A Formula of Particles Oscillating in Electromagnetic and Weak Fields Giving Energy and Decay of Particles, Beta Decay and Energy Levels of Hydrogen Atom

Kh. M. Mannan

Affiliation not available

April 17, 2023
A Formula of Particles Oscillating in Electromagnetic and Weak Fields Giving Energy and Decay of Particles, Beta Decay and Energy Levels of Hydrogen Atom

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ABSTRACT: A formula for the mass-energy and the decay of particles is obtained from a model of particles oscillating in the electromagnetic and in the weak fields. For some selected displacements of the particle, important results are obtained. From this formula it is suggested that particles acquire their masses while massless gauge bosons (W±) get their masses by Higgs mechanism. Mass-energies and decay of particles are obtained from the decay of a W± boson coupled to the electromagnetic or the weak fields in which the particles oscillate. In the centre of mass (CM) frame, the spin of a particle is shown as localized circular motion of the centre of charge around the CM.
confined within the reduced Compton wavelength for the particle. Decay constants of pseudo-mesons are obtained from zero point energies of the oscillators. The beta decay and kinetic energies of beta particles are obtained from this formula. Dimension (d) of a three-dimensional harmonic oscillator explains the build up of charge (Q) by the oscillation of quarks constituting a particle. Particle appearance quantum number (N) gives parity (P) of particles. Discrete energy levels for the hydrogen atom are obtained from the same formula with appropriate conditions.

1. INTRODUCTION

In the Standard Model (SM) [1] Higgs mechanism [2,3] gives masses to the W± and Z0 bosons by spontaneous symmetry breaking. In the current picture of Higgs mechanism W± and Z0 bosons and quark masses are generated by interactions with the vacuum field [4,5]. In the for the flavour changing transition probability between one quark to another [6] and, meson mass eigenstates are obtained by mixing quark flavour eigenstate [7]. In this work a formula for the mass-energies of hadrons and leptons (excluding neutrinos) are obtained. It is suggested that particles when oscillate in the electromagnetic or in the weak fields get their masses while massless gauge bosons (W±) acquire their masses by Higgs mechanism. This formula also gives the decay of hadrons. Spins of particles are shown to arise from the centre of charge of the rest mass-energy circulating around the centre of mass confined within the reduced Compton wavelength. This formula is applicable in calculating masses of particles arising from strong interactions via π0, ρ0, or ω0 coupled to the strong field by strong coupling constant αs. This formula explains the energetics of beta (β±) decay, and the maximum kinetic energy KE(β±)max of the beta particles. Dimension (d) of a 3-dimensional harmonic oscillator shows how charge (Q) in a particle builds up from the oscillation of the quarks constituting a particle. The quantum number N for the distance at which a particle appears gives the parity (P). The decay constants of pseudoscalar mesons are obtained from the zero point energy of the harmonic oscillators. Mass-energies of particles and their decay, beta decay and the discrete energy levels for the hydrogen atom obtained from one formula shows the importance of the work. For the first time important properties of particles, beta decay and decay of particles are explained in terms of a 3D quantum mechanical harmonic oscillator.

2. OSCILLATION OF PARTICLES AND HIGGS MECHANISM, SPIN AND PARITY
2.1. OSCILLATION OF PARTICLES AND HIGGS MECHANISM

From uncertainty principle the reduced Compton wavelength of a particle is thought of as the fundamental limitation on measuring the position of a particle i.e. this is the minimum deviation away from the equilibrium point at which a particle appears. Let us consider a particle at a distance \( X_p \geq \lambda_{pc} \) when,

\[
\lambda_{pc} = \frac{\hbar}{(m_0 p c)}
\]

where, \( \lambda_{pc} \) is the reduced Compton wavelength of the particle, \( \hbar \) the reduced Planck constant, \( m_0 p \) rest mass of the particle and \( c \) velocity of light. The Einstein-de-Broglie equation gives

\[
h\omega_0 p = m_0 p c^2
\]

when de Broglie suggested that \( \omega_0 p \) is the natural circular frequency of a particle. The particle property arises from the localized vibrational energy \( (h\omega_0 p) \) divided by \( c^2 \) so that the inertial mass \( m_0 p \) is given by [8,9]

\[
m_0 p = (h\omega_0 p)/c^2
\]

and, for the corresponding Compton wavelength,

\[
(\lambda_{pc} \omega_0 p) = [(h \omega_0 p) / (m_0 p c)] = c
\]

In the celebrated Schrödinger equation for wave number \( k = (1/\lambda_{pc}) \), Einstein-de-Broglie equation (2) is retained as

\[
m_0 p c^2 (1+2\alpha(0)) = 2h\omega_0 p
\]

when, Coulomb potential energy \( V(1/k) = \alpha(0)m_0 p c^2 \) and \( \alpha(0) \) is the fine structure constant.

From relativity \( E = \pm m_0 p c^2 \) where, \( \pm \) means a particle and an antiparticle respectively. When a \( W^\pm \) boson is displaced by a distance \( X_{\text{disp}} = D.\lambda w \), mass-energy state \( (\pm m_0 p c^2) \) of a particle appears at a distance \( X_p = N.\lambda_{pc} \) where, \( N \) and \( D \)'s are integers, and \( \lambda w \) the reduced Compton wavelength of the boson. These particles are attracted by the respective Coulomb force \((\pm F_p)\) between the particle and the boson. For neutrons a \( W^+ \) boson attracts a d-quark.

Particles/antiparticles are put into orbital motion by the emission of a \( W^- \) or a \( W^- \) boson which provides the centripetal force and for orbital motion centripetal force equals the Coulomb force. The charged \( W \) boson is connected with an oppositely charged or a neutral particle as if by a spring and this is put into oscillation due to the emission of a \( W^\pm \) boson from the particle.

Let us consider that the mass-energy state for a particle appears at a distance \( X_p = N.\lambda_{pc} \) away from the point of equilibrium when a \( W^\pm \) boson pops up at a
distance $X_{\text{disp}} = D \lambda w$. The particle is put into oscillation in a Coulomb field by the emission of a $W^\pm$ boson from the particle. For simple harmonic oscillation, the spring constant is $K$ given by $K = m_0 p (\omega_0 p)^2 = - (F_p / X_{\text{disp}})$. Let the displacement of the particle be $X_{\text{disp}} = D \lambda w$ and this is in the opposite direction of the Coulomb restoring force ($F_p$). For spring constant $K$ and from equation (4) as $\lambda p c_\omega p = c$, we get,

$$m_0 p c_2 = [(\alpha(0)) / (N D)] \cdot (M_w)$$

(6)

when, $M_w = m_w c^2 = (c\hbar) / (\lambda w)$, $N$ the particle quantum number, $D$ boson displacement quantum number, $\alpha(0) = [(q^2) / (4\pi \varepsilon_0 c \hbar)] = 1 / (137.03599)$, the fine structure constant [10], $q$ charge, and $\varepsilon_0$ permittivity of free space. Equation (6) tells that the rest mass-energy $(m_0 p c^2)$ of a particle is proportional to $(\alpha / (N D))$, the constant of proportionality being mass-energy $M_w = \pm m_w c^2$ of the charged boson $W^\pm$. The constant of proportionality imparts the properties of a gauge boson $(M_w)$ to the rest mass-energy of a particle. It is suggested that while a massless gauge $(W^\pm)$ boson acquires mass by Higgs mechanism [2,3], then any particle as well acquires mass in accordance with equation (6).

### 2.2. COUPLING CONSTANT

The charged current electroweak interaction is caused by the emission and absorption of $W^\pm$ bosons. In the beta decay $n(udd) \rightarrow p(uud) + W^- \rightarrow \bar{\nu} e (p(udd) + e^- + (\text{anti}) \nu e)$, Coulomb force between the quark $d(Q = -1/3)$ and $W(Q = -1)$ boson pushes the neutron into oscillation.

The coupling constants $\alpha(0)$ and $\alpha_w$ are used for the electroweak forces. In the strong interaction exchange of $\pi$-meson between nucleons takes the place of $W$ boson. At low energy when neutral current is zero, a gauge coupling constant $\alpha_w$ for weak charged-current interaction is obtained by Kushtan [11] as:

$$\alpha_w = (G_F M_w 2) / (4\pi 2^{1/2})$$

(7)

where, $G_F = $Fermi's constant$ = 1.166$ GeV$ ^2$, $M_w = 80.399$ GeV [10], so that $\alpha_w = 4.244 \times 10^{-3}$.

### 2.3. SPIN OF PARTICLES

A particle subjected to a restoring force undergoes a time varying displacement;
When $X_{\text{disp}} = T \cdot \lambda_{pc}$ instead of $X_{\text{disp}} = D \cdot \lambda w$ in equation (6), we get

$$m_0 p c^2 = \left[ \frac{\alpha(0) M w}{(T^2 D)} \right] = \left[ (\pm F p) \cdot (T \cdot \lambda_{pc})^2 \right] / (T^2 \cdot \lambda_{pc})$$

$$= (F p) \cdot \left[ (\lambda_{pc} / T) \right] \tag{8}$$

In the orbitting charge model for particles [Rivas,12], it is considered that all charges are concentrated in a point known as centre of charge (CC) which is spinning around the centre of mass (CM) at the constant speed of light (c) while for CM observer CM is at rest. The appearance of the CC is expressed in terms of an angle of rotation $T \cdot (2 \pi)$ after which the wavefunction of the particle returns to itself when $T$ is the number of rotations by an angle of $2 \pi$. Also, $T = (1 / S_z)$ where $S_z$ is the z-component of the spin. From equations (2) and (8) as $T \cdot m_0 p c^2 = (\lambda_{pc} / T)$, when $(\text{lop} \cdot \text{wop})$ is the angular momentum for rotation around the centre of charge, we get the Z-component of spin as

$$\text{lop} \cdot \text{wop} = (1 / T) \cdot h = S_z \cdot h \tag{9}$$

Thus, $T=2$, means $S_z=(1/2)$, the angle of rotation being $2 \cdot (2 \pi)$; $T=0$ means $S_z=0$ for rotation invariance; $T=1$ means $S_z=1$, the angle of rotation being $(2 \pi)$; $T=1/2$ means $S_z=2$ for rotation being $(\pi)$.

In the CM frame, from equations (8) and (9) when the centripetal force $[(m_0 p c^2) / \text{RCC}]$ balances the Coulomb force $F p$, the CC rotates in a non-radiating circular orbit of radius $\text{RCC} = \lambda_{pc} / T$ with velocity of light. For spin $\frac{1}{2}$ particles $\text{RCC} = (\lambda_{pc} / 2)$. This local microscopic circulatory motion of the CC confined within the reduced Compton wavelength $\lambda_{pc}$ is presumed to be the basis of spin and magnetic moment for particles. In the inertial reference frame the CC moves around the CM in a helical path with constant velocity $v=c$, while the CM moves with velocity $v<c$ perpendicular to the plane of rotation along the axis of the helix [12].

2.4. PARITY

The angular momentum quantum number [L] of a particle is given by $L = N - 1$ as in equation (6). Particle Data Group [13] gives parity (P) as

For mesons: $P = (-1)^{L+1}$

For baryons: $P = (-1)^L \tag{10}$

3. ZERO POINT ENERGY AND DECAY CONSTANTS FOR PSEUDOSCALAR MESONS
In Quantum Field Theory (QFT), each point in a field is a quantum harmonic oscillator. In QFT a field arises from the vibration of inter-connected springs and balls filling the vacuum space [14]. Excitations of the field gives elementary particles. The energy eigenvalues (E<sub>n</sub>) of a 3-D isotropic quantum mechanical harmonic oscillator at energy level n is given by [15]:

\[
E_n = \hbar \omega_0 (n + \frac{d}{2})
\]  

(11)

when, the position for the 3-dimensional oscillator is given by x<sub>1</sub>, x<sub>2</sub>, and x<sub>3</sub>. Mass-energy and kinetic energy of particles is obtained by transition to lower energy states(m) for m<n. From equations (6) and (11) the mass-energy and decay formula for a particle is given as:

\[
E_n = (m_0 p_0 c^2) \lambda n = \left(\frac{\alpha M_w}{N_2 D}\right)(n + d/2)
\]  

(12)

where, \(\hbar \omega_0 = (\alpha M_w)/(N_2 D)\), M<sub>w</sub> mass-energy of the W± boson, n the energy level of the particle of mass-energy=(m<sub>0</sub> p<sub>0</sub> c<sup>2</sup>) , and \(\alpha\) the coupling constant for the strong, weak or the electromagnetic fields in which the particle oscillates. The zero point energy (ZPE) from equation (12) is \(E_0 = [(\alpha M_w)/(N_2 D)](d/2)\) when d is the dimension of the oscillator. According to QFT, there is ZPE for strong, weak and electromagnetic interactions. ZPE drives the Compton oscillation of the wavepacket representing a particle at n=0 [16,17].

The charged pion decay amplitude \(<M^2>\) is given by [18]

\[
<M^2> = \left(\frac{1}{16}\right)[(g w^2 . f m^2)/(Mw^2)]^2 . mL(m p\pi - m l^2) |V_i j|^2
\]  

(13)

where, gw is the weak charge, fm charged pion decay constant, mL mass of lepton, mπ mass of pion, and Vi j KM matrix. In the SM the decay constant is determined from experiment. For quark-antiquark coupled to the W± boson, the lepton weak vertex factor is \((-igw/(8)^{1/2}.\gamma^u(1-\gamma^5))\) when, \(\gamma^u\) gives a vector current (V) and, \((\gamma^u.\gamma^5)\) gives the axial vector current (A). For themW interaction the pure vertex factor is given by \([(-igw/(8)^{1/2}).f m].p\pi\mu\) when, \(p\pi\) is the momentum of \(\pi\) [19]. Taking zero point energy equal to the energy of the weak interaction as obtained from the vertex factor, from equations (12) and (13) we get,

\[
[(gw)/(8)^{1/2}].f m = (\alpha M_w)/(N_2 D)
\]  

(14)

4. DIMENSION (d) AND OSCILLATION OF QUARKS

The values for dimension (d) are obtained considering that quarks constituting a particle also oscillate and this oscillation builds up charge Q in a particle. It was found that the charge Q is explained by the number of quarks (Nq) constituting a
particle and dimension \( d \) of the oscillator by the relation as given below:

\[ Q = \pm (Nq - d) \quad \text{(15)} \]

where, the + sign is for the particle and the – sign for the anti-particle. For neutral particles as \( \Sigma Q_{\text{quark}} = 0 \), we get \( Nq = d \). These values of \( d \) are used in calculating the hadron masses from equation (12). The significance of dimension \( d \) in building up charge in a particle in accordance with equation (15) is outlined below:

(a) For the mesons, when \( d = 1 \) the oscillating quarks are coupled as \( x_1 = x_2 \) giving \( Q = \pm 1 \). For baryons \( d = 1 \) gives \( x_1 = x_2 = x_3 \) i.e. all the quarks are coupled giving \( Q = \pm 2 \). (b) For mesons, when \( d = 2 \), \( x_1 \neq x_2 \) i.e. the quarks are not coupled when oscillation giving \( Q = 0 \). For baryons when \( d = 2 \), \( (x_1 = x_2) \neq x_3 \) gives \( Q = \pm 1 \) where two of the oscillating quarks are coupled and the the third quark oscillates being uncoupled to the two coupled quarks. (c) For baryons \( d = 3 \) means \( x_1 \neq x_2 \neq x_3 \) i.e. all the quarks oscillate randomly as they are uncoupled giving \( Q = 0 \).

From equation (15) changes in \( Q, Nq \) and \( d \) are given as

\[ \Delta(Q) = \pm (\Delta(Nq) - \Delta(d)) \quad \text{(16)} \]

For flavour changing weak charged-current interactions when \( \Delta(Nq) = 0 \), we get \( \Delta d = -\Delta Q \) i.e. dimension of the QMHO increases with decrease in \( Q \). Flavour change produces change in charge and dimension of a particle. When \( \Delta Nq = 0 \) and \( \Delta d = 0 \), we get \( \Delta Q = 0 \). From equation (15) and Gell-Mann Nishijima formula, isospin \( I_3 \) in this model is given as

\[ I_3 = \pm ((Nq - d) - \frac{Y}{2}) \quad \text{(17)} \]

and hypercharge \( Y \) is given in the SM as:

\[ Y = S + C + B + T + B \quad \text{(18)} \]

when, strangeness \( (S = -1) \), charm \( (C = 1) \), bottomness \( (B = 1) \), topness \( (T = 1) \), and baryon number \( (B = 1) \) are the quantum numbers in the Standard Model. Weak hypercharge \( Y_w = 2(Q - I_3) \) plays the same role in weak interactions as the charge \( Q \) in electromagnetic interaction.

5. RESULTS AND DISCUSSIONS

5.1. PARTICLES AND THEIR DECAY

From equation (12) we get mass-energy formula for particles as
\[ E_n = \left( \frac{\alpha M_w}{N^2 D} \right) (n + d/2) \quad (19) \]

The decay process of a particle is explained by the transition to the lower mass-energy state \( m \) for \( m < n \). From these transitions the kinetic energy (KE) of a neutrino is inferred.

5.1.1. MASS-ENERGY OF \( (\pi^\pm) \) MESONS AND THEIR DECAY

Mass-energy

The \( \pi^\pm \) decays by weak interaction into a lepton-neutrino pair \([20,21]\) as shown below:

\[ \pi^+ \rightarrow \mu^+ + \nu_\mu \quad (\text{muon neutrino}) \quad (20) \]

For weak interaction \( \alpha = \alpha_w = 4.244 \times 10^{-3} \), and for \( N = 1, D = 11, d = 1, n = 4 \), the mass-energy of \( \pi^\pm \) is 139.58(139.57) MeV.

Decay

For \( E_4 \rightarrow 3 \) decay, as \( \Delta d = 0 \) and \( \Delta Nq = 0 \), we get \( \Delta Q = 0 \), so that the emitted particle arising from this transition is uncharged e.g. a neutrino, and neglecting mass-energy of the neutrino, the KE(\( \nu_\mu \)) = \( E_4 - E_3 \approx 31.0 \) (29.8) MeV \([22]\). The particle arising from \( E_3 \rightarrow X = \infty \) has \( Q = \pm 1 \) and, KE(\( \mu^+ \)) = \([E_3 - m_\mu c^2] = (108.56 - 105.66) \) MeV = 2.90(4.12) MeV \([22]\).

From equations (15) to (18), \( I_3 = \pm 1 (\pm 1) \), and \( Q = \pm 1 (\pm 1) \), and \( P = -1 (1) \) when the SM values are given in brackets. Fig.1 depicts the energy level diagram for the decay of \( \pi^\pm \) meson.

<table>
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<th>N</th>
<th>D</th>
<th>n</th>
<th>En MeV</th>
<th>( \Delta E ) MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11</td>
<td>4</td>
<td>139.58</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>11</td>
<td>3</td>
<td>108.56</td>
<td>31.00</td>
</tr>
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<td>11</td>
<td>x</td>
<td>0(Xp=∞)</td>
<td>108.56</td>
</tr>
</tbody>
</table>

Fig.1. Energy level diagram for the decay of \( \pi^\pm \) meson into \( \mu^+ \) and \( \nu_\mu \).

5.1.2. MASS ENERGY OF \( K^\pm \)

Charged kaons decay by weak interaction as shown in the Feynman diagrams \([20,21]\). For \( \alpha = \alpha_w \), and \( N = 1, D = 10, d = 1, \) and \( (n+d/2) = 14.5 \), mass-energy of \( K^\pm \) is
obtained as 494.74 (493.70) MeV.

Present Model values of isospin \(I_3\), charge \(Q\), and Parity \(P\) are given with SM values in the bracket as \(I_3=\pm 1/2(\pm 1/2)\), \(Q=\pm 1(\pm 1)\), and \(P=-1(-1)\).

5.1.3. NEUTRAL KAON AND STRONG COUPLING CONSTANT \(\alpha_s\)

Neutral kaons are produced in strong interaction given by \(\pi(ud) + p(uud) \rightarrow K^+ (us) + K^0 (sd) + p(uud)\) and they decay via weak interactions \([20,21]\). These suggest that mass-energies of \(K^0\) obtained from equation (19) for both electroweak and strong interactions are the same so that

\[
\alpha_s = \frac{(m_0 p_c c^2)_{\text{em}} (D_s)}{(M_s (n_s + d/2))} \quad (21)
\]

when, subscripts s and em stand for strong and electroweak interactions respectively, \(M\) mass-energy of the exchange particle for strong interaction \((\pi^0 = 135\text{MeV})\). For electroweak interaction, \(\alpha(0)_{\text{em}} M_w = 586.7\text{MeV}, \) \(D_{\text{em}} = 13\), \((n_{\text{em}} + d/2) = 11\) gives \((m_0 p_c c^2)_{\text{em}} = 496.44 \text{MeV}\) so that from equation (20) for \(D_s = 1, (n_s + d/2) = 31\), we get, \(\alpha_s = 0.1186(0.1184)\) \([10]\).

5.1.4. MASS-ENERGY AND DECAY OF \(\Sigma^0\) AND \(\Lambda^0\) BARYONS

Decays of \(\Sigma^0\) and \(\Lambda^0\)

The decays \(\Sigma^0 \rightarrow \Lambda^0 + \nu\) and \(\Lambda^0 \rightarrow p + \pi^-\) are explained by weak interactions \([20,21]\). For \(N=1, D=9, d=3, (n+d/2)=30+1.5\), and \(\alpha = \alpha_w = 4.24\times 10^{-3}\), the mass-energy of \(\Sigma^0\) is 1193.68 (1192.64) MeV, and, \(\Sigma^0\) decays to \(\Lambda^0\) of mass-energy 1115.68 (1115.68) MeV when \(KE(\Lambda^0)_{\text{max}} = 2.21\text{MeV}\) for \((n+3/2) = 29.5\) from the transition \(E_{30} \rightarrow E_{28}\). From equation (15) as \(\Delta N_q = 0\) and \(\Delta d = 0\), the emitted mass-energy does not carry any charge. It is suggested that a gamma ray(\(\nu\)) of energy 75.79 MeV is emitted. The \(\Sigma^0\) is an excited state of \(\Lambda^0\) as there is no change in flavour when \(\Delta d = 0\) in \(\Sigma^0 \rightarrow \Lambda^0\) decay and strangeness quantum number (S) is conserved in this decay.

Decay of \(\Lambda^0\)

For \(\alpha_w = 4.24\times 10^{-3}\), \(N=1, D=9, d=1,\) and \((n+1/2) = 4.5\), \(E_4 = 170.525\text{MeV}\) giving \((E_2 - E_4) = 1115.68 - 170.525 = 945.155\text{MeV}\). From equation (16), \(\Delta Q = (\Delta N_q) - (\Delta d) = -1 + 2 = +1\) giving \(Q = +1\) for the emitted particle. The \(KE(p^+)_{\text{max}} = (945.155 - 938.27)\text{MeV} = 6.89\text{MeV}\), and for the transition to \(X_p = \infty\) gives \(KE(\pi^-)_{\text{max}} = (170.525 - 139.57)\text{MeV} = 30.96\text{MeV}\) when \(\Delta Q = -1\) for conservation of charge. The predicted values for the KE max’s of the decay products need experimental confirmation.
5.2. BETA (β±) DECAY

The beta (β±) decay process is explained as weak interaction from equation (12) with the help of Feynman diagrams [20,21]. In the weak charged current interaction quarks change flavour by emitting or by absorbing a virtual W± boson [23]. For example a neutron becomes a proton by emitting a virtual W- boson which decays into β- particle and an anti-neutrino (νe) [24,25]. The processes of β- decay are shown below:

\[ n \rightarrow p + W^- \rightarrow p + e^- + \nu e (\text{antineutrino}) \]  

(22)

The flavour change at quark level is given by

\[ d \rightarrow u + W^- \rightarrow u + e^- + \nu e \]  

(23)

The β- decay is given by

\[ p \rightarrow n + e^- + \nu e \ (\text{neutrino}) \]  

(24)

At quark level this is explained as

\[ u \rightarrow d + W^+ \rightarrow d + e^+ + \nu e \]  

(25)

The emitted positron annihilates with the nearest shell electron giving rise to two photons as \( 2m_e c^2 = 2h\gamma \).

The mass-energy difference \( (\Delta M_0 c^2) = [m(Z,A)-m(Z\pm1,A)]c^2 \) between the parent and daughter atoms gives the released energy (Q) as

\[ Q(\beta^-) = [m(Z,A)-m(Z+1,A)]c^2; \]

\[ Q(\beta^+) = [m(Z,A)-m(Z-1,A)-2m_e]c^2 \]  

(26)

In equation (25), \( Q(\beta^-) \) gives KE’s of the β- particles and gamma energy (\( \mathcal{E}_{\text{gam}} \)) from excited nuclei neglecting the energy of neutrinos. Equation (15) gives \( d=1 \) for a point like beta particle. From equation (12) for beta decay the atomic mass-energy difference \( (\Delta M_0 c^2) \) between the parent and daughter atoms considering weak interaction (\( \alpha = \alpha w \)) and for the Xdisp=(D.\( \alpha w \cdot \lambda w \)), we get,

\[ \mathcal{E}_n = [\Delta M_0 c^2]n = [(\alpha w^2 \cdot Mw)/(N^2. D)](n+1/2) \]  

(27)

The KE\( \text{max} \) for β-decays arise from \( [\mathcal{E}_n - \mathcal{E}(X=\infty)] \) transitions and for β-decays arising from collisions with excited daughter nuclei. Excited nuclei come to lower energy states by emitting gamma rays of energy \( \mathcal{E}_\gamma = (\mathcal{E}_n - \mathcal{E}_m) \) where \( n>m \). The proposed model accurately explains β± decay energies.
5.2.1. SINGLE β- DECAY TO GROUND STATE

A neutron outside the nucleus is not stable and decays as

\[ 1n^0 \rightarrow ^1p + \beta^- + \nu_e \text{ (anti neutrino)} \]  \hspace{1cm} \text{(28)}

As \((mn-mp).c^2 = 1.292 \text{ MeV [10]}\), maximum calculated kinetic energy of \(\beta^-\) is \(KE(\beta^-)_{\text{max}} = 0.782 \text{MeV}\). As mass of a neutrino is very small, it is neglected. From equation (27), for \(N=1, D=12, (n+1/2)=6.5\), as \(\Delta \epsilon = \epsilon_6 - \epsilon(X_p=\infty) = KE(\beta^-)_{\text{max}} = 0.784(0.781) \text{MeV}\) when the calculated value of KE is given in bracket.

5.2.2. SINGLE β- DECAY TO AN EXCITED STATE

For this process:

\[ E = \beta^- + KE(\beta^-)_{\text{max}} + \epsilon_{\text{gam}} \]  \hspace{1cm} \text{(29)}

\(\epsilon_{\text{gam}}\) is the energy of gamma ray. For example the process as

\[ ^{198}\text{Au} \rightarrow ^{198}\text{Hg} + \beta^- + KE(\beta^-)_{\text{max}} + \epsilon_{\text{gam}} \] \hspace{1cm} \text{(30)}

(i) For \(N=1, D=9\) and \((n+1/2)=8.5\) gives \(KE(\beta^-)_{\text{max}} = 1.37(1.37) \text{MeV}\).

(ii) For \(N=1, D=9, (n+1/2)=6.5, \epsilon_6 = \epsilon_{\text{gam}} = 1.05(1.09) \text{MeV}; KE(\beta^-)_{\text{max}} = (\epsilon_8 - \epsilon_6) = 0.32(0.285) \text{MeV}\).

(iii) For \(N=1, D=9, (n+1/2)=2.5, \epsilon_2 = \epsilon_{\text{gam}} = 0.40(0.41) \text{MeV}; KE(\beta^-)_{\text{max}} = (\epsilon_8 - \epsilon_2) = 0.97(0.96); \epsilon_{\text{gam}} = (\epsilon_4 - \epsilon_2) = 0.63(0.65) \text{MeV}\). All observed values are given in brackets.

5.2.3. POSITRON (β+) EMISSION

This is given as:

\[ p \rightarrow n + \beta^+ + KE(\beta^+)_{\text{max}} + \epsilon_{\text{gam}} \] \hspace{1cm} \text{(31)}

For the \(\beta^+\) emission as shown below [26] :

\[ ^{15}\text{O} \rightarrow ^{15}\text{N} + \beta^+ + KE(\beta^+)_{\text{max}} \] \hspace{1cm} \text{(32)}

For \(N=1, D=5, (n+1/2)=9.5\): \(KE(\beta^+)_{\text{max}} = 1.73(1.735) \text{MeV}\).
5.3. LEPTONS OTHER THAN NEUTRINOS

**Electron:** From equation (19) for \( N=10, D=10, d=1, (n+d/2)=1.5 \) and \( \alpha \omega M_w = 341.2 \text{MeV} \), giving for electron \( m_e c^2 = 0.512 \text{(0.511)MeV} \), and KE plus mass-energy of \( \nu e = 0.8 \text{KeV} \). At \( n=0 \), we get the vacuum expectation value for the electron as 0.17MeV i.e. one-third of its mass-energy.

**Muon:** For muon, \( N=1, D=8, d=1, n=2, \) and \( \alpha = \alpha w, E_2 = 106.625 \text{(105.66)MeV} \), and \( E_1 = 63.975 \text{MeV} \).

As charge must be conserved, one of the muon decay products must be an electron so that the muon decay is as given below:

\[
\mu^- \rightarrow e^- + \nu e + \nu \mu
\]  
(33)

From equation (15) as \( \Delta Q=0 \) for \( \Delta d=0 \), charges for mass-energy states 1 and 2 are the same, and the particle arising from \( E_2 \rightarrow E_1 \) transition is thus neutral. We assign \( E_1-E_2=42.65 \text{MeV} \) as the mass-energy plus KE of muon neutrino(\( \nu \mu \)). The KE of the electron comes from the transition \( E_1 \rightarrow X(p)=\infty = 63.98(50.0) \text{MeV} \) when calculated value is given in bracket. The charge at \( n=1 \) is \( Q_1=(Nq-d^1)=(0-1)=-1 \) as \( Nq=0 \) considering leptons as poinlike particles.

**Taun:** For the tauon, \( N=1, D=7, d=1, n=36, E_{36}=1778.3(1776.8) \text{MeV} \), K.E. of \( \nu \tau < 1.4 \text{MeV} \).

5.4. THE HYDROGEN ATOM

For neutral hydrogen atom \( (Q_p+Q_e)=0 \) gives \( d=Nq=2 \) and the factor \( (n+d/2)=(n+1)=n \). Replacing MW by \( m_e c^2 \), \( X_{\text{disp}}=-(2n)(\alpha - \lambda e) \) and \( N=n \), from equation (6) the energy of electron in the hydrogen atom \( (E)H_n \) is obtained for \( \alpha = \alpha (0) \) as

\[
(E)H_n = \left[ \alpha (0)^2 m_e c^2 \right] \left[ (n+1)/(N^2.D) \right] \\
=-\left[ e^4 m_e / (8 \epsilon_0 h^2) \right] [1/n^2] \\
\]  
(34)

Equation (34) means that when an electron oscillating in an electromagnetic field, say, due to a proton, is coupled to the electromagnetic field by coupling constant \( \alpha (0) \) and when it is displaced by \( X_{\text{disp}}=-(2n)(\alpha - \lambda e) \), then discrete energy levels same as those in the Bohr’s model appear.
From equation (16) as $\Delta d=0$ and $\Delta Nq=0$, then $\Delta Q=0$ for any change in $n$, so that the energy absorbed or released is due to an uncharged particle and conservation of spin means this particle is a photon. The radius of the $n$-th stable Bohr orbit is given by

$$R_n=n. \lambda DB$$

(35)

where, $\lambda DB$ is the reduced de Broglie wavelength. Applying condition (34), radii of stable atomic orbitals are obtained. Also we get,

$$\lambda DB = \alpha^{-1}. \lambda e$$

(36)

From equations (35) and (36), we get, $R_1=0.529x10^{-10}m(0.529x10^{-10}m)$ when the value obtained from Bohr’s theory is given in bracket.

5.5. DECAY CONSTANTS OF PSEUDOSCALAR MESONS

Quark-antiquark annihilates via a virtual $W^+$ boson to the final $l+\nu$ state for $\pi^+$, $K^+$, $D^+$ and $D_s^+$ pseudoscalar mesons are reported by the Particle Data Group (PDG) from experimental data of various sources with radiative corrections [27]. There are considerable differences in the radiative corrections from different groups in these results. Values of decay constants for these pseudoscalar mesons obtained from equation (14) with PDG values in the bracket are given below:

For $\pi^+$ decay as $N=1$, $D=11$, we get $f_{\pi}=133.3(131.1)\text{ MeV}$;

similarly, for $K^+$ decay, as $N=1$ and $D=10$, $f_K=146.4(156.1)\text{ MeV}$;

For $D^+$ decay, as $N=1$, $D=7$, $f_D=209.2(206.7\pm11)\text{ MeV}$;

For $D_s^+$ decay, as $N=1$, $D=6$, $f_{D(s)}=244.1(257.6)\text{ MeV}$.

7. CONCLUSIONS

(1) The present model of particles oscillating in different fields due to Coulomb force, explains mass-energies of particles and their decay, beta decay and kinetic energies of beta particles, mass-energies of leptons excluding neutrinos and other properties of hadrons from a single formula. (2) Dimension (d) of the quantum mechanical harmonic oscillator explains build up of charges due to oscillation of the coupled and uncoupled quarks constituting a particle. (3) Unlike SM where there are 19 free parameters in calculating masses, in the model
presented here masses are obtained by one constant($\alpha$) and three hand selected quantum numbers (D,d,n) having physical significance. (4) The $\beta^\pm$ process is explained by the change in difference between the atomic masses of the parent and daughter atoms for $X_{\text{disp}}=(D\alpha w+\lambda w)$ and $\alpha=\alpha w$. (5) The energies of the orbital electron of the hydrogen atom are obtained from the formula presented in this work.

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A FORMULA OF PARTICLES OSCILLATING IN ELECTROMAGNETIC AND WEAK FIELDS GIVING ENERGY AND DECAY OF PARTICLES, BETA DECAY AND ENERGY LEVELS OF HYDROGEN ATOM:

PHYSICS