alluvial patch persisting only in a topographic low area in the bedrock ($t = 4 \, \text{h}$). The flow resistance decreases as bedrock gets exposed because of the underlying linear relationship between the fraction of bedrock cover and grain roughness. Unlike grain roughness, the sediment transport roughness is larger where grain movements actively take place. The sediment transport roughness tends to be higher over the bedrock surface partially covered with sediment because the sediment transport rate over the smoother bedrock surface, where the flow velocity is faster, is higher than over the rougher alluvial surface. The sediment transport roughness contributes to the formation of alluvial patches in the bedrock channel by reducing grain entrainment over the mixed bedrock-alluvial surface. These results indicate that separate accounting of grain and sediment transport roughness produces morphodynamic predictions that better match observations of persistent alluvial patterns in mixed bedrock-alluvial channels.

Figure 12. The time evolution of bed exposure for Run 2-B2 without sediment transport roughness from flow resistance ($k_t = 0$). Colorbar indicates alluvial cover thickness, and the white area is exposed to the bedrock surface.
6 Conclusions

In this study, we have developed a two-dimensional morphodynamic model to explore bar formation and migration in an alluvial channel and sediment transport mechanisms in a mixed bedrock-alluvial channel without sufficient sediment supply. Comparisons of model predictions with experimental observations from the free bar test show that the model predicts bed morphology and bedform development similar to the observations of the flume experiment. However, some discrepancies raise the need for proper tuning of model parameters and initial bottom perturbation.

The model predicts the flow field and sediment distribution patterns in mixed bedrock alluvial channels reasonably well. Numerical experiments show that the inclusion of bedform roughness and a shear stress correction for near-bed sediment transport is critical to be able to replicate the alluvial patterns over bare bedrock observed in flume experiments. The evolving interactions between the alluvial and bedrock bed surface, flow field, and sediment transport simultaneously modify the degree of sediment cover and bed topography. The numerical model presented in this study captures the behavior associated with the bedrock alluviation process and can be used to extend its applicability to various flow and sediment supply conditions with different channel slopes and antecedent topography.

Future work could explore the mechanisms of sediment pattern formation in mixed bedrock-alluvial rivers and characterize the effects of the channel slope, initial sediment cover thickness, difference between the grain and bedrock roughness, and bedrock configuration.

Acknowledgments
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Open Research

The code demonstrating the 2D morphodynamic model and numerical data of test cases shown herein are archived at https://github.com/rcemorpho/morph2d.

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