STOCHASTIC INVERSION WITH MAXIMAL UPDATED DENSITIES FOR STORM SURGE WIND DRAG PARAMETER ESTIMATION

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The QoI Map

The main problem with using the SIP framework to solve PIPs is data assimilation - how do we incorporate new data to solve the PIP and reduce uncertainty (i.e. variance) in our parameter estimate? We propose the usage of Data-Constructed QoI Maps to functionally assimilate data and reduce variance as more data is collected. Assume we have:

- M₁ devices collecting data over space and time, each taking Nᵢ measurements.
- Arbitrary ordering of these n = ∑Nᵢ Nᵢ data points \( x_j \) that equals the i'th measurement datum, \( M_{\lambda_i} = (\lambda_1, \lambda_2, \ldots, \lambda_i) \) is the i'th measurement sample.

We can define a matrix \( X \in \mathbb{R}^{n×m} \) of Z-scored residuals for a sample set \( s \) samples component-wise as

\[
x_i = \frac{x_i - \mu}{\sigma}
\]

Letting \( p^{(\ell)} \) be the \( \ell \)th principle component of \( X \), we define Q₁-copy component-wise as

\[
Q^{(\ell)}(x_j) = \sum_{i=1}^{n} \frac{\langle x_j, p^{(\ell)} \rangle}{\|p^{(\ell)}\|^2} p^{(\ell)}
\]

Properties of Q₁

- Ordering of measurements in \( X \) is irrelevant, since PCA does not depend on column order.
- It can be shown that for the Q₁-copy map, the observed distribution \( p^{(\ell)} \) is always a stationary \( N(0, 1) \) distribution no matter how many data points collected.
- We only take up to the first \( m \) components that capture a user-specified percentage of variance in the original data set \( X \). We expect the number of components, \( m \), to be equal to the dimension of our parameter space, \( n \), however if data is not sensitive to all parameters present in our inverse problem, this may not be the case. In these examples, we turn to the diagnostic \( E(x) \) as an important measure of the quality of the updated density and thus the reliability of \( N^{(0)} \).

ADCIRC Problem Set-Up

Shinnecock Inlet Simulated Extreme Event

- Used well known test mesh based on the Shinnecock Inlet on the Outer Barrier of Long Island, NY, USA.
- External forcing using tides, constant air pressure of 1013 millibars, and winds computed from a 0.25° hourly CFISv2 10-m wind fields for a period of 16 days (29 December 2017 - 31 January 2018).
- Winds are modified for the purposes of the numerical experiment to simulate a more extreme (Category 4) event, with winds scaled radially down to zero from the point of interest.
- Modified ADCIRC to include a parameterized form of the Garnett wind drag law, with a slope (\( \lambda \)) and a cut-off (\( \lambda_0 \)) parameter.

\[
C_D = \min \left[ \frac{1}{2}, \frac{3}{2}, \frac{1}{2} \lambda \right]
\]

- Constructed a "true" signal by collected water elevation data at an artificial recording station and populated each measurement with i.i.d. \( N(0, \sigma^2) \) noise, using \( \sigma = 0.05 \).
- Goal: Estimate wind drag parameter values that produced the "true" signal.

Conclusions

- \( \text{QoI}_{\text{pred}} \) map can effectively estimate both parameters provided the data used in the map exhibit sensitivity to those parameters.
- The diagnostic \( E(x) \) gives us a specific metric to determine the quality of reconstructed distributions for each parameter.