Analytical beampattern synthesis for symmetric nonuniform array based on superposition principle

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Abstract

In this letter, an analytical method for the beampattern synthesis of symmetric nonuniform array is proposed. This method consists of two steps. In the first step, it acquires a real symmetric excitation by the convex optimization method to attain a pencil beam. In the second step, it superposes the pencil beams pointing in different directions to synthesize the prescribed beampattern. Numerical results are provided to verify the effectiveness of the proposed method.
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Introduction: Beampattern synthesis of array antenna has a wide range of application scenarios, such as radar, remote sensing, satellite communications, etc. [1-3]. In recent years, researchers have proposed many effective algorithms to synthesize the expected beampattern. These methods can be roughly divided into three categories. The first category is based on the convex optimization [4-6]. Fuchs proposed two methods. One is based on sequential convex optimization [4], the other one is based on semidefinite relaxation [5]. Both methods can synthesize the prescribed beampattern, such as flat-top pattern and cosecant squares pattern. Hu et al proposed one method based on magnitude least squares and semidefinite relaxation (SDR) [6]. It reduces the computation through nonuniform angular sampling. The second one focuses on the evolutionary algorithms [7, 8]. Boeringer et al. and Jiang et al. proposed two methods based on particle swarm optimization (PSO) [7] and genetic algorithm (GA) [8]. Both methods can also attain various beampatterns. However, the above-mentioned methods are time-consuming. The third one focuses on the analytical methods. Three analytical methods based on superposition principle are proposed [9-11]. They first attain some pencil beams using the Taylor method, and then superpose them to achieve the shaped beampattern. These methods can synthesize the beampatterns in real time. And they are very attractive in engineering.

With the development of antenna technique, the nonuniform arrays have been more attractive [12]. They can use fewer elements to realize similar performance with uniform array. That means the cost can be reduced. Besides, they own more degree of freedom, leading to better performance in some ways. Here, we only consider a special nonuniform array, namely symmetric nonuniform array, and talk about the beampattern synthesis for it. Unfortunately, the performance of all above-mentioned methods will decrease for the symmetric nonuniform arrays. The convex optimization methods and evolutionary algorithms require large computation. Especially, the analytical methods will fail to synthesize the expected beampattern, because the Taylor method is no longer suitable for the symmetric nonuniform arrays.

To facilitate the beampattern synthesis of the symmetric nonuniform arrays, we propose a novel method. This method first acquires the real symmetric excitation by the convex optimization method. Then, it attains some pencil beams pointing in different directions and superposes them to synthesize the prescribed beampattern. It is worth mentioning that the real symmetric excitation acquired by the convex optimization method can be stored in advance, the following process can be implemented in an analytical way, and the method is very attractive in engineering. Numerical results are provided to verify the effectiveness of the proposed method.

Notations: Vectors and matrices are represented by boldface lowercase and upper letters, respectively. The transpose and the conjugate transpose (Hermitian) operators are denoted by \( \mathbf{a}^\top \) and \( \mathbf{a}^\dagger \). \( i = \sqrt{-1} \), and \( | \cdot | \) means the magnitude of a complex number. Finally, \( \circ \) represents the Hadamard product of two vectors.

Symmetric nonuniform linear array: Consider a symmetric nonuniform linear array consisting of \( N \) elements, and the element positions can be denoted by \( x_0, x_1, \ldots, x_{N-1} \), with the following relationship:

\[
x_n = x_{N-1-n}, \quad n = 0, 1, \ldots, N-1
\]  

And its array factor can be written as:

\[
S(\theta) = \mathbf{w}^\dagger \mathbf{a}
\]

where \( \mathbf{a} = \begin{bmatrix} e^{j2\pi x_0 \sin \theta}, e^{j2\pi x_1 \sin \theta}, \ldots, e^{j2\pi x_{N-1} \sin \theta} \end{bmatrix}^\dagger \), \( \mathbf{w} = \begin{bmatrix} w_0, w_1, \ldots, w_{N-1} \end{bmatrix}^\dagger \) and \( \lambda \) is the wavelength.

Notably, if the excitation is conjugate symmetric, i.e.,

\[
w_n = w_{N-1-n}, \quad n = 0, 1, \ldots, N-1
\]

the array factor will be real-value.

\[
S(\theta) = w_0 e^{j2\pi x_0 \sin \theta} + w_1 e^{j2\pi x_1 \sin \theta} + \ldots + w_{N-1} e^{j2\pi x_{N-1} \sin \theta}
\]

Then, \( \text{Re}(w_n) = \text{Re}(w_{N-1-n}) \), \( n = 0, 1, \ldots, N-1 \)

\[
\text{Im}(\mathbf{w}) = 0
\]

Pencil beam synthesis with real symmetric excitation: For the uniform array, the Taylor method can be used to attain the pencil beam with a real symmetric excitation. However, this method is not suitable for the symmetric nonuniform array. To address this issue, we propose a method based on convex optimization. First, we can formulate the following optimization problem:

\[
\min_w \eta \quad \text{s.t.} \quad \mathbf{w}^\dagger \mathbf{a}_0 = 1
\]

\[
|\mathbf{w}^\dagger \mathbf{a}_0| \leq \eta, \quad \theta \in \Theta_M
\]

\[
\text{Re}(w_n) = \text{Re}(w_{N-1-n}), \quad n = 0, 1, \ldots, N-1
\]

\[
\text{Im}(\mathbf{w}) = 0
\]

where

\[
\mathbf{Hw} = 0
\]

Moreover, the original problem can be rewritten as:

\[
\min_w \eta \quad \text{s.t.} \quad \mathbf{w}^\dagger \mathbf{a}_0 = 1
\]

\[
|\mathbf{w}^\dagger \mathbf{a}_0| \leq \eta, \quad \theta \in \Theta_M
\]

\[
\mathbf{Hw} = 0
\]

\[
\text{Im}(\mathbf{w}) = 0
\]

Obviously, the problem above is convex, and the global optimal solution can be attained via CVX [13].

Beampattern synthesis based on superposition principle: After the real
symmetric excitation $\mathbf{w}_n$ and the corresponding pencil beam $S_n(\theta)$ are attained, the pencil beams pointing in different directions can be achieved with the Hadamard product of the real symmetric excitation and the complex vector $\Theta = [e^{j\theta_1}, e^{j\theta_2}, \ldots, e^{j\theta_N}]^T$. It is worth mentioning that the newly generated excitation is conjugate symmetric, leading to that the corresponding pencil beam is real-valued. Moreover, the real-valued pencil beams can be superposed to synthesize the prescribed beampattern.

Suppose that there are $N$ pencil beams $S_1(\theta), \ldots, S_N(\theta)$, pointing to the different angles $\theta_1, \ldots, \theta_N$. The prescribed beampattern can be considered as the sum of all the pencil beams with a specific weight $b_1, \ldots, b_N$. And the synthesized beampattern can be represented as:

$$G(\theta) = b_1 S_1(\theta) + b_2 S_2(\theta) + \ldots + b_N S_N(\theta)$$

(9)

Notably, the different angles $\theta_1, \ldots, \theta_N$ can be obtained by sampling the mainlobe region, and the weight $b_1, \ldots, b_N$ can be determined by the amplitude of the prescribed beampattern at the $\theta_i$. Finally, the corresponding excitation is

$$\mathbf{w} = b_1 \mathbf{w}_1 \odot \Theta_1 + b_2 \mathbf{w}_2 \odot \Theta_2 + \ldots + b_N \mathbf{w}_N \odot \Theta_N$$

(10)

It is worth mentioning that the real symmetric excitation $\mathbf{w}_n$ acquired by the convex optimization method can be stored in advance, the following process can be implemented in an analytical way, and the method is very attractive in engineering.

Numerical results: To verify the effectiveness of the proposed method, we have done some contrast experiments. First, the geometry of the symmetric nonuniform array is generated. To simultaneously avoid the grating lobe and the mutual coupling of the neighboring elements, the element spacing is limited to 0.4 λ to 0.6 λ. The generated geometry is shown as Fig. 1. Then, the beampattern synthesis is conducted for the generated array, including the pencil beam, the flat-top beampattern, and the cosecant squares beampattern. All simulations are operated on a 64-bit Windows personal computer with Intel Core (TM) i5-7500 CPU and RAM 4 GB, employing MATLAB (Version R2012b, 64bit).

For the generated symmetric nonuniform array, the real symmetric is obtained using the method proposed. The mainlobe is set to [-5°, 5°], [-10°, 10°], respectively. The pencil beams and the corresponding amplitudes of excitations are shown in Figs. 2a-2b. Both pencil beams point at 0 degrees and have low SLLs. With respect to the mainlobe of [-5°, 5°], the half-power beamwidth is 5°, and the corresponding SLL is -16.6 dB. And with respect to the mainlobe of [-10°, 10°], the half-power beamwidth is 7°, and the corresponding SLL is -39.6 dB. Obviously, the mainlobe is wider, the SLL is lower. To get the lower SLL, we choose the wider mainlobe in the following experiments.

As mentioned above, the pencil beams pointing to different directions can be superposed to obtain the prescribed beampattern. Here, we use this method to synthesize the flat-top beampattern. The mainlobe region is set to [-20°, 20°]. Because the half-power beamwidth is about 7°, we sample 7 angles uniformly. The comparison with the existing methods [5, 6] is shown in Fig. 3. It can be seen that the synthesized beampattern of the proposed method has the lowest SLL of -20dB. The running times are listed in Table I. And the proposed method has the shortest time of 0.07s whereas the other two methods take more than 100 times as long. The reason is that the aforementioned real symmetric excitation is stored in advance and the final excitation is obtained analytically.

Besides, the cosecant squares beampattern is synthesized. The mainlobe region is set to [5°, 35°] and we sample 5 angles uniformly in this case. The synthesized cosecant beampatterns by the three methods are shown in Fig. 4. The proposed method has a SLL about -25dB lower than other methods. The running times are listed in Table I. Obviously, the proposed method is still more efficient.

Conclusion: In this letter, we propose an analytical beampattern synthesis method for the symmetric nonuniform array. It first attains a real symmetric excitation to achieve a pencil beam via the convex optimization method. Then, it superposes the pencil beams pointing in different directions to synthesize the shaped beampattern analytically. Numerical results of the symmetric nonuniform linear array are provided to verify the effectiveness.

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References


