Couple effect of Joule heating and multiple slips on an unsteady electromagnetic nanofluid towards a stagnation point: A statistical inspection

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Couple effect of Joule heating and multiple slips on an unsteady electromagnetic nanofluid towards a stagnation point: A statistical inspection

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Abstract:
This research is focused on the examination of an unsteady flow of an electromagnetic nanofluid close to a stagnation point over an expanded sheet kept horizontally. The Buongiorno’s nanofluid model is revised with the combined influence of the externally applied electric and magnetic fluxes. Moreover, the underneath surface offers multiple slips into the nanofluid flow. The leading PDE’s are renovated to the non-linear ODE’s by the assistant of similarity transformations. Thus, the outcomes are received numerically by using the RK-6 with Nachtsheim-Swigert shooting technique. The enlistment of the outcomes for the momentum, energy, and concentration profiles along with the skin-friction coefficient $C_p^*$, Nusselt number $N_u^*$, and Sherwood number $S_h^*$ for several parametric values are presented in a graphical and tabular form and discussed in detail. The variation of streamlines with respect to the unsteadiness parameter is also recorded. Statistical inspection reveals that the flow parameters are highly correlated with the wall shear stress, wall heat and mass fluxes. Findings indicate that the escalation of electric flux tries to intensify the hydrodynamic boundary layer meanwhile the magnetic flux assists to stabilize the growth by reducing it for both the steady and unsteady flow patterns. Influence of velocity slip parameter $\xi$ from 0.0 to 1.5 causes the reduction in $N_u^*$ by 16.98% for steady flow while 60.27% for time dependent flow case.

Keywords:
Nanofluids; Stagnation point flow; EMHD Flow; Unsteady flow; Slip effects

2000 Mathematics Subject Classification: 76W05, 76V05
Introduction:

A fluid with the blend of metallic or non-metallic nanoparticles or nanotubes (diameter between 1-50nm) within it is commonly recognized as Nanofluids. The idea of mixing nanoparticles with an ordinary base fluid to enhance its thermal conductivity, long term stability, homogeneity and minimal clogging rate was pioneered by Choi [1]. Later, Buongiorno [2] designed a remarkable nanofluid model with the consideration of the moment of the nanoparticles, by incorporating the Brownian motion, thermophoresis, inertia and nanoparticles volume fraction, which is still helping the researchers who are studying with nanofluids. Owing to its widespread flow features, in recent days ample applications of nanofluids can be found in the fields of nano-drug delivery systems, biomedical industry [3], advanced solar energy systems [4], automobile industry [5], advanced microelectronics cooling [6], geothermal energy systems [7], nuclear power plants [8], etc. Recent developments in nanofluid drifts over different geometrical structures embedded in absorbent medium can be found in the study of Kasaeian et al. [9]. Nadeem et al. [10] obtained the numerical solution of a non-Newtonian nanofluid drift incorporating Brownian diffusion and thermophoresis effect. nanoparticles Diameter effect on Magnetohydrodynamic nanofluid flow under the course of melting heat transportation was revealed by Giri et al. [11]. Recently, Chakraborty et al. [12] demonstrated heat transportation phenomena of a hybrid nanofluid flow crosses an exponentially stretched sheet drenched in a non-Darcy absorbent medium.

The research on the stagnation point flow has inward a substantial notice owing to its immense applications in aerodynamic engineering and polymer industries. Such applications include convection cooling process of metal sheets, production of gas turbine blades, manufacturing artificial fibbers, polymer sheets extrusion and many more hydrodynamic mechanisms. The occurrence of Stagnation point flow is owing to the stretching surface and a jet-like flow impinges on the surface either vertically or obliquely. Orthogonally impinges viscous fluid flow close to a stagnation point was first coined by Hiemenz [13]. Mustafa et al. [14] renovated the research of Mahapatra et al. [15] with the consideration of Buongiorno’s nanofluid model and produced analytical solution of a stagnation point flow of a nanofluid passes a linearly expanded surface. Chakraborty et al. [16] derived a semi analytical solution of an orthogonally impinges MHD Jeffrey nanofluid flow close to a stagnation point with melting heat effects. Halim et al. [17] reported in their study that the amplification of stagnation parameter aids higher heat transportation rate for nanofluid flow under both actively and passively controlled normal mass fluxes. Recently, Yahaya et al. [18] examined the stability of a hybrid nanofluid drift near a stagnation point of a convectively heated disc. A computational framework on Maxwell nanofluid double diffusive flow towards a stagnation point was reported by Vijay and Sharma [19]. More studies in this regard can be found in the researches [20-24].
Electromagnetohydrodynamics (EMHD) is an extensive area of research owing to its plentiful applications in microfluidics and nanofluidics, especially for flows through nano-channels, fabrication of microelectronic chips, controlling electrolyte flows, etc. In many mechanical systems electromagnetic body forcers are widely used to control the flow features by activating a boundary layer. For classical magnetohydrodynamic (MHD) flow control, the drift of electrically conducting fluids like semiconductor melts of $\sigma \sim 10^6 \text{ S/m}$ and liquid metals, can be influenced by an externally applied magnetic flux of potency about $\sim 1 \text{T}$. Whereas, for weaker electrically conducting materials like saltwater $\sigma \sim 10 \text{ S/m}$, the quantity of induced current owing to the externally applied magnetic flux is very less, therefore the requirement of an external field is necessary for the effectual control of such flows. In EMHD flow, the pressure gradient originated by the boundary layer flow is influenced by a wall-parallel Lorentz force, caused by the combined externally applied electric and magnetic field. Thus a stabilize boundary layer flow with a restricted growth can be easily obtained. Ayub et al. [25] inspected the EMHD nanofluid drift passing over a Riga plate and they received a velocity growth with the enhancement of external magnetic flux. Mabood et al. [26] observed the double stratification effect on EMHD flow of a non-Newtonian micropolar nanofluid through a needle with Robinson’s condition. Mumraiz et al. [27] deliberated that the momentum profile of $\text{Al}_2\text{O}_3$-$\text{Cu}/ \text{H}_2\text{O}$ augments with the externally applied electric field for an EMHD flow. It is found from the study of Daniel et al. [28] that in the attendance of the magnetic flux and thermal radiation, the momentum boundary layer width grows rapidly with the escalation of external electric field. The combined upshot of electric and magnetic flux postpone the rate of progression of a hybrid nanofluid in the direction of a stagnation point was reported by Zainal et al. [29]. Khashi’ie et al. [30] deliberated an unsteady flow feature with dual solutions of an EMHD flow of a hybrid nanofluid over Riga plate. Current developments invoking the study of EMHD flow of different nanofluid models can be found in [31-35].

Earlier researchers of fluid dynamics are mainly concerned with the usual no-slip conditions at the solid boundaries. But with the advancement of technologies, later it was understood that this no-slippage phenomena is not valid for all cases of fluid flows, especially for the drifts through micro-electro-mechanical-systems (MEMS), where the characteristic length of the systems are very thin or the flow pressure is insignificant. Also, when the flows occur over some lubricated surfaces with higher velocity then slip cannot be neglected. Navier [36] observed that for such flow situations, the tangential component of fluid velocity at the wall is directly proportional to the tangential stress, and he developed a set of boundary conditions to understand these slippage characteristics. Of late, Martin and Boyd [37] extended the work of Navier [36] for Blasius boundary layer flow problems. Das et al. [38] reported in their research that the augmentation of velocity slip parameter escalates the thermal boundary layer width faster for $\text{Al}_2\text{O}_3$-$\text{Cu}/ \text{H}_2\text{O}$. Hydrodynamic slip and temperature jump effects on CNT-nanofluid flow over an expanded sheet was observed by Kundu et al. [39]. A numerical
computation of partial slip effects on Buongiorno nanofluid model through a rotating disc was
analyzed by Mustafa [40]. Abbas et al. [41] revealed in their research that the Maxwell nanofluid
velocity near the boundary region boosts with the second order hydrodynamic slip effects in
attendance of Soret and Dufour effects. The impression of multiple slips on different nanofluid flows
with exciting flow characteristics can be obtained in the researches [42-46].

Inspired by the new trends of researches in the field of stagnation point flow; here we
endeavour to analyze comparative flow features between a steady and unsteady
electromagnetohydrodynamic flow of a nanofluid close to a stagnation point with multiple slip effects
using Buongiorno’s [2] model. The mutual effect of transverse electric and magnetic flux is imposed
normally to the course of the flow, which induces wall-parallel Lorentz force within the boundary
region. The energy equation experiences Joule heating for the presence of external electric and
magnetic flux. The governing equations are renovated into non-linear ODE form with a suitable
similarity transformation then tackled numerically by RK-6 with Nachtsheim-Swigert shooting
technique. The numerical results for momentum, energy and concentration under the action of
different pertinent parameters are vividly presented with graphs and Tables. Comparisons with the
earlier researches [14, 15, 20] under the identical presumptions are also produced to substantiate the
accuracy of our present code; and they are found in excellent harmony. As per our best knowledge, no
such research article yet has not been published, hence our problem is new.

2. Flow Analysis:

2.1 Model assumptions and problem formulation:

An electromagnetohydrodynamic (EMHD), time reliant, 2D, viscous, incompressible, laminar
drift of a nanofluid, close to a stagnation point over a horizontal expanding sheet is posted here. The
physical replica and the co-ordinate system are framed as the $x$-axis along the surface and the $y$-axis is
kept normally to the surface, which is portrayed in Fig. 1. A jet like flow impinges orthogonally to the
surface near the stagnation point and then spread out. The sheet is elongated with a stretching velocity
$U_w(x,t) = \frac{cx}{(1-bt)}$, while the free stream velocity of the nanofluid is $U_e(x,t) = \frac{ax}{(1-bt)}$. Here,
c, a, b are the constants with $b$ represents the unsteadiness of the flow with dimension (time)$^{-1}$. It is
noteworthy that for time dependent drift $bt < 1$ with the presumption that $b > 0$ accelerates the outer
potential flow, $b < 0$ accelerates the inner potential flow and $b = 0$ correlates the steady flow. Also,
for $t < 0$ there is no flow and the flow starts at $t = 0$. 


An external transverse electric flux $\vec{E} = (0, 0, -E)$ and magnetic flux $\vec{B} = (0, B, 0)$ take the special form $B = \frac{B_0}{\sqrt{1-bt}}$ and $E = \frac{E_0}{\sqrt{1-bt}}$ [28, 29] are imposed orthogonally to the sheet, with $B_0, E_0$ are the constant magnetic and electric potencies. This combined electro-magnetic flux induces a wall-parallel Lorentz force and Joule heating within the flow, which aids to stabilize the boundary layers growth. The fluid temperature $T(x, y, t)$ and the nanoparticles concentration $C(x, y, t)$ are imagined to be constant at the wall with the form $T_w$ and $C_w$ respectively, meanwhile the ambient values are $T_\infty$ and $C_\infty$ with the consideration that $T_w \gg T_\infty$ and $C_w \gg C_\infty$. More basic assumptions are also perceived to form the governing equations of the flow are as follows:

- Buongiorno’s [2] nanofluid model is considered with the combined effects of Brownian motion and thermophoresis of nanoparticles.
- Magnetic Reynolds number is thought to be insignificant; hence the induced magnetic flux becomes negligible in contrast to externally applied magnetic flux.
- The sizes of the nanoparticles (diameter < 50 nm) are uniform and spherical, and they do not agglomerate.
- Nanoparticels and the base liquid are in same thermal state and $c$ are incorporated.
- There is no add-on chemical reaction.

With the aforementioned suppositions and employing the usual boundary layer approximations to the governing equations of continuity, momentum, energy and nanoparticles concentration for an EMHD boundary layer flow towards a stagnation point becomes [14, 15, 20, 28, 29, 31]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial U_e}{\partial t} + U_e \frac{\partial U_e}{\partial x} + v \frac{\partial^2 u}{\partial y^2} + \frac{\sigma_f}{\rho_f} \left( EB - B^2 \right) \left( u - U_e \right)$$

(2)

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_f \frac{\partial^2 T}{\partial y^2} + \tau \left[ D_b \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_f \frac{\partial T}{\partial y}}{T_e} \right] + \frac{\sigma_f}{\rho C_p} \left( (u - U_e) B - E \right)^2$$

(3)

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_b \frac{\partial^2 C}{\partial y^2} + \frac{D_f \frac{\partial T}{\partial y}}{T_e} \frac{\partial^2 T}{\partial y^2}$$

(4)