Power Allocation for Joint Sum Rate and Fairness Optimization in Downlink NOMA Networks

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Abstract

Non-Orthogonal Multiple Access (NOMA) is an essential enabling technology that is expected to help future broadband wireless networks meet their higher system throughput requirements. However, in addition, NOMA should also aim to provide a desired trade-off between system throughput and user fairness, as fairness is an equally important aspect that should go hand in hand with system throughput. In order to achieve such a desired trade-off, in this paper, we derive optimal power allocation (PA) coefficients at the NOMA transmitter, by formulating and solving a joint sum rate and fairness optimization problem. To the best of our knowledge, such a work is missing in the literature. Additionally, along with the usual transmitter power budget and Quality of Service (QoS) constraints, we also consider the minimum transmit power gap between users, that is required for successful signal decoding of a user in a SIC receiver, a constraint ignored at large in the literature. The weighted sum method is used to convert the joint objective optimization problem into a single-objective optimization problem to make it analytically solvable and also provide the desired trade-off between the conflicting objectives. As part of the validation process, we present simulation results and also compare the performance of the derived system with a system that considers only the sum rate objective.
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Summary
Non-Orthogonal Multiple Access (NOMA) is an essential enabling technology that is expected to help future broadband wireless networks meet their higher system throughput requirements. However, in addition, NOMA should also aim to provide a desired trade-off between system throughput and user fairness, as fairness is an equally important aspect that should go hand in hand with system throughput. In order to achieve such a desired trade-off, in this paper, we derive optimal power allocation (PA) coefficients at the NOMA transmitter, by formulating and solving a joint sum rate and fairness optimization problem. To the best of our knowledge, such a work is missing in the literature. Additionally, along with the usual transmitter power budget and Quality of Service (QoS) constraints, we also consider the minimum transmit power gap between users, that is required for successful signal decoding of a user in a SIC receiver, a constraint ignored at large in the literature. The weighted sum method is used to convert the joint objective optimization problem into a single-objective optimization problem to make it analytically solvable and also provide the desired trade-off between the conflicting objectives. As part of the validation process, we present simulation results and also compare the performance of the derived system with a system that considers only the sum rate objective.

KEYWORDS:
Non-orthogonal multiple access (NOMA), Sum rate, Fairness, Power Gap, Power allocation (PA), Resource allocation (RA), Multi-objective optimization (MOO), Successive Interference Cancellation (SIC)

1 | INTRODUCTION

Future broadband wireless networks need to be capable of meeting the enormous demand for high-speed data transmission. Non-orthogonal multiple access (NOMA) has been recognized as one of the important enabling elements to satisfy such a demand. The fundamental idea of NOMA is that several users are served in a single time-frequency block, in contrast to the orthogonal multiple access (OMA) method, which serves a single user in each resource block. As a result, NOMA is found to give a higher data rate and spectrum efficiency than OMA. Therefore, in recent years, many researchers in academia and industry have been inspired to further investigate ways to maximize the sum rate of the NOMA networks.

Despite the importance of the sum rate, user fairness is an equally important factor in NOMA networks. When there is a possibility that users with poor gains may be unable to access the channel, we can take advantage of fairness at the cost of reduced throughput. In addition, as NOMA aims to serve multiple users through non-orthogonal resource allocation, it is desirable to realize trade-off between sum rate and user fairness. Designing such a trade-off between the sum rate and fairness in a
NOMA network is highly dependent on distributing the total transmit power among users at the BS NOMA transmitter. However, the distribution of the total transmit power should be done in a way that in addition to the network and user constraints, NOMA technology’s constraint should also be satisfied.

To this end, in addition to the users’ QoS and transmitter power budget constraints, we should also consider the co-channel interference between NOMA multiplexed users at the receiver. Since NOMA multiplexes users on the same resource block, it results in co-channel interference between the multiplexed users. Hence, successive interference cancellation (SIC) may help decrease interference at the receivers. However, for the SIC to be appropriately executed at the user’s end, the NOMA transmitter should ensure a sufficient gap between the users transmit power. In this respect, this research aims to find the optimal PA coefficients among multiplexed users, for downlink NOMA networks to jointly optimize the sum rate and user fairness, while satisfying users’ QoS, transmit power budget and users minimum transmit power gap constraints.

The rest of the article is organized as follows. Section 2 discusses the related literature, motivation, and summary of our contributions. Section 3 presents the downlink NOMA system model and the multi-objective optimization (MOO) problem formulation. Section 4 derives the optimal PA coefficients and provides an iterative algorithm for the solution of the MOO problem. The proposed MOO problem’s simulation results and related discussions are presented in Section 5. Finally, the paper is concluded in section 6.

2 RELATED WORK

We begin with sum rate maximization articles, particularly those that deal with the SIC constraint in addition to other constraints. To maintain consistency in discussion, in this paper, if the SIC process at the receiver is imperfect, then interference from multiplexed co-users with lower channel coefficients is not completely eliminated when decoding a user’s signal. Henceforth, imperfect SIC in this paper means there are some residues of user interference even after successive interference cancellation. The optimal PA for maximizing the weighted sum rate in a downlink multi-carrier NOMA network with imperfect SIC in addition to power budget and power order constraints are investigated in the manuscript. To optimize the sum rate for downlink NOMA networks, the study proposed an optimal PA scheme that considers the imperfect SIC along with the power budget and the minimum rate constraints. The study optimizes user clustering and PA for downlink and uplink NOMA networks to maximize the sum rate. This study includes a sufficient gap between users transmitting power constraint in addition to power budget, QoS, one user can be allocated to at most one cluster while at least two users are grouped into one cluster, and total frequency resource constraints. The authors of maximize the sum rate of a two-user downlink NOMA network while considering the imperfect SIC along with QoS constraints. The sum rate is maximized by calculating the optimal improper Gaussian signaling circularity (IGS) coefficient. The study comes with an optimization problem that solves for precoding matrix and PA to optimize system throughput by employing imperfect SIC for downlink MIMO NOMA with power budget and minimum rate requirement for weak user constraints. The research proposes a MOO problem for optimally allocating power in a downlink transmission NOMA network in order to maximize the sum rate while minimizing transmit power under the constraint of the minimum power gap among users, along with QoS and power budget constraints. For multicarrier NOMA networks, the research obtain subchannel and power to maximize the sum rate while fulfilling SIC, minimum user rate, the maximum number of users in a subchannel, and power budget constraints. The research explores the performance of a NOMA-based satellite-terrestrial system in terms of outage performance, ergodic sum rate, and system throughput in the presence of imperfect SIC and CSI while satisfying the QoS requirement.

We now examine sum rate studies that ensure a desired fairness level without addressing the SIC constraint while considering other constraints. In the authors optimize the PA to maximize the average sum rate for two-user downlink transmission with delay-tolerant transmission over fading channels, incorporating peak and average power and minimum individual rate constraints. Sum rate maximization is achieved for both complete and partial CSI, and fairness is ensured by including a minimum achievable ergodic rate requirement. In a downlink transmission NOMA network studies the maximization of fairness in terms of rate under full CSI and outage probability under average CSI with optimal PA. The research employs zero-forcing (ZF) and minimum mean square error (MMSE), and the paper employs MRC and MIMO SIC receiver algorithms to solve the joint fairness and sum rate optimization problem for uplink transmission NOMA networks. The authors in offer a method for concurrently improving fairness and sum rate through optimal PA, subband assignment, and user grouping. The author investigated proportional fairness scheduling in a two-user downlink NOMA network in the research. The author demonstrated that the proportional fairness
scheduling approach optimizes both the sum rate and the least normalized rate with optimal PA, resulting in proportional fairness and a slight variation in transmission rates. The optimal PA optimization problem in the paper\cite{22} is intended to optimize the instantaneous sum rate with $\alpha$-fairness for the downlink NOMA network under the power budget constraint. The user rates are updated based on the instantaneous CSI. The work\cite{23} formulates an optimization problem for a downlink multicarrier NOMA network to optimize fairness and EE amongst users in terms of PA and subcarriers. The optimization problem has QoS requirements, the transmission power of the BS, power budget for each subchannel, and user limit on the subcarrier constraints. First, worst-case user first subcarrier allocation (WCUFS) algorithm is presented to allocate subcarriers. Then, optimal PA is achieved to improve the EE further and guarantee maximum fairness for NOMA users. The authors offer a water-filling-based joint PA and a proportional fairness scheduling approach in\cite{24} to optimize the achievable rate by a quasi-optimal repartition of transmission power among subbands while assuring a high resource allocation fairness. The paper\cite{20} proposed a joint NOMA and TDMA scheme in the Industrial Internet of Things (IoT), which allows several sensors to communicate in the same time-frequency resource block using the NOMA method. Time slot allocation, power control, and user scheduling are optimized simultaneously to maximize the system $\alpha$-fair utility with minimum rate, available transmission time allocated to all sensors, and transmit power constraints.

Some researchers have studied the sum rate that ensures user fairness while considering the SIC constraint in addition to other constraints. The research\cite{10} addresses a MOO problem to jointly optimize sum rate and fairness for designing optimal beamforming in a MISO NOMA network by incorporating SIC power order in addition to the total transmit power constraints. The objective of the study\cite{15} is to maximize the fairness in the data rates of different users while optimizing PA in a NOMA network by taking the minimum gap among users’ powers as well as QoS, power budget constraints into consideration. The article\cite{17} investigates the optimal PA for maximizing energy efficiency (EE) for MIMO NOMA networks with SIC power order as well as weak user QoS requirements, user fairness, and maximum transmission power constraints.

### 2.1 Motivation and Contributions

The majority of research on NOMA till now has been on single-objective optimization, either of the sum rate or fairness. However, in many situations, multiple system objectives need be optimized simultaneously. Quite often, it is essential to employ MOO to jointly optimize both sum rate and user fairness with respect to PA in order to make effective decisions in the face of trade-offs between these two equally important but conflicting objectives. Also, since NOMA is based on the idea of multiplexing users in the same time and frequency, it can cause a significant amount of multiple access interference during the decoding process at each intended receiver. In order to handle this multiple access interference, SIC is an important process at the receivers in a NOMA network. But, to make the SIC process successful, the transmit power of all the users must satisfy the minimum required gap. However, to the best of our knowledge, there is no work till now that addresses the joint optimization of sum rate and users fairness while also considering the minimum power gap constraint. Even though\cite{10} jointly maximizes the sum rate and user fairness, it does so with respect to beamforming vector and not users’ power allocation, and without considering the power gap constraint. Only a few work\cite{21,23,13,15} guarantee a minimal gap between users’ transmission powers, but these papers do not address the MOO problem for joint sum rate and fairness maximization. Hence, to the best of our knowledge, a work that considers the joint optimization of sum rate and users fairness under the transmit power budget, users QoS and minimum gap between users transmit power constrains is missing in the literature. In order to fill this important gap in the literature, we investigate the MOO problem for joint maximization of sum rate and user fairness through optimal PA, while adhering to a sufficient power gap constraint between the users’ transmit powers in addition to the transmit power budget and users’ QoS constraints. In this regard, the main contributions of the paper are summarized as follows:

- We formulate and study the MOO problem to achieve optimal PA in downlink NOMA networks. The MOO problem jointly optimizes the sum rate and user fairness while meeting the minimum transmit power gap between users, power budget, and QoS requirements through optimal PA.

- We run Monte Carlo simulations to verify our analytical expressions and the proposed iterative algorithm. Our simulations show that the proposed algorithm requires only a few iterations to converge to the optimal PA coefficients.

- We show through our derived analytical expressions and simulations the trade off between the sum rate and user fairness as the weighting coefficient that indicates the amount of importance given to each objective is varied. This also helps us to compare the proposed method’s performance with single objective benchmark scheme that considers only the sum rate objective in a network.
• Using our investigations, we also show that as the minimum power gap increases, except for the weakest user, other users restrictions increase with their power. Thus, the sum rate performance degrades with the increase in the minimum power gap. Hence, we conclude that there is an optimal power gap that needs to be chosen for the best performance of the network.

While our investigation of the MOO problem provides a lot of insights into how we can bring a meaningful trade-off between sum rate and user fairness under the said constraints, it has to be noted that, the formulated problem poses sufficient challenges to solve because of the complicated nature of objective functions and the additional power gap constraint that is included in our work. In order to circumvent the challenges, as will be seen in the subsequent sections, we judiciously include a new assistance variable to convert the original problem into a solvable problem. Further, we adopt the weighted-sum method, which allows the MOO problem of maximizing sum rate and user fairness to be linearly combined into a single-objective optimization problem\[10,22,23\] using a weighting coefficient that enables a desired trade-off between these conflicting objectives. Following that, we derive the optimal solution to our MOO problem using a Lagrange dual decomposition method and Karush–Kuhn–Tucker (KKT) conditions. Hence, overall, considering the importance of the problem that is formulated, the challenges that such a problem formulation poses to get solved and the numerous insights that our investigation provides, this work can be considered important for the literature.

### 3 SYSTEM MODEL AND PROBLEM FORMULATION

This section discusses the downlink NOMA network system model and formulation of the MOO problem for joint sum rate and user fairness maximization using optimal PA. Consider a downlink NOMA-based wireless network with a BS that communicates with $M$ users, as shown in Fig. 1. We assume a single input and single output (SISO) system where the BS uses a single antenna to transmit and the users use a single antenna to receive. NOMA is based on the idea that users can share time-frequency resources by splitting them into power-domain and code-domain\[7,8\]. This results in power-domain NOMA and code-domain NOMA mechanisms. We utilize power-domain NOMA user multiplexing, which enables us to serve all users at the same time across the same frequency band\[12,2,5,14,21\]. NOMA employs superposition coding at the transmitter, and at the receiver, it uses SIC decoding\[5,6,12\]. In superposition coding, all user signals are multiplied with separate PA coefficients, then combined at the BS. As a result, the transmitted signal at BS that uses superposition coding can be expressed as,

$$X = \sum_{m=1}^{M} \sqrt{a_m} P \ S_m,$$

*Figure 1* The system model illustrating the SIC process of downlink transmission NOMA network.
where $S_m$ is the symbol of user $m$, $\alpha_m$ is PA coefficient for user $m$, and $P$ is total transmit power of BS. The received signal at user $m$ is given by,

$$Y_m = h_m X + \eta_m,$$

where, $h_m$ is the complex channel coefficient between BS and user $m$, $\eta_m$ is complex additive white gaussian noise (AWGN), represented by $\eta_m \sim \mathcal{CN}(\mu, \sigma^2)$ with zero mean ($\mu = 0$) and variance $\sigma^2$.[14,15] We assume the channels between the BS and all served users to have rayleigh fading with independent distribution,[10] represented by $h_m = g_m d_m^{-pl}$ and $g_m$ is the small scale fading parameter that follows a complex Gaussian distribution i.e., $g_m \sim \mathcal{CN}(0, 1)$ where $m \in (1, 2, \cdots, M)$, $d_m$ is the distance between BS and user $m$, and $pl$ is the path loss exponent.[12,13] Without loss of generality, we sorted the channel gains between the BS and all users in ascending order as $|h_1| \leq |h_2| \leq \cdots \leq |h_M|$. Further, we assume that the channel state information (CSI) is perfectly known at the BS.[14,15]

SIC is used to decode data in the sequence of decreasing power levels on the receiver side. First, the data corresponding to the user who provided the most power is directly decoded. Then, the user who provided the next greatest power is decoded by removing interference from the previously decoded user. This procedure is repeated until all of the user’s data has been decoded.[6,15] The SIC process for the downlink transmission of the NOMA network is illustrated in Fig. 1. After the SIC process as described above, we can obtain the signal $Z_m$ at user $m$ given by,

$$Z_m = h_m \sqrt{\alpha_m P} S_m + h_m \sum_{n=m+1}^{M} \sqrt{\alpha_n P} S_n + \eta_m,$$

The signal-to-interference and noise ratio (SINR) as seen by user $m$ can be written as,[14,15]

$$SINR_m = \frac{\alpha_m P |h_m|^2}{\sum_{n=m+1}^{M} \alpha_n P |h_n|^2 + \sigma^2}.$$ 

Hence, the achievable rate for user $m$ is given by,[14,15]

$$R_m = \log_2(1 + SINR_m) \quad \text{bps/Hz}.\quad (5)$$

Fairness measurements are tools for determining the amount of fairness. The fairness measurements can be characterized as quantitative or qualitative based on their quantitative ability. Quantitative measurements include Jain’s Fairness Index (JFI) and Entropy, while qualitative measures include max-min and proportional fairness.[24,25] JFI and Entropy measures do not help identify resources that have been treated unfairly. Additionally, JFI and Entropy measures need complete information on resource allocation to obtain fairness.[22] In max-min fairness measures, maximizes the user service by providing the lowest service to the maximum allocated resource. It may result in network inefficiency. max-min fairness does not measure individual fairness and the amount of fairness.[23] On the other hand, an $\alpha$ - fair utility function is a generalized form of the fairness function. Compared to the fairness mentioned above, an $\alpha$ - fair utility function measures individual fairness and does not require complete information. An $\alpha$ - fair utility function uses a single scalar to represent various levels of user fairness.[10,12,25] As a result, in this study, we prefer to use the $\alpha$ - fair utility function, which may be represented as,[10,12,25]

$$U(R_m) = \begin{cases} \frac{R_m^{\alpha-1}}{1-\alpha}, & \text{if } x \geq 0, \ x \neq 1, \\ \ln(R_m), & \text{if } x = 1 \end{cases}.$$ 

Where $x$ represents different fairness levels of achievable rate $R_m$. We incorporate the constraint of minimum power gap for successful SIC into our optimization problem, which can be represented as follows,[6,13,15]

$$\alpha_m P |h_{m+1}|^2 - \sum_{i=m+1}^{M} \alpha_i P |h_{i+1}|^2 \geq \phi, \text{ for } m = 1, 2, \cdots, M - 1.$$ 

where $\phi$ is the minimal power gap between users to implement the SIC procedure successfully. From Eqn. (7), primary decoded users must be given more power than later decoded users to ensure successful SIC in NOMA networks. Furthermore, there should be a reasonable difference in transmission power between users.
Next, we frame the MOO problem to determine the operating point that simultaneously optimizes the sum rate and user fairness using optimal PA for downlink NOMA networks. This MOO problem can be written as \( P1 \),

\[
(P1) : \max_{\alpha_m} \left( \sum_{m=1}^{M} R_m, \sum_{m=1}^{M} U(R_m) \right),
\]

s.t.

\[
\alpha_m P|h_{m+1}|^2 - \sum_{i=m+1}^{M} \alpha_i P|h_i|^2 \geq \phi, \text{ for } m = 1, 2, \ldots, M - 1. \tag{8b}
\]

\[
R_m \geq R_{\text{min}}, \forall m. \tag{8c}
\]

\[
\sum_{m=1}^{M} \alpha_m P \leq P_{BT}. \tag{8d}
\]

\[
\alpha_m \geq 0, \forall m. \tag{8e}
\]

Where Eqn. (8a) denotes objective functions to be maximized simultaneously, the first function is the sum rate, and the second function is the sum utility. The constraint in Eqn. (8b) ensures minimum power gap for successful SIC process in NOMA. In the context of QoS, the constraint in Eqn. (8c) implies the requirement that the data rate of each user must not fall below the minimum user data rate \( R_{\text{min}} \). The constraint in Eqn. (8d) ensures that the network’s overall transmit power does not exceed BS’s total power budget \( P_{BT} \). Finally, the constraint in Eqn. (8e) assures that each user has a non-negative transmit power.

## 4 SOLUTION OF MULTI-OBJECTIVE OPTIMIZATION PROBLEM

This section will provide the optimal solution for an \( M \) users downlink NOMA network in terms of both sum rate and user fairness maximization. Later in this section, we also discuss benchmark schemes. The weighted sum and \( \varepsilon \)-constraint methods are the two most often adopted classical methods for solving MOO problems.\(^{26}\) In the weighted sum method, a weighting coefficient linearly combines the MOO problem into a single-objective optimization problem. The \( \varepsilon \)-constraint method optimizes one objective function while constraining the others to be less than or equal to a defined numerical value.\(^{26}\) The \( \varepsilon \) vector in the \( \varepsilon \)-constraint method must be carefully chosen to fall within the minimum and maximum values of the objective function, which might make this approach challenging. However, the weighted sum method is more straightforward to implement than the \( \varepsilon \)-constraint method. Furthermore, we may make a trade-off between multiple objectives using the weighting coefficient method. This is also consistent with our aim of reaching optimal PA to achieve a balanced trade-off while simultaneously optimizing sum rate and user fairness. Thus, we use the weighted sum method to solve the MOO problem \( P1 \). This results in a MOO problem \( P1 \) being converted into a single objective optimization problem, which is as follows:\(^{10,22,23}\)

\[
(P2) : \max \omega \sum_{m=1}^{M} R_m + (1 - \omega) \sum_{m=1}^{M} U(R_m),
\]

s.t.

\[
\text{subject to } (8b), (8c), (8d), (8e). \tag{8c}
\]

where \( \omega \) is a weighting coefficient, such as \( 0 \leq \omega \leq 1 \). The \( \omega \) shows a tradeoff between two objectives, i.e., higher \( \omega \) values support maximizing the sum rate, while lower \( \omega \) values favor maximizing user fairness. Note that the fairness objective function is neither convex nor concave with regard to \( \alpha \). Hence, the optimization problem \( P1 \) becomes non-convex. Therefore, the solution for \( P1 \) may not be global. We recognize that the fairness objective function can be converted to a concave function (concavity proof is shown in appendix A) by creating a new assistant variable \( t_m \) for the user \( m \),\(^{27}\) and then \( P2 \) is written as,

\[
(P3) : \max \omega \sum_{m=1}^{M} R_m + (1 - \omega) \sum_{m=1}^{M} U(t_m),
\]

s.t.

\[
\text{subject to } (8b), (8c), (8d), (8e).
\]

\[
t_m \leq R_m, \forall m. \tag{9a}
\]

As stated in\(^{27}\) when the weighted objective function in \( P3 \) is to be maximized, \( U(\cdot) \) must be an increasing function, and \( t_m \) must be equal to \( R_m \). Then, \( P2 \) and \( P3 \) would have the same solution. As a result, instead of \( P2 \), we can solve \( P3 \). We may now use the Lagrange dual decomposition method to solve the optimization problem \( P3 \)\(^{21,15,28} \). As a result, the following
Lagrangian function is obtained as,

\[
L(\alpha_m, \lambda_m, \beta_m, \theta_m, \xi_m) = \omega \sum_{m=1}^{M} R_m + (1 - \omega) \sum_{m=1}^{M} U(t_m) + \theta_m(P_{BT} - \sum_{m=1}^{M} \alpha_m P)
\]

\[
\sum_{m=1}^{M} \beta_m(R_m - R_{min}) + \sum_{m=1}^{M-1} \lambda_m \left( \alpha_m P|h_{m+1}|^2 - \sum_{i=m+1}^{M} \alpha_i P|h_{i+1}|^2 - \phi \right) + \sum_{m=1}^{M} \xi_m(R_m - t_m).
\]

(10)

There are two sub-problems in solving the above Lagrangian function. The first is a problem of application layer optimization with variable \( t \), and the second is a problem of physical layer optimization with variable \( \alpha \). Hence, the problem of application layer optimization may be expressed as,

\[
T_1 = \max_t g_1(t_m),
\]

(11)

where,

\[
g_1(t_m) = (1 - \omega) \sum_{m=1}^{M} U(t_m) - \sum_{m=1}^{M} \xi_m t_m.
\]

(12)

and the physical layer optimization problem can be written as:

\[
T_2 = \max_{\alpha_m} g_2(\alpha_m),
\]

(13)

where,

\[
g_2(\alpha_m) = \omega \sum_{m=1}^{M} R_m + \sum_{m=1}^{M} \beta_m(R_m - R_{min}) + \theta_m(P_{BT} - \sum_{m=1}^{M} \alpha_m P) + \sum_{m=1}^{M-1} \lambda_m \left( \alpha_m P|h_{m+1}|^2 - \sum_{i=m+1}^{M} \alpha_i P|h_{i+1}|^2 - \phi \right) + \sum_{m=1}^{M} \xi_m R_m
\]

(14)

In Eqn. (6), the \( \alpha \)-fair utility function has two cases: \( x = 1 \) and \( x \geq 0 \). As a result, for \( x = 1 \) and \( x \geq 0 \), we can study the optimal solution of \( T_1 \) of Eqn. (11) independently as follows.

**Case I** \((x = 1)\): In this case, \( U(t_m) = \ln(t_m) \) is a strictly concave function of \( t_m \). Therefore, the first term \((1 - \omega) \sum_{m=1}^{M} U(t_m)\) is a strictly concave function of \( t_m \). The second term \( \sum_{m=1}^{M} \xi_m t_m \) in (12) is a linear function of \( t_m \). Hence, \( g_1(t_m) \) is a concave function of \( t_m \). Thus, by using the KKT conditions\(^{35}\), one may derive the closed-form formula for \( t_m \) in case I by obtaining the first partial derivative of \( g_1(t_m) \) with respect to \( t_m \) and equating to zero. Hence,

\[
\frac{\partial g_1(t_m)}{\partial t_m} = \frac{\partial}{\partial t_m} \left( \frac{\omega}{t_m} - \frac{\xi_m}{\omega} \right) = 0 \implies t_m^* = \frac{\omega}{\xi_m}.
\]

(15)

**Case II** \((x \geq 0)\): \( g_1(t_m) \) is a concave function of \( t_m \), as demonstrated in Appendix A. As a result, the closed-form formula for \( t_m \) in case II can be computed by applying the KKT conditions, as shown in Appendix C. Therefore, we get \( t_m^* \) as,

\[
t_m^* = \sqrt{\frac{1 - \omega}{\xi_m}}.
\]

(16)

To derive optimal PA coefficients \( \alpha_m^* \), we first demonstrate in Appendix B that \( g_2(\alpha_m) \) is a concave function of \( \alpha_m \). As a result, as seen in Appendix D, the closed-form expression for \( \alpha_m^* \) can be determined using the KKT conditions as stated in Eqn. (17) with \([\alpha]^+ = \max(0, \alpha)\) and \( T_m \) as given in Eqn. (18).

\[
\alpha_m^* = \left[ \frac{(\omega + \beta_m + \xi_m) P|h_m|^2 - T_m \left( \sum_{m=m+1}^{M} \alpha_m P|h_m|^2 + \sigma^2 \right)}{T_m P|h_m|^2} \right]^+, \quad (17)
\]

\[
T_m = \theta_m P - \lambda_m P|h_{m+1}|^2 + \sum_{i=1}^{m-1} \left( \frac{\lambda_i P|h_{i+1}|^2}{\sum_{j=i+1}^{M} \alpha_j P|h_j|^2 + \sigma^2} \right) \left( \frac{\omega + \beta_i + \xi_i}{\sum_{j=i+1}^{M} \alpha_j P|h_j|^2 + \sigma^2} \right)^2
\]

(18)

Now, using a sub-gradient method, the Lagrange multipliers can be calculated and updated iteratively, as described below\(^{35}\).

\[
\lambda_m(t + 1) = \left[ \lambda_m(t) + \delta_1(t) \left( \phi - \alpha_m P|h_{m+1}|^2 + \sum_{i=m+1}^{M} \alpha_i P|h_{i+1}|^2 \right) \right]^+, \quad \text{for } m = 1, 2, \ldots, M - 1,
\]

(19)

\[
\beta_m(t + 1) = \left[ \beta_m(t) + \delta_2(t)(R_{min} - R_m) \right]^+, \quad \forall m,
\]

(20)
\[\theta_m(t+1) = \left[ \theta_m(t) + \delta_3(t) \left( \sum_{m=1}^{M} \alpha_m P - P_{BT} \right) \right]^+, \quad (21)\]

\[\xi_m(t+1) = \left[ \xi_m(t) + \delta_4(t)(t_m - R_m) \right]^+. \quad (22)\]

where \(\delta\) denotes the step size, and \(t\) represents the iteration index. The subgradient method uses a variety of different step size rules. We pick the step size in accordance with the rule of diminishing step sizes described in \(29\). Once the equations are derived, we can present an iterative algorithm 1 for obtaining the optimal PA coefficients for the MOO problem \((P1)\) of jointly maximizing the sum rate and user fairness.

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**Algorithm 1** Iterative Optimal Power Allocation Algorithm

1. Requires: \(P, \sigma^2, P_{BT}, R_{min}, M, pl, x, t, \omega,\) and \(N;\)
2. Requires: \(h, \alpha, \lambda, \beta, \theta, \xi,\) and \(\delta\) for all users;
3. Calculate initial rates and user fairness for \(M\) users;
4. for until sum rate/user fairness repeats do
5. for \(m = 1\) to \(M\) do
6. \(t = t + 1;\)
7. Update \(T_m(t)\) using Eqn. \(18;\)
8. Update \(a_m(t)\) using Eqn. \(17;\)
9. Update \(t_m(t)\) using Eqn. \(15\) if \(x = 1\) else update \(t_m(t)\) using Eqn. \(16\) if \(x \geq 0;\)
10. Update \(\lambda_m(t)\) using Eqn. \(19;\)
11. Update \(\beta_m(t)\) using Eqn. \(20;\)
12. Update \(\theta_m(t)\) using Eqn. \(21;\)
13. Update \(\xi_m(t)\) using Eqn. \(22;\)
14. Update rates and user fairness for \(M\) users;
15. end for
16. end for

Now we study the time complexity of our proposed algorithm. Since algorithm 1 has \(M\) users, it takes \(M\) worst-case computation time to update the sum rate and fairness at line 3. It is important to note that line 8 takes the most computing time inside the loop. The worst-case time computation of \(T_m\) in Eqn. \(18\) is \(M^2\). Line 8 updates \(a_m\) using Eqn. \(17\), which depends on \(T_m\). Thus, the worst-case time computation of \(a_m\) is \(M^3\). Since the inner loop (line 5 to line 15) repeats \(M\) times, the inner loop’s worst-case complexity is \(M^4\). Assume that the total number of iterations required for algorithm convergence is \(T\). As a result, our proposed method’s worst-case run time complexity for \(M\) users is \(\Theta(M^3 T)\).

### 4.1 Benchmark: Single-objective optimization

It is worth noting that our proposed MOO problem \((P1)\) jointly optimizes the sum rate and user fairness. In certain circumstances, it may be necessary to optimize only the sum rate of a NOMA network in order to decrease computing complexity. Consequently, subsection \(4.1\) provides the single-objective optimization problem \((P4)\) by omitting the user fairness objective of the MOO problem \((P1)\). As a result, section III-B of work \(15\) provides a benchmark for the MOO problem \((P1)\). In \((P4)\), we find the optimal PA for maximizing sum rate while keeping the minimum transmit power gap between users, QoS, and power budget constraints in mind. As a result, the optimization problem \((P4)\) may be stated as follows:

\[(P4) : \max_{a_m} \sum_{m=1}^{M} R_m,\]

\[\text{s.t.} \ (8b), \ (8c), \ (8d), \ (8e).\]

We solve the optimization problem \((P4)\) using the same steps and method as in section \(4\) due to the similarities in solving the optimization problem. As a result, all steps and explanations are ignored.
5 | RESULTS

This section discusses the simulation results for our proposed MOO problem. Additionally, we compare the simulation results to the benchmark method. The performance parameters for all users are obtained in the simulation across $10^5$ channel gain realizations of Rayleigh fading, and then we take the average of these performance parameters. Table 1 lists the values of the simulation parameters used to generate the plots unless otherwise stated. We have set the user’s minimum power gap to 1.5.

$(\phi = 1.5)$. The proposed solution ensures power gaps of constraint (8b) for given user’s channel conditions. Table 2 shows the proposed MOO problem’s initial and converged system parameter values. In our simulations, we randomly initialize the PA coefficients as $\alpha_1 = \alpha_2 = \alpha_3 = 0.1$, as shown in Table 2, regardless of our MOO problem’s objectives and constraints. On the other hand, our method guarantees that the optimal PA coefficients converge to $\alpha_1 = 0.6501$, $\alpha_2 = 0.1066$, $\alpha_3 = 0.0222$, resulting in a total transmit power of 7.79 W that meets all constraints of (P1) while also concurrently optimizing sum rate and fairness. Table 2 shows that, as expected by NOMA principles, the user with the worst channel conditions receives the most transmit power. In contrast, the user with the best channel conditions receives the least transmit power. Additionally, we can observe in Table 2 that randomly initialized PA coefficients provide rates that are not ensuring the QoS constraint. On the other side, once

<table>
<thead>
<tr>
<th>System Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel realization ($N$)</td>
<td>$10^5$</td>
</tr>
<tr>
<td>Number of users ($M$)</td>
<td>3</td>
</tr>
<tr>
<td>Total transmit power from BS ($P$)</td>
<td>10 W</td>
</tr>
<tr>
<td>Total power budget of BS ($P_{BT}$)</td>
<td>3 W</td>
</tr>
<tr>
<td>Variance of AWGN noise ($\sigma^2$)</td>
<td>1</td>
</tr>
<tr>
<td>The minimum required rate for QoS ($R_{min}$)</td>
<td>1 bps/Hz</td>
</tr>
<tr>
<td>Minimum power gap among different users ($\phi$)</td>
<td>1.5</td>
</tr>
<tr>
<td>Weighting coefficient ($\omega$)</td>
<td>0.5</td>
</tr>
<tr>
<td>Fairness level ($x$)</td>
<td>0.75</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>System Parameter</th>
<th>Initial value</th>
<th>Converged value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>0.1</td>
<td>0.6501</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.1</td>
<td>0.1066</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>0.1</td>
<td>0.0222</td>
</tr>
<tr>
<td>Transmit power from BS to user 1</td>
<td>1 W</td>
<td>6.50 W</td>
</tr>
<tr>
<td>Transmit power from BS to user 2</td>
<td>1 W</td>
<td>1.07 W</td>
</tr>
<tr>
<td>Transmit power from BS to user 3</td>
<td>1 W</td>
<td>0.22 W</td>
</tr>
<tr>
<td>Total transmit power from BS</td>
<td>3 W</td>
<td>7.792 W</td>
</tr>
<tr>
<td>$R_1$</td>
<td>0.35 bps/Hz</td>
<td>1.68 bps/Hz</td>
</tr>
<tr>
<td>$R_2$</td>
<td>0.75 bps/Hz</td>
<td>1.38 bps/Hz</td>
</tr>
<tr>
<td>$R_3$</td>
<td>4.23 bps/Hz</td>
<td>2.34 bps/Hz</td>
</tr>
<tr>
<td>$SR$</td>
<td>5.33 bps/Hz</td>
<td>5.40 bps/Hz</td>
</tr>
<tr>
<td>$JFI$</td>
<td>0.51</td>
<td>0.95</td>
</tr>
</tbody>
</table>
the algorithm converges, all user rates satisfy the QoS constraint. Those rates converge very near to one another, demonstrating the effectiveness of our proposed method in terms of user fairness while simultaneously maximizing the sum rate.

Figures 2 and 3 show graphically how each user’s transmit power and rate change with each iteration of our algorithm. The iteration index is shown on the horizontal axis, while the parameter of interest is displayed on the vertical axis. Fig. 2 shows that user 1’s transmit power begins at 1 W and eventually converges to 6.50 W as the number of iterations approaches 35. We may draw similar conclusions from Fig. 2 for users 2 and 3. As a result, the optimal transmit power of users 1, 2, and 3 is 6.50 W, 1.07 W, and 0.22 W, respectively, resulting in a total transmit power of 7.79 W, which is less than the power budget constraint ($P_{BT} = 10$ W). Thus it ensures power budget constraint. In Fig. 3, individual rates converge to optimal values of 1.68 bps/Hz, 1.38 bps/Hz, and 2.34 bps/Hz for users 1, 2, and 3. Note that all user’s optimal rates are higher than $R_{min} = 1$ bps/Hz, which is expected given constraint (8c) in the MOO problem (P1). The proposed method only needs about 35 iterations to converge once the first random PA coefficients are given, proving that it has a fast convergence rate and is effective enough to be implemented in practical networks like 5G networks.

Fig. 4 illustrates the sum rate versus the number of iterations for various weighting coefficients ($\omega$) in the proposed and benchmark methods. The $\omega$ allows for a choice between the sum rate and fairness. Higher $\omega$ values support increasing the sum
rate, while lower \( \omega \) values support increasing user fairness. Thus, when seen in Fig. 4 as \( \omega \) grows, so does the sum rate. When \( \omega = 1 \), the sum rate is entirely favored, and fairness is completely ignored. Hence, benchmark and the \( \omega = 1 \) case are the same. As a result, the \( \omega = 1 \) and benchmark plots intersect, as illustrated in Fig. 4.

5.1 Discussion on Fairness

The \( \alpha \)-fair utility function measures the fairness of a single user. However, if we wish to measure fairness across all network users, JFI is a widely used quantitative measure of fairness. Thus, we can use JFI in simulation to determine system fairness.\(^{12,20,30,31,32}\) JFI in terms of achievable rates, which is defined as follows:\(^{33}\)

\[
JFI = \frac{\left(\sum_{i=1}^{M} R_i\right)^2}{M \sum_{i=1}^{M} R_i^2},
\]

As shown in Eqn. (24), JFI accepts continuous values within the range \([1/M; 1]\). \( JFI = 1 \) denotes the fairest rate allocation, ensuring that all users get the same rate allocation. \( JFI = 1/M \), on the other side, implies the least fair rate allocation, in which all rate allocations are allocated to a single user. Fig. 5 depicts JFI versus iteration count for various weighting coefficients \( (\omega) \) in the proposed and benchmark methods. The JFI for \( \omega = 0.5 \) is equal to 0.51 for initially randomly chosen PA coefficients.
and grows to 0.95 when iterations exceed 35. This indicates clearly that our proposed MOO method maximizes user fairness. In Fig. 5 when ω increases, the fairness decreases. Fig. 5 shows that plots overlap for ω = 1 and benchmark method, which is expected. Fig. 6 depicts the achieved sum rate and JFI over the weighting coefficient. ω is indicated on the horizontal axis,

![Figure 6 Sum rate and JFI against weighting coefficient in downlink NOMA network.](image)

the sum rate is shown on the left side of the y-axis, and JFI is indicated on the right side of the y-axis. For problem (P1), as anticipated, at ω = 0, the maximum sum rate is achieved at the cost of the lowest fairness. Furthermore, when ω = 1, maximum fairness is achieved at the cost of the lowest sum rate. However, the BS can choose an appropriate value for the weighting coefficient ω in order to achieve a good balance between the sum rate and fairness. As shown in Fig. 6, choosing ω = 0.52 results in a good trade-off between these performance parameters. Fig. 7 depicts the proposed method’s sum rate versus iteration count for different power gaps (φ). When φ increases, the sum rate decreases for the proposed method. This is because, when φ

![Figure 7 Sum rate vs. the number of iterations for various power gaps in downlink NOMA network.](image)

increases, the PA coefficients decrease (except for the PA of the weakest user). The reason for the decrease in PAs is that users, excluding the weakest, must meet the SIC process’s power gap criterion (8b). The weakest user is provided the most power. Hence, this user directly decoded in the NOMA network’s SIC procedure. Therefore, the weakest user is exempt from meeting the power gap condition between decoded and non-decoded users. As a result, φ does not affect the PA coefficient of the weakest user. Fig. 8 illustrates the proposed method’s transmit power versus total power budget (P_BT) for various weighting coefficients (ω). According to constraint (8d), when P_BT increases, transmit power for all users should increase. However, because of the minimum power gap constraint (8b), all users except the weakest have limitations when it comes to increasing transmit power.
As a result, we can see in Fig. 8 that the transmit power for the weakest user grows rapidly, while the transmit power for other users increases gradually. Fig. 9 shows the sum rate against the total power budget for different weighting coefficients in the proposed method. According to the discussion mentioned above, when $P_{BT}$ increases, so does the transmit power for all users.

As a result, the network’s overall rate increases.

6 | CONCLUSIONS

This research presented a MOO method for investigating the trade-off between sum rate and user fairness in downlink communication NOMA networks incorporating the minimum power gap for successful SIC constraint. The study looks at a novel approach and demonstrates that the proposed method performs well in downlink NOMA networks. First, we formulated the MOO problem for jointly maximizing sum rate and user fairness while optimizing PA under minimum power gap among users, transmit power, and QoS requirement constraints. Then, we converted a MOO problem to a single-objective optimization problem using the weighted sum method. In order to solve the optimization problem, we used the Lagrange dual decomposition method and the KKT conditions. Finally, simulation results show how downlink NOMA networks can maximize the sum rate and be fair to all users while maintaining a quick convergence rate for the proposed method. We also compared our method’s performance to that of benchmark methods.
We plan to keep studying the implementation of our proposed method for downlink transmission in NOMA-based heterogeneous networks in the future.

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Author contributions
Sachin Trankatwar: Conceptualization, Methodology, Investigation, Simulation, Writing - original draft, Formal analysis. Prashant Wali: Conceptualization, Writing – review and editing, Supervision.

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Conflict of interest
The authors have no conflict of interest or competing interest to declare for the publication of this paper.

APPENDIX

A PROOF OF $g_1(t_m)$ IS CONCAVE FUNCTION

We prove the concavity of $g_1(t_m)$ in this appendix. This is accomplished by demonstrating that the Hessian matrix is negative definite. Let $\Delta_i$ represent the determinant of the leading principal sub-matrix of first $i$ rows and $i$ columns of the Hessian matrix $A$. If $\Delta_i > 0$, for even $i \in \{1, 2, \cdots, M\}$ and $\Delta_i < 0$, for odd $i \in \{1, 2, \cdots, M\}$, then we say $A$ is negative definite. First term in eqn. (12) can be written as,

$$U_{\text{sum}} = (1 - \omega) \sum_{m=1}^{M} \frac{t_m^{1-x}}{1-x}, \ x \geq 0.$$

The Hessian matrix for $U_{\text{sum}}$ can then be calculated as follows:

$$A = \begin{bmatrix}
\frac{\partial}{\partial t_1} \frac{\partial U_{\text{sum}}}{\partial t_1} & \frac{\partial}{\partial t_1} \frac{\partial U_{\text{sum}}}{\partial t_2} & \cdots & \frac{\partial}{\partial t_1} \frac{\partial U_{\text{sum}}}{\partial t_M} \\
\frac{\partial}{\partial t_2} \frac{\partial U_{\text{sum}}}{\partial t_1} & \frac{\partial}{\partial t_2} \frac{\partial U_{\text{sum}}}{\partial t_2} & \cdots & \frac{\partial}{\partial t_2} \frac{\partial U_{\text{sum}}}{\partial t_M} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial}{\partial t_M} \frac{\partial U_{\text{sum}}}{\partial t_1} & \frac{\partial}{\partial t_M} \frac{\partial U_{\text{sum}}}{\partial t_2} & \cdots & \frac{\partial}{\partial t_M} \frac{\partial U_{\text{sum}}}{\partial t_M}
\end{bmatrix}$$

After solving, we get,

$$A = \begin{bmatrix}
-\frac{(1-\omega)x}{t_1^{(x+1)}} & 0 & \cdots & 0 \\
0 & -\frac{(1-\omega)x}{t_2^{(x+1)}} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & -\frac{(1-\omega)x}{t_M^{(x+1)}}
\end{bmatrix}$$

The determinant of a diagonal matrix is the product of its diagonal elements. Hence, we get, $\Delta_i > 0$, for even $i \in \{1, 2, \cdots, M\}$, and $\Delta_i < 0$, for odd $i \in \{1, 2, \cdots, M\}$. Hence, Hessian matrix $A$ is negative definite. Hence, $U_{\text{sum}}$ function is concave. It is worth noting that in Eqn. (12), the second term $\sum_{m=1}^{M} \xi M m$ is a linear function of $t_m$. As a result, it is strictly concave-convex. Hence, $g_1(t_m)$ is a concave function.
B PROOF OF \( g_2(\alpha_m) \) IS CONCAVE FUNCTION

In this appendix, we prove the concavity of the \( g_2(\alpha_m) \) function. Since the channels are sorted as \(|h_1| \leq |h_2| \leq \cdots \leq |h_M|\), then we can express the sum rate as,

\[
R_{\text{sum}} = \sum_{m=1}^{M} \log_2 \left( 1 + \frac{\alpha_m P|h_m|^2}{\sum_{\iota=1}^{M} \alpha_{\iota} P|h_\iota|^2 + \sigma^2} \right).
\]

Denoting Hessian matrix for \( R_{\text{sum}} \) as, \( B_{M \times M} = [b_{ij}] \). After solving, we get the coefficients of the Hessian matrix \( B \) as shown below,

\[
b_{11} = b_{12} = b_{13} = \cdots = b_{1M} = -C_{11},
b_{21} = b_{31} = b_{41} = \cdots = b_{2M} = -C_{11},
b_{22} = b_{23} = b_{24} = \cdots = b_{2M} = -C_{11} + C_{12} - C_{22},
b_{32} = b_{42} = b_{52} = \cdots = b_{3M} = -C_{11} + C_{12} - C_{22},
b_{33} = b_{34} = b_{35} = \cdots = b_{3M} = -C_{11} + C_{12} - C_{22} + C_{23} - C_{33},
b_{43} = b_{53} = b_{63} = \cdots = b_{4M} = -C_{11} + C_{12} - C_{22} + C_{23} - C_{33},
\]

\[ \vdots \]

\[
b_{MM} = -C_{11} + C_{12} - C_{22} + C_{23} - C_{33} + \cdots + C_{(M-1)(M-1)} - C_{(M-1)M} + C_{MM}. \]

By performing elementary matrix operations to reduce the matrix to upper triangular form, we get,

\[
B = \begin{bmatrix}
-C_{11} & -C_{11} & -C_{11} & \cdots & -C_{11} \\
0 & C_{12} - C_{22} & C_{12} - C_{22} & \cdots & C_{12} - C_{22} \\
0 & 0 & C_{23} - C_{33} & \cdots & C_{23} - C_{33} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & C_{(M-1)M} - C_{MM}
\end{bmatrix}
\]

Since \(|h_1| \leq |h_2| \leq \cdots \leq |h_M|\), therefore \( C_{12} < C_{22}, C_{23} < C_{33}, \cdots, C_{(M-1)M} < C_{MM} \). Therefore, \( \Delta_i > 0 \) for even \( i \in \{1, 2, \cdots, M\} \) and \( \Delta_i < 0 \) for odd \( i \in \{1, 2, \cdots, M\} \). As a result, Hessian matrix \( B \) is negative definite. Hence, \( R_{\text{sum}} \) is concave function. As, \( R_{\text{sum}} \) is concave function, therefore first term \( \omega \sum_{m=1}^{M} R_m \), second term \( \sum_{m=1}^{M} \beta_m (R_m - R_{\min}) \), and fourth term \( \sum_{m=1}^{M} \varphi_m R_m \) are concave function. Also, it is important to note that in eqn. (14), the third term \( \sum_{m=1}^{M} \lambda_m \left( \alpha_m P|h_{m+1}|^2 - \sum_{\iota=m+1}^{M} \alpha_{\iota} P|h_{\iota}|^2 - \phi \right) \) and the fourth term \( \theta_m (P_{BT} - \sum_{m=1}^{M} \alpha_m P) \) are linear function of \( \alpha \). Hence, the third and fourth terms in eqn. (14) are concave-convex. Therefore, \( g_2(\alpha_m) \) is concave function.

C DERIVATION OF A CLOSED-FORM EXPRESSION \( t_m \)

The equation for \( g_1(t_m) \) is,

\[
g_1(t_m) = (1 - \omega) \sum_{m=1}^{M} U(t_m) - \sum_{m=1}^{M} \varphi_m t_m.
\]

The first partial derivative of \( g_1(t_m) \) with regards to \( t_m \) is given as,

\[
\frac{\partial g_1(t_m)}{\partial t_m} = (1 - \omega) \sum_{m=1}^{M} t_m \left( t_m - 1 \right) - \sum_{m=1}^{M} \left( \varphi_m t_m \right),
\]

By solving the partial derivative and equal to zero, we may get the optimal value of \( t_m \) in equation (16).

D DERIVATION OF A CLOSED-FORM EXPRESSION \( \alpha_m \)

The equation for \( g_2(\alpha_m) \) is given as,

\[
g_2(\alpha_m) = \omega \sum_{m=1}^{M} R_m + \sum_{m=1}^{M} \beta_m (R_m - R_{\min}) + \theta_m (P_{BT} - \sum_{m=1}^{M} \alpha_m P) + \sum_{m=1}^{M-1} \lambda_m \left( \alpha_m P|h_{m+1}|^2 - \sum_{\iota=m+1}^{M} \alpha_{\iota} P|h_{\iota}|^2 - \phi \right) + \sum_{m=1}^{M} \varphi_m R_m
\]
To get a closed-form expression for $\alpha_m$, compute the first partial derivative of $g_2(\alpha_m)$ concerning $\alpha_m$, as shown below,

$$
\frac{\partial g_2(\alpha_m)}{\partial \alpha_m} = \sum_{i=1}^{m-1} \left( \frac{-(\omega + \beta_i + \xi_i)}{\log 2} \right) \times \left( \frac{\alpha_i P^2 \left| h_i \right|^2}{\sum_{n=i}^{M} \alpha_n P \left| h_i \right|^2 + \sigma^2} \right) + \lambda_i P \left| h_{i+1} \right|^2 \right)

+ \frac{(\omega + \beta_m + \xi_m)}{\log 2} \frac{P \left| h_m \right|^2}{\sum_{n=m}^{M} \alpha_n P \left| h_m \right|^2 + \sigma^2} - \theta_m P - \lambda_m P \left| h_{m+1} \right|^2.

We may now solve the partial derivation and equal it to zero to get the optimal solution $\alpha_m^*$ for user $m$, as shown in Eqn. (17).

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