Adjoint-state waveform inversion using the S-net system for tsunami source imaging and early warning

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April 4, 2023

Abstract

We explore the potential of the adjoint-state full waveform tsunami inversion method for tsunami warning and source imaging using S-net, an array of ocean bottom pressure gauges. Compared to finite-fault tsunami source inversions, the method we use does not require as densely gridded Green’s functions to obtain a high resolution result, thus reducing computation time. What is required is a dense instrument network with good azimuthal coverage. The S-net pressure gauges fulfill this requirement and reduce the data collection time, thus making it possible to invert the recordings for the tsunami source and issue a timely tsunami warning. We apply our method to synthetic waveforms of the 2011 Mw 9.0 Tohoku earthquake and tsunami as well as data from the 2016 Mw 6.9 Fukushima earthquake. The results of the synthetic tests show that using the first 5 minutes of the waveforms, the adjoint-state inversion method achieves good performance with an average accuracy score of 93\%, with the error of predicted wave amplitudes ranging between -5.6 to 1.9 m. Our application to the 2016 Fukushima earthquake shows the required waveform duration to achieve accurate inversions for smaller events is longer than that of a larger event. However, using the first 25 minutes of the waveforms, the inversion yields a tsunami source that is sufficient for making accurate predictions of arrival times and amplitudes. Assuming a uniformly distributed fault slip, we estimated a stress drop for the latter event to be 4.6 MPa, which is in line with estimations from recent studies.
Adjoint-state waveform inversion using the S-net system for tsunami source imaging and early warning

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Key Points:

• We use an adjoint-state tsunami inversion method that images tsunami sources directly from ocean bottom pressure gauge (OBPG) recordings.
• The OBPG recordings of S-net provide sufficiently uniform azimuthal coverage within a short collection time for accurate adjoint inversions.
• Our case study of the 2016 Fukushima earthquake shows seafloor deformation consistent with shallow slip on a steep normal fault.

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Abstract

We explore the potential of the adjoint-state full waveform tsunami inversion method for tsunami early warning using S-net, an array of off-shore ocean bottom pressure gauges. Compared to finite-fault tsunami source inversions, the method we use does not require as densely gridded Green’s functions to obtain a high resolution result, thus reducing computation time. What it does require is a dense instrument network with good azimuthal coverage. The density and coverage of the S-net pressure gauges fulfill this requirement and reduce the data collection time, thus making it possible to invert the recordings for the tsunami source and issue a timely tsunami warning. We apply our method to synthetic waveforms of the 2011 $M_w$ 9.0 Tohoku earthquake and tsunami with triggered secondary sources as well as the data from the 2016 $M_w$ 6.9 Fukushima earthquake. The results of the synthetic tests of the 2011 $M_w$ 9.0 earthquake show that using the first 5 minutes of the waveforms, the adjoint-state inversion method achieves good performance with an average accuracy score of 93%, with the largest error of predicted wave amplitudes ranging between -5.6 to 1.9 m. The secondary sources are clearly resolved using the first 20 mins of the waveforms. Our application to the 2016 Fukushima earthquake shows the required waveform duration to achieve accurate adjoint inversion for smaller events is longer than that of a larger event. However, using the first 25 minutes of the waveforms, the inversion yields a tsunami source that is sufficient for making accurate predictions of arrival times and amplitudes. Assuming a uniformly distributed fault slip, we estimated a stress drop for the latter event to be 4.6 MPa, which is in line with estimations from recent studies.

Plain Language Summary

Issuing timely tsunami warnings is essential in allowing at-risk populations to have adequate evacuation time. However, traditional frameworks for issuing warnings rely on accurate models for the earthquake that caused the tsunami. These models can be inaccurate in the immediate aftermath of an earthquake and can lead to underestimated wave heights and late wave arrival time predictions. To address this issue, we have developed a method that can yield the tsunami source without the need for an earthquake model, simply relying on the wave height data recorded by pressure sensors at the bottom of the ocean. This can also be useful when tsunami-causing events other than earthquakes (e.g. volcanic eruptions) occur. We test this method on recorded data from the 2016 Fukushima earthquake’s tsunami as well as synthetic waveforms from the 2011 Tohoku tsunami. Our results for the former show accurate predictions of wave arrival times and heights when we use the first 25 minutes of the wave height data. For the latter, predictions are accurate when we use the first 5 minutes of the data.

1 Introduction

Fast and accurate estimation of tsunami source is essential for tsunami early warning. Compared to the methods of tsunami source estimation based on seismic waves, the methods that utilize tsunami waves are more accurate and are particularly useful in scenarios such as submarine volcanic eruptions and landslides. However, the low density of tsunami stations limits the application of the tsunami observation for tsunami early warning.

After the 2011 Tohoku earthquake, an offshore deep-ocean observation network, S-net (Seafloor Observation Network for Earthquakes and Tsunami) was constructed, which covers the major potential regions of tsunami sources along the Japan Trench. The high density and large coverage of the S-net pressure gauges shorten the data collecting time, which makes it possible to invert the recordings for the tsunami source and issue a tsunami warning. Several methods based on tsunami recordings have been developed and tested using synthetic waveforms and recordings of S-net pressure gauges (e.g. Aoi
The issue with using traditional finite-fault tsunami source inversion methods is that to attain a high resolution, one needs densely-gridded Green’s functions, which is not always possible to implement in the immediate aftermath of a tsunami. Pre-calculated Green’s functions can curtail this issue. However, covering a large expanse of the ocean requires a coarse grid or a large amount of spacing within the Green’s functions, leading to a heavy computational burden. For example, if we mesh the entire north-eastern Pacific Ocean from the Aleutian Islands to the coast of the State of Washington with a 1 arc-minute spacing, the total number of grids will be about 2000000, which requires several months or even years to calculate the Green’s functions with hundreds of CPU cores. On the other hand, the adjoint-state tsunami inversion is suitable for real-time application because it recovers accurate initial water elevation in only a few iterations (Zhou et al. (2019)). So, the adjoint-state full-waveform inversion of the tsunami source has two major benefits compared to the traditional finite-fault tsunami source inversion method: high computational efficiency and high resolution. This allows the tsunami source image to be used quickly and directly in forward tsunami simulation, producing accurate forecasts of tsunami height and arrival time.

However, an ideal adjoint state inversion needs dense near-source stations with uniform azimuthal coverage, which makes it difficult to be applied to early warning in most subduction zones. S-net fits the requirement of the adjoint state inversion method very well and provides an opportunity to test the performance of the method to be used for early warning. The large scale and near-equal spacing of the S-net stations guarantees the uniform azimuthal coverage within a short time (within ~ 30 min). Moreover, the offshore ocean bottom pressure gauges (OBPGs) are preferred for the adjoint state method compared to tide gauges for three main reasons: (1) ocean-bottom pressure gauges do not get affected by wave reflections off the coast as much as tide gauges (2) wave speeds are not as affected by the resolution of bathymetry data and mesh size for computation as tide gauges, (3) the nonlinear effect of propagating water waves is negligible since the OBPGs are located at large depths.

In this study, we applied the adjoint-state inversion method to synthetic waveforms of the 2011 $M_w$ 9.0 Tohoku earthquake and tsunami in addition to the S-net data of the 2016 $M_w$ 6.9 Off Fukushima earthquake, the largest tsunami event in the near field so far. S-net is designed to monitor the tsunami generated by megathrust earthquakes like the Tohoku earthquake. Here we applied our adjoint state method to synthetic waveforms to demonstrate its effectiveness using the OBPGs network for rapid source imaging and early warning purposes. We also add multiple secondary sources to the Tohoku earthquake to test its resolvability for possible triggered submarine landslides. We briefly describe our method in section 2 and the pre-processing of our data in section 3. In section 4, we report the results of applying our adjoint method to synthetic waveforms of the 2011 $M_w$ 9.0 Tohoku earthquake and recorded tsunami waveforms of the 2016 $M_w$ 6.9 Fukushima earthquake. In section 5, we discuss what our results tell us about the accuracy of the adjoint inversion method for large and small tsunamigenic earthquakes as well as its potential in tsunami early warning systems. We also show the results of our inversion for the source parameters of the 2016 event and compare them to that of other studies.

2 Method

The adjoint-state tsunami source inversion method is a numerical technique, developed by Zhou et al. (2019), citation hereafter referred to by Z19, to solve for the tsunami source (initial water elevation) by minimizing the difference (misfit) between the observed

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and predicted tsunami waveforms. It is inspired by the full waveform inversion methods in exploration seismology, which takes advantage of the rich information embedded in the full wave field. Extensive synthetic tests conducted in exploration geophysics show that the adjoint-state method improves the resolution and balances the spatially unevenly distributed recordings (Virieux and Operto (2009); Feng and Schuster (2017); Sun et al. (2016)).

A key component of the adjoint-state method is the use of time-reversal imaging to obtain an initial model for the tsunami initial water elevation (IWE). Time-reversal imaging of the tsunami source was first done by Hossen et al. (2015) and involves reversing observed waveforms (how wave heights vary in time at a particular location) in time and back-propagating them using station locations as point sources. The idea is that the resulting wavefields (how wave height varies with space) at the end of the reversed time series will constructively interfere at the location of the original source, with some amplitude and locational distortion depending on station density and azimuthal coverage.

Following the workflow from Z19, time-reversal imaging is first applied to our observed waveforms to obtain the initial model of the tsunami source. The source is then forward propagated to station locations and subtracted from observed waveforms to obtain the data residue. The misfit between observations and predictions is then calculated using the L2-norm misfit functional (equation 5 in Z19). Next, we minimize this misfit with each iteration as much as possible. Plessix (2006), Tarantola (1986) and Tromp et al. (2004) showed that the gradient of the misfit function (equation 6 in Z19) with respect to source parameters is equivalent to the adjoint wavefield, obtained by back-propagating the data residue at each station to the original source via time-reversal imaging. The gradient is then used to update the source model using a conjugate direction scaled by a certain step length, as done in the conjugate gradient method (equation 9 in Z19). The conjugate method is a numerical method of gradient descent that minimizes the difference between observed and predicted waveforms. The size of the step length is itself scaled (by empirically obtained values of 2 or 0.5) at each iteration depending on which value results in smaller data residue after propagating the new source. With a new source obtained, the algorithm is repeated for the specified number of iterations, usually until convergence of the residue occurs.

For the time-reversal and the forward simulations, we use the Parallel Cornell Multi-grid Coupled Tsunami modeling package (PCOMCOT; Liu et al. (1998); An et al. (2014); X. Wang and Liu (2006); Zhu et al. (2021)) in the spherical coordinate system. This package uses finite difference methods to solve the shallow water equations in Cartesian or spherical coordinate systems. The calculation region (138° E to 148° E, 32° N to 45° N) covers the entire region of S-net and is divided into 400 by 520 grids. We use a calculation grid size of 90 arcseconds, which shortens the calculation time to less than 5 s for 1 forward simulation with 16 CPU cores and yields relatively accurate results in the open sea, which is less sensitive to the bathymetries compared to coastal regions.

3 Data Processing

3.1 Observed and synthetic recordings of S-net

The S-net is a seafloor observation network which records both water pressure and seismic signals. It was installed starting in 2016 in response to the devastation caused by the 2011 Tohoku tsunami. These sensors measure the absolute water pressure and are manufactured by Paroscientific, Inc. (e.g. Pulster et al. (2009); Watts and Kontoyianis (1990)). The water pressure is transferred to the pressure sensors which are sealed in a metal house via a diaphragm made of hard rubber. The observed pressure is then time-stamped based on GPS information and transmitted to a landing station for con-
version and final transmission to the data center (Aoí et al. (2020)). The S-net is capable of observing tsunamis with amplitudes less than 1 cm (Kubota et al. (2018)).

At the time this study was conducted, the largest recorded tsunami event close to the network was the 2016 $M_w$ 6.9 Fukushima earthquake. The locations of all currently installed stations that are used for the synthetic test are shown in Figure 1. The stations located at the outer-rise region were not yet installed in 2016, so we only use data from 125 stations for the analysis of the 2016 $M_w$ 6.9 Off Fukushima earthquake. To understand the performance of the adjoint approach using S-net for future events on the scale of the 2011 Tohoku tsunami, we generated the synthetic tsunami waves for this event at the locations of all currently installed S-net stations. In our first synthetic test, we adopt a slip model of the 2011 $M_w$ 9.0 Tohoku earthquake (Figure 2) inferred from tsunami data (Fujii et al. (2011)). In the second synthetic test, in addition to the signal of the Tohoku earthquake, we added multiple secondary sources representing triggered submarine landslides assuming they emerge simultaneously with the mainshock. Each secondary source is a binary square source with an uplift and a subsidence of 20 meters on each side. The side length of the square is 20 km (Figure 2). We use bathymetry with a resolution of 15 arcsec, which is finer than the 90 arcsec resolution used in the adjoint inversion. This difference in resolution is designed to test the effect of bathymetry error on the accuracy of the adjoint inversion.

### 3.2 Waveform window selection

We previously proposed to select a window which contains only the first complete peak and trough for the adjoint inversion to avoid artifacts caused by the reflected wave at the tide gauges near the coast (Z19). However, since the S-net OBPGs are located in the open sea, there are no reflected waves within 10 min of the tsunami origin time at most of the stations. We confirmed this by running inversions on our synthetic data with and without the peak-and-trough window selection method. The results of this confirmation (Figure S1) show that the method of choice makes little difference in misfit and IWE results. Consequently, we opted to use all the available waveforms (locations shown in Figure S2) from the origin time to $t_0$, where $t_0$ is the time when the tsunami warning is issued. We discuss the shortest $t_0$ that achieves an accurate warning in section 5. Using all the available waveforms is also a simpler strategy for automated implementation since we do not need to consider a specific window for each station.

### 3.3 Filtering and artifacts removing

One issue with the recordings of S-net ocean bottom pressure gauges is that the initial waveforms of the stations close to the source are contaminated by tsunami-irrelevant steps (Kubota et al. (2018); Kubota et al. (2021); Nakata et al. (2019)). These tsunami-irrelevant steps can be caused by tilt due to ground shaking or long-term mechanical drift. Moreover, the tsunami signal mixes with seismic waves and Earth’s tides. Therefore, after resampling the waveforms to 2 Hz, we low-pass filter (100 s) them to remove the seismic signal following Kubota et al. (2018). Then, the tsunami-irrelevant steps are removed if the time derivative at a particular time is greater than 0.0005 m/s. We show examples of this correction in Figure S3c. Finally, a high-pass filter (1000 s) was applied to remove tidal effects. Details about this processing can be found in the supplementary information (Text S1, Figure S3).

### 3.4 The estimation of the coseismic displacement at the stations

Tsunami waves are a type of surface gravity wave. The backward propagations in the adjoint state inversion process require the histories of the water surface elevation at each station. As shown in Figure 3, the water elevation (black lines) is the sum of the water column height (blue lines) and the vertical seafloor displacement. However, the
OBPGs measure the in-situ pressure which can be converted to the height of the water column above a station. We assume the water elevation and water column height are equal when the seafloor displacement is small compared with the tsunami signal. Since the coseismic seafloor displacement attenuates rapidly over distance, this assumption is valid for distant stations (station c in Figure 3). In the case of the Tohoku earthquake, for the stations that are more than 50 km away from the principal-slip region (Slip $> 5$ m), the coseismic displacement is relatively small ($< 1$ m) compared to the tsunami height ($> 4$ m). However, for the stations within the principal-slip zone, we need to add the coseismic displacement to the measured water column height to obtain the water elevation (stations a and b in Figure 3). We assume the IWE at the beginning of the tsunami propagation is equivalent to the vertical coseismic displacement at each station. The coseismic deformation can be estimated with the permanent change of equilibrium pressures (Figure 2b) before and after the earthquake. When a long enough recording after the earthquake is available, this is not difficult to estimate as shown by Tsushima et al. (2012) and Inoue et al. (2019). However, for warning purposes, we need to estimate the coseismic deformation in a short time. For the stations within the principal-slip region, since the primary tsunami signal passes by quickly (in the first 5 to 10 min), we can assume the equilibrium point is reached at the end of the available recordings and estimate the coseismic displacement. Assuming the available window is $t_0$ min long, we take the average tsunami amplitudes within the $t_0$-1 to $t_0$ min time window as the new equilibrium water elevation at a station. We test different window lengths in the later sections to find the shortest window that is accurate enough for warning purposes. It is important to note that stations that are not within the principal-slip region do not need such correction since the local coseismic displacement is small. Even if such correction is desired, it can not be carried out since the wave height may not have reached the assumed equilibrium point in the time window of interest (e.g. station c in Figure 3).

Figure 3b demonstrates the effectiveness of our correction method when we manually select stations located within the principal-slip region and correct coseismic deformation only for those stations. However, for early warning purposes, it is necessary to automatically identify stations within the principal-slip region. Our analysis revealed that when the peak of the first 5 min waveform is within the first $t_0$-1 min minutes, we can successfully identify the station as being within the principal-slip region. This empirical criterion yields satisfactory correction results for waveforms of various lengths and can be easily automated. Figure S4 shows the results for a 5-minute waveform case, and we utilized this criterion in the Tohoku and Fukushima event applications. In future work, we could further enhance the accuracy of our approach by combining our method with that of Inoue et al. (2019), who proposed a more reliable method to identify the uplifting area according to systematic studies of the patterns of pressure histories recorded by S-net.

We note this procedure based on wave-height equilibrium for estimating coseismic deformation is only a first-order approximation for conducting time reversal imaging to obtain the starting model of the adjoint inversion. In each adjoint iteration, we actually use the inverted IWE from the previous iteration as the seafloor displacement at each station. We update the coseismic deformation iteratively according to the inversion result of the IWE. We correct the pressure recording and calculate data residual in the ith iteration based on the inverted IWE of the iteration $i$-1:

$$\delta d_i = d - (d_{obs} + c_{i-1})$$  \hspace{1cm} (1)$$

where $\delta d_i$ is the data residue at the $i$th iteration for a station. $d$ is the predicted waveform, which contains the coseismic deformation. $d_{obs}$ is the observed waveform, which does not contain the coseismic deformation. $c_{i-1}$ is the coseismic deformation of the $(i - 1)$th iteration at the station. It is considered the same as the inverted IWE of the $(i - 1)$th iteration at the location of this station, which is updated according to the adjoint wave-
field as described in the method section. $c_0$ is estimated from the last minute of the available waveforms, as we introduced. With further iterations, the IWE, and consequently the coseismic deformation, will converge, reducing potential errors from the initial estimation.

4 Results

We test the performance of the adjoint state inversion method on synthetic and observed data of large tsunami events near the Japan Trench. We choose to adopt synthetic data of the 2011 Tohoku earthquake due to it being the largest tsunamigenic earthquake ever recorded in Japan. We also applied our method to data from the 2016 Fukushima earthquake, which generated the largest tsunami recorded by S-net to date. In order to understand the method’s capability of delivering rapid warnings, we evaluate the accuracy of the IWE and waveform predictions using different window lengths and iteration numbers. Quantitatively, the accuracy can be described using the variance reduction (VR) between predicted and observed waveforms for each test. This formula has been used in various forms in seismic (Shao and Ji (2012)) and tsunami (Kubota et al. (2020)) studies to assess the fit between observations and predictions. The formula we use to evaluate waveform prediction accuracy is shown in equation 2 and is the explicit form of the formula in Kubota et al. (2020):

$$(VR)_{\text{waveform}} = \left(1 - \frac{\sum_{i=1}^{N_s} \sum_{j=1}^{N_t} (x_{\text{obs},i,j} - x_{\text{pred},i,j})^2}{\sum_{i=1}^{N_s} \sum_{j=1}^{N_t} (x_{\text{obs},i,j})^2}\right) \times 100(\%) \quad (2)$$

where $x_{\text{obs},i,j}$ and $x_{\text{pred},i,j}$ are the observed and predicted water elevation values at the $j$th data sample of the $i$th station, respectively. $N_s$ is the total number of stations and $N_t$ is the total number of data samples at a particular station.

The formula we use to evaluate the accuracy of our IWE predictions in the synthetic test is a modified version of equation 2:

$$(VR)_{\text{IWE}} = \left(1 - \frac{\sum_{i=1}^{N_s} (x_{\text{obs},i} - x_{\text{pred},i})^2}{\sum_{i=1}^{N_s} (x_{\text{obs},i})^2}\right) \times 100(\%) \quad (3)$$

where $x_{\text{obs},i}$ and $x_{\text{pred},i}$ are the synthetic and predicted IWE values at the $i$th location in the simulation region, respectively. $N_s$ is the total number of IWE values. Notice that in both equations (2) and (3), the variance between observations and predictions is normalized by the sum of the squares of observations at a station or the synthetic IWEs. We do not apply equation 3 to our obtained IWE values from the 2016 Fukushima event because there is no ground truth for that application. We will, however, compare our results to those obtained by Kubota et al. (2021) and the Japanese Meteorological Agency (2017) (JMA).

4.1 Application to synthetic waveforms of the 2011 $M_w$ 9.0 Tohoku earthquake

Figure 4 shows the IWE results produced when we applied the adjoint technique to our synthetic data of the 2011 $M_w$ 9.0 Tohoku earthquake and tsunami. We notice that the pattern and amplitudes of the tsunami sources resolved using waveforms of different lengths are very similar, all of which reproduce the input model with a waveform VR ($(VR)_{\text{waveform}}$) greater than 0.85 and a model VR ($(VR)_{\text{IWE}}$) greater than 0.7 after 5 iterations (Figure 7(a)). The most obvious difference between the input and resolved IWE is the presence of artifacts at the outerise region (~1º east of the Japan trench), ~2 m uplift is seen in Figures 4F-M (longer-length waveforms) and ~2 m subsidence is
seen in Figures 5C-D (shorter-length waveforms). These artifacts are not seen in the input model (Figure 4A). Since the adjoint technique's success relies on high station density with good azimuthal coverage, we attribute these artifacts to the lack of stations to the east of the Japan trench.

Figure 4 reveals that our estimated tsunami source results change significantly in the first 5 iterations of our inversion. The smearing of the input model <0.2° east of the Japan trench in the initial guess (first column from the left) decreased substantially in the first 5 iterations. The decreases are most apparent when using waveforms of length 6 or 8 min, where we see that the >2 m of uplift decreases to <1 m. The $(VR)_{\text{waveform}}$ (Figure 7a) also shows that the similarity between the synthetic and predicted waveforms increases rapidly in the first 4 iterations and then plateaus. So, we opted to use the result after 5 iterations as our final result for tsunami early warning. However, to model the tsunami source accurately for purposes of source parameter inversions, we would have to conduct at least 10 iterations of our inversion (Figure 7c) to see the convergence of $(VR)_{\text{W,E}}$.

Figure 4 also shows how the inversion results change with increasing iterations for various time window lengths. We notice that the residual (Figure 4 rightmost column) between the synthetic (input) data and our results is minimal as we increase the length of waveforms used in the inversion. For example, when using 4-min waveforms, the area of >2 m residual (Figure 4E) is very large compared to that at 8-mins (Figure 4M). Also, generally, the longer the waveforms we used, the higher both VR values converge to (Figures 7a and 7c). This is because longer waveforms carry more information and reach more sensors, which improves the coverage of stations used in the inversion. Comparisons between observed and predicted waveforms are shown in Figure S5.

### 4.2 Application to synthetic waveforms with secondary sources added

We then test the resolvability of the adjoint method for possible triggered submarine landslides. Secondary sources are important to model since they can increase the run-up of the initial tsunami waves. Moreover, since they can lead to later-arriving tsunami waves, having the capability of resolving these sources can improve a warning system's tsunami forecasts. We model these landslides as secondary, binary sources with uplift and subsidence of 20 m on each side (Figure 2). Since the landslides are much smaller in area than the uplift/subsidence region of the primary source, they need more time to propagate to a sufficient number of stations for accurate resolvability. Given this, we apply the inversion to longer length waveforms (15, 20, and 25 mins). We use 6 sources to test the resolvability at different locations surrounding the principal-slip region.

Figure 5 shows that 4 out of 6 sources are clearly resolved when 20 min waveforms are available. The two sources near the trench are not resolved very well, possibly due to the reflection effect of the trench. The minimum and maximum amplitudes are -16.32 and 16.37 m after 5 iterations, which are reasonably accurate compared to the maximum amplitude of the input model (20 m). When using 15 min waveforms, the minimum and maximum amplitudes obtained are -14.66 and 12.85 m after 5 iterations, with only 2 input sources being resolved relatively clearly. In contrast, using 20-min waveforms resolves the uplift and subsidence of 3 sources and the subsidence of a fourth source relatively clearly. Using 25-min waveforms improves the resolvability of the fourth source, with the two sources near the trench still not being resolved very well, possibly due to reflection effects of the trench. Overall, using 25-min waveforms resolves the secondary sources best despite having very similar results to the result using 20-min waveforms. These results show that 15-min waveforms are not long enough to resolve most secondary sources we modeled but a minimum waveform length of 20-min waveforms is sufficient for that purpose.
4.3 Application to S-net data of the 2016 $M_w$ 6.9 Fukushima earthquake

In order to understand the performance of the adjoint method on real data, we applied it to S-net data of the 2016 $M_w$ 6.9 Fukushima earthquake. Figure 6 shows that using the waveforms of the first 15 min, the inversion results of the Fukushima earthquake show much smaller amplitudes compared to the result when we use longer waveforms. This is because only three stations contain the first peak when only 15 min of data is available (Figure S6a). This means that the inversion is not fed enough information to obtain an accurate enough IWE. Examining Figure S6, we see that the data fitting using only the first 15 min is much worse than that of 25 min, where more than 8 stations with complete first peaks with amplitude larger than 0.05 m are available. This, combined with the results from Figure 6 indicate that when we use waveforms that are 25-60 min long, our data fitting improves immensely compared to using 15-min waveforms.

Further examining Figure 7, we notice that the initial guess is improved greatly after 5 iterations (using 25-60 min waveforms) of our inversion but does not change that much after 5 iterations. This means that the inversion converged after 5 iterations, which is consistent with our application to the synthetic data of the 2011 Tohoku event. As a result, we use the 5-iteration result as our best output for tsunami warning. Moreover, our subsidence region and our boundary between uplift and subsidence match those obtained by Kubota et al. (2021) and JMA, respectively (Figure 6 - 40-min and 60-min waveform results).

5 Discussion

5.1 The required waveform duration for large and small earthquakes

One important finding in our study is that the required waveform duration to achieve accurate adjoint inversion for larger events is shorter than that of a smaller event. For instance, the $M_w$9.0 earthquake requires a time window of 5 min to achieve an accurate result while the smaller $M_w$6.9 Fukushima needs a window of at least 25 min long. A number of factors can be used to explain this difference. The first factor involves the rupture size. For self-similar earthquakes, the rupture area scales with $M_w^{2/3}$. This means that the corresponding area of seafloor deformation in a larger magnitude earthquake is usually bigger compared to that of a smaller magnitude event. Thus the number of near-field stations (located directly above the rupture zone of the earthquake) is also greater for larger events. Since the rupture propagation ($\sim$2-3 km/s) is significantly faster than tsunami waves ($\sim$0.2-0.3 km/s), the near-field stations receive the tsunami signal faster and directly capture the coseismic displacement. A greater number of the near-field stations allow the major pattern of the tsunami source to be constrained faster and more accurately. With the Tohoku earthquake application, $>25$ stations recorded tsunami wave arrivals in the first 5 mins after the origin time. Examining the IWE results in Figure S7, we notice that the inversion of 5-min data for Tohoku resulted in an IWE that resembles the synthetic data quite well ($\langle VR\rangle_{IWE}$ converging to $\sim$83%, Figure 7c). In contrast, only 3 stations recorded arrivals in the first 15 mins of the Fukushima application, which is insufficient to capture the major pattern of the IWE given its much smaller scale and higher resolution is needed. So we observe that the IWE does not match the expected uplift-subsidence pattern obtained by other studies until at least 25-mins waveforms are used (Figure 6). For the $M_w$6.9 Fukushima, the source size is expected to be smaller than 50 km, which is close to the interstation distance of the S-net array. As a result, very few near-field stations are directly above the peak slip point (distance of closest station to the epicenter is $\sim$30 km). This phenomenon of larger earthquakes being forecasted more accurately than the smaller ones is also reported by Mulia and Satake (2021) and can also be used to explain why the synthetic test with secondary sources requires waveforms of at least the first $\sim$20 minutes to clearly resolve those synthetic landslides (Fig-
ure 6). Namely, the secondary sources have fewer stations located above their deformation zones and so more time is required for their tsunami waves to propagate to enough stations.

The second factor involves the ocean depth at the location of rupture. Compared to the Tohoku earthquake, the 2016 Fukushima earthquake is located in a region with a shallower ocean depth (~226 compared with ~4400 m), so the speed of the tsunami wave propagation is ~47 m/s (compared to ~209 m/s). This means that the arrival time of the tsunami wave at the closest station is about 10 min. So, we have to wait longer for the tsunami wave to reach enough stations in a smaller earthquake scenario when it is located in a region with a shallow ocean depth.

Another factor is that the smaller-earthquake application’s waveforms have a lower signal to noise ratio. Figure 6 shows that using the waveforms of the first 15 min, the inversion results of the 2016 Fukushima earthquake show much smaller amplitudes and are very scattered compared to the result when we use longer waveforms. Figure S6 (a) shows that only three stations show the first peak clearly within the first 15 min. In addition to the longer tsunami propagation time (due to shallower depth) and absence of sufficient near field stations, this is also because the signals of these stations are noisier due to the incomplete removal of the tsunami-irrelevant steps (e.g. settlement of the instrument after deformation). In contrast, the data fitting using the first 25 min is much better, with 8 stations having complete first peaks and fewer stations being affected by the contamination of the steps. Although the two earthquake applications differed in the time window length required for an accurate result, the number of iterations (5) until variance reduction convergence is the same (see Figure 7).

5.2 Potential for tsunami early warning

An effective tsunami early warning system is dependent on time management. So, one needs to ensure that the minimum amount of time is spent on each step (data processing, inversion, prediction and information dissemination) in order to allow for as much time as possible for evacuation. Assuming the data processing including filtering, detiding and removal of tsunami-irrelevant steps is done instantaneously after the data is collected, the total tsunami warning time would be the sum of the data collection and computation times for the adjoint inversion and final forward simulation of the obtained IWE. Since the data collection time is much longer than the computation time, it is critical to use the minimum collection time that results in a reliable IWE estimate (5 and 25 min for Tohoku and Fukushima, respectively). For real-time application, the inversion using the waveforms within 5, 6, 7, 8, 10, 20, 25, and 30 min could be calculated as soon as the data is collected, which yields consistently-updating results that could be integrated into current tsunami warning systems. Since the computation time increases with the number of iterations in our inversion, we seek to minimize the number of iterations used to obtain a reliable IWE estimate. In most of our inversions, we find the variance reduction between observed and predicted waveforms plateaus after ~5 iterations (Figure 7), with each iteration taking ~16 seconds to complete. Although some cases require up to 10 iterations for convergence, for a preliminary estimation of the IWE for early warning purposes, we propose to use the result after 5 iterations to shorten the calculation time. This would then be updated with more accurate inversion results as longer length waveforms become available and more time is allowed for further iterations to run. The inversion itself takes about 1 min and 20 s to run using 16 CPU cores (AMD 3990 X @ 2.9 GHz) if we use the waveforms within the first 25 min from all of the S-net stations and terminate the inversion after 5 iterations using a calculation grid resolution of 90 arcsec. If we use the first 5 mins of waveform, the inversion takes 45 s. These processing times can potentially be reduced with the addition of more computational power. Assuming the final forward simulation that is used to predict the arrival times and amplitudes takes about 40 seconds, the total time for issuing the warning is about 7 min for
large earthquakes like the 2011 $M_w$ 9.0 Tohoku earthquake and about 27 min for smaller
earthquakes located in shallow regions with a smaller tsunami wave speed. For the 2011
Tohoku earthquake, the first coastal arrival based on eyewitness accounts and clocks stopped
by tsunami inundation was at about 23 min (Figure 1 in MUHARI et al. (2012)). This
means that a hypothetical warning issued after using our inversion technique will be ab-
sent from a blind zone assuming a similar scenario were to occur. When applying the
adjoint method to tsunami warning in the real world, we propose to invert the waveforms
for the source and then issue an update to the source model every 5 min.

To further evaluate the accuracy of the inversion results, we run a forward simu-
lation of tsunami waves using the inverted model after 5 iterations and compare the am-
plitude and time of the first peak of the predicted wave with the recordings at coastal
tide gauges and GPS buoys (Figure 8, Figure S8). Note that for the Tohoku event, we
utilize tide gauge recordings based on forward modeling the synthetic data shown in Fig-
ure 4A. This is to ensure that most inaccuracies we see in our inversion result’s forward
modeling are due to errors in our inverted source model as opposed to differences between
the synthetic and true source models. For the synthetic test of the Tohoku earthquake,
if we use the model inverted from the waveforms available within 5 min, the errors of the
amplitudes range between $-5.55$ and $1.93$ m. The largest discrepancy, with a magnitude
of $-5.55$ m, corresponds to station Soma. The predicted amplitude at this station is $8.01$
m, while the actual peak amplitude is $13.56$ m (Figure S9a). It’s likely that this under-
estimation of the amplitude is due to insufficient constraints for smaller scale features
when only 5 min waveforms are available. The use of 8 min waveforms probably provides
more accurate resolution of small scale features, including small patches with high IWE,
leading to a more accurate prediction of the peak amplitude at station Soma of $12.34$
m (Figure S9b). Other potential sources of error in the waveform modeling may include
artifacts resulting from reflections off the Japan Trench or inaccuracies in the bathymetry
data. Following Tsushima et al. (2009)), we estimate the average accuracy score of the
amplitude. This score is defined similarly to equation (3) in this paper, except $z_{\text{mod}}$ and
$z_{\text{pred}}$, are replaced by the $A_{\text{obs}}$, and $A_{\text{pred}}$, respectively. $A_{\text{obs}}$, is defined as the maxi-
mum positive amplitude of the first tsunami wave observation at the ith coastal tide sta-
tion and $A_{\text{pred}}$, is the maximum positive amplitude of the predicted waveform. When
using the first 5-min and 8-min waveforms, the average accuracy score of the amplitudes
is 93% and 98% respectively. For the 8-min waveforms, the amplitude errors range be-
tween $-1.92$ and $1.35$ m. For the Fukushima earthquake, the errors of the amplitudes range
between $-0.20$ and $0.22$ m using 25-min waveforms, with an average accuracy score of
78%. The errors of the arrival times range between $-9.11$ and $14.61$ min when using the
first 5 min for the Tohoku event and 25 min for the Fukushima earthquake. Since $\sim 85$
% of our predicted arrival times are $\pm 4$ mins of the true arrival time, they are accurate enough
for warning purposes.

5.3 Comparison of the $M_w$ 6.9 earthquake result with previous results

The USGS-derived focal mechanism indicates that the Fukushima earthquake oc-
curred on a normal fault. This indicates that the subsidence region in our IWE result
(Figure 6) corresponds to the hanging wall moving down (and the uplift shows the foot
wall moving up) on a North-east to South-west trending fault trace. Our obtained tsunami
source distribution of the $M_w$ 6.9 Fukushima earthquake (using 60-min waveforms af-
ter 10 iterations) shows a major subsidence region with a peak of $\sim 1.3$ m and a region
of uplift with a peak of $\sim 0.65$ m. The location and range of the subsidence is consis-
tent with the IWE inferred from S-net data by Kubota et al. (2021), which resolved the
subsidence region with a peak of $\sim 2.0$ m. Such subsident deformation is expected for
the normal-faulting Fukushima event and corresponds to the hanging wall moving down
(and the uplift shows the foot wall moving up) on a North-east to South-west trending
fault trace. However, the associated uplift in the IWE caused by the foot wall moving
up is not necessarily expected and is dependent on the estimated depth of the slip. Al-

though the region of uplift is not clear in Kubota et al. (2021)’s result, a finite fault slip model obtained by Japanese Meterological Agency (2017) inverted from seismic waves predicts a region of uplift (also shown in Figure 3 of Nakata et al. (2019)). The sharp boundary between the subsidence region (with a peak of ~ 2.4 m) and an uplifted region (with a peak of ~ 0.5 m) in JMA’s result is very close to the location and strike of the boundary line resolved by our method (Figure 6 green dashed line). However, none of the results in the tsunami-based inversions we compare to in Table S1 predict this uplifting region. The existence of a discernable uplifting region is related to the dip angle of the fault and the depth distribution of the slip: If the dip angle changes from 49 to 59º, the maximum of predicted uplift will increase from 0.13 to 0.28 m. A shallower slip distribution would also cause more clearly seen uplifting, as well as a sharper boundary between the uplifting and subsidence region. Thus the uplifting region we resolve suggests the existence of a minor slip patch at shallow depth. The dip angle of this patch is probably steeper than the average dip angle of the entire fault. This steepness could be explained by dynamic triggering on a splay fault which has been shown to occur in tsunamiigenic earthquakes (Wendt et al. (2009); van Zelst et al. (2022)).

Our resolved IWE also allows us to model the rupture extent and location. Assuming a rectangular fault with uniform slip distribution with the fault orientation fixed to the USGS W-phase solution (strike=42°, dip angle=49°, rake=-101°), we searched for the fault parameters based on the IWE achieved using 60-min waveforms after 10 iterations. We performed a grid search for 5 parameters: the longitude, latitude and the depth of the center of the fault, the slip, and the fault width. Assuming a moment magnitude of 2.484×10¹⁹ N.m (from USGS W-phase inversion) and a rigidity of 30 GPa, we estimate the length of the fault and build a uniform slip model for each combination of parameters. The best fitting model to our IWE is located at 141.537ºE, 37.243ºN, with a centroid depth of 8 km. The average slip for a model at this location is 3 m, with a width of 12 km. The fault length was then estimated to be 23 km. The comparison between our results and those obtained by other authors (including the Geospatial Information Authority (2017)) are summarized in Table S1. Compared to the average of each parameter of the other results, the center of our obtained fault is located ~5.4 km to the southeast and is ~1 km shallower (Figure S10b, Table S1). Table S1 shows that the width and length of our rectangular fault model are ~0.6 km and 3.7 km shorter than the average, respectively. Most of our obtained fault area overlaps with the average obtained by other studies (Figure S10a). Finally, our slip is ~0.78 m more than the average of the others. These differences are small and are close to the size of the grid we use (~5 km). We show the seafloor displacement predicted from our best uniform fault model in Figure S10c. This shows that, compared to our obtained seafloor displacement in Figure 6, the seafloor displacement derived from our best fault model is shifted slightly south. Our grid search demonstrates that the IWE obtained with the adjoint inversion method can be used to characterize the earthquake source relatively accurately.

With these obtained parameters, the stress drop was estimated to be 4.6 MPa using:

\[ \Delta \sigma = \frac{cM_0}{(LW)^{1.5}} \]  

where \( \Delta \sigma \) is the stress drop, \( c \) is a constant (=8/3, when Poisson’s ratio is 0.25, Kanamori and Anderson (1975)), and \( L \) and \( W \) are the lengths and widths of our rectangular fault, respectively. Our estimation of the stress drop is very close to Kubota et al. (2021)’s estimation (2021) of 4.2 MPa. According to those authors, the shear stress change caused by the 2011 Tohoku earthquake (a thrust fault earthquake) at the epicenter of the 2016 Fukushima event is ~2 MPa. This is smaller than our estimated stress drop of the 2016 Fukushima event (a normal fault earthquake). They explained this stress discrepancy by suggesting that the Tohoku earthquake amplified already existing horizontal extensional stresses in the shallow (<15 km) portion of the bending continental
plate (Figure 6 in Kubota et al. (2021)), leading to an increase in normal-fault earthquakes at Fukushima. Thus, our estimated stress drop showing the mismatch helps to confirm their hypothesis that the extensional stress was concentrated in the shallow region of the crust near the 2016 Fukushima event prior to the 2011 Tohoku earthquake.

6 Conclusions

In this study, we evaluated the potential of the application of the adjoint-state full waveform tsunami inversion method for tsunami early warning using S-net OBPGs. We use a time-derivative-based algorithm to remove the tsunami-irrelevant step functions in OBPG recordings and an automatic procedure to extract the coseismic deformation for stations close to the source. We apply the adjoint inversion method to synthetic waveforms of the 2011 $M_w$ 9.0 Tohoku earthquake and recordings of the 2016 $M_w$ 6.9 Fukushima earthquake. Using all available S-net stations, our result shows that the tsunamis associated with the $M_w$ 6.9 and $M_w$ 9.0 earthquakes can be accurately predicted 27 min and 7 mins, respectively, after they occurred. Triggered secondary sources of the $M_w$ 9.0 event were accurately resolved using 20-min waveforms. These results benefit from the fast estimation of the coseismic deformation beneath the closest stations to the epicenter. Moreover, we showed that tsunamis caused by $M_w < 7$ earthquakes tend to require longer waveforms to capture the major pattern (which has a much smaller scale) and achieve an accurate result than larger events. One plausible explanation is that the smaller rupture area leads to a longer data collection time. This is because tsunami waves need to travel a further distance to arrive at enough stations for an accurate inversion result to be obtained. Another explanation is that the Fukushima earthquake has low signal-to-noise ratio at OBPG recordings near the principal-slip region due to the tsunami irrelevant contamination. This means that waves need to travel a further distance such that this low SNR effect is minimized. Our calculated accuracy scores for the 25-min and 5-min waveform windows for Fukushima and Tohoku were 78 and 93%, respectively. The predicted amplitude and timing of first arriving waves at most of our selected coastal tide gauges and GPS buoys for both applications had errors of ~2 to 1 m and ~±4 mins, which are small enough for warning purposes. Our estimation of the fault parameters and stress drop associated with the 2016 event are generally consistent with those obtained by other studies. However, we resolve an uplifting region in our IWE that is possibly explained by shallow slip on a steeper fault plane. Overall, our study reveals that tsunami sources and triggered secondary sources can be accurately resolved using the adjoint inversion of OBPGs which thus has the potential to be integrated into a tsunami warning system.

Data and code availability

We download the S-net OBPG data (https://www.seafloor.bosai.go.jp) from Japan’s National Research Institute for Earth Science and Disaster Prevention (NIED) website (https://doi.org/10.17598/NIED.0007). The waveforms of GPS buoy stations that are used to evaluate the performance of tsunami prediction are digitized from Nakata et al. (2019) for the Fukushima earthquake. For the Tohoku event, the location of the stations are from the Sea Level Station Monitoring Facility and Intergovernmental Oceanographic Commission of UNESCO (www.ioc-sealevelmonitoring.org). COMCOT software package is downloaded from https://github.com/AndybnACT/comcot-gfortran. The General Bathymetric Chart of the Oceans (www.gebco.net) provided our high resolution bathymetry data. The adjoint inversion code is available on github (https://github.com/xieyqgeo/adjoint_tsunami_inversion).
Acknowledgments

This work is supported by the NSF CAREER grant EAR-1848486 and the Leon and Joanne V.C. Knopoff Fund. We thank Chao An for valuable discussions about tsunami wave propagation simulation. We thank Hiroaki Tsushima for sharing their understanding and processing method about the tsunami-irrelevant steps.

References


Figure 1. Map showing the S-net stations (yellow circles) and the Japan Meteorological Agency epicenters (red stars) for the synthetic test (larger star) and the 2016 Fukushima earthquake (smaller star).
Figure 2. (a) The IWE predicted from the slip model of the 2011 Mw 9.0 Tohoku earthquake (Fujii et al. (2011)) with secondary sources added. Red star denotes JMA epicenter of the 2011 event. The horizontal and vertical axes correspond to degrees of longitude and latitude respectively. The contours indicate different uplift/subsidence levels due to the Tohoku earthquake. The green circles indicate S-net stations, with the named ones being used in (b). (b) Examples of the waveforms from labeled stations in (a) and the correction for coseismic deformation for the synthetic test without the secondary sources.
Figure 3. Illustration of coseismic effect at stations in the principal-slip region. Modified from Inoue et al. (2019). A) shows water (blue) and ocean bottom (orange) behavior at 6 time steps with ocean bottom pressure gauges indicated by the yellow triangles and labeled a, b and c. Stations a and b are located within the principal slip region while station c is located far away from it. The blue and black arrows indicate pressure-derived water column height (blue line in B) and water elevation (black line in B) with respect to the location of the ocean floor prior to an earthquake, respectively. B) shows the water height recordings at each station at each time step. Red-dashed line in both A) and B) corresponds to the height of the water column prior to the earthquake.
Figure 4. The result of the synthetic test using the stations available within 4, 6, and 8 min from the second row to the third row, respectively. The first row shows the input model (synthetic data) from Figure 2a with the secondary sources removed. In the second row and beyond: the first, second, third, and fourth columns are the initial model, the results after 5 iterations, the results after 50 iterations, and the misfit between the inverted and input model (inversion result IWE minus input model IWE) after 5 iterations respectively. Each plot is labeled with a letter in the top right for reference. The contour lines and colors represent the uplift or subsidence of the water surface. The horizontal and vertical axes correspond to degrees of longitude and latitude respectively.
Figure 5. The result of the synthetic test with multiple secondary sources using the stations available within 15 (b), 20 (c) and 25 min (d) after 5 iterations. Input model shown in (a). The horizontal and vertical axes correspond to degrees of longitude and latitude respectively.
Figure 6. The result of the $M_w$ 6.9 Fukushima earthquake using the stations available within 15, 25, 40 and 60 min from the first row to the fourth row. The left, middle and right columns are the initial model, the results after 5 iterations and the results after 10 iterations. Contour lines show uplift or subsidence at intervals of 0.5 m. The red dashed lines indicate the major subsidence region with amplitude larger than 0.2 m, resolved by Kubota et al. (2021) using S-net data. The green dashed lines indicate the boundary between subsided and uplifted regions of the IWE predicted from the finite fault slip model of JMA (JMA, 2017b) inverted from seismic waves. The horizontal and vertical axes correspond to degrees of longitude and latitude, respectively.
Figure 7. The change of variance reduction with each iteration. The variance reduction of waveforms of (a) the synthetic test of the $M_w$ 9.0 Tohoku earthquake and (b) the Fukushima earthquake. (c) The variance reduction of the IWE model of the synthetic test.
Figure 8. The accuracy of the predicted arrival time and amplitudes at tide gauges and GPS buoys using the waveform from the first 5 min of the synthetic test for Tohoku event and 25 min of the Fukushima event. (a) Correlation between predicted and recorded arrival times of the first peak for the two applications. (b) Correlation between the predicted and recorded amplitude of the first peak for the two applications. (c) The error distribution of the arrival time. The red curve is the fitted Gaussian distribution. The dashed lines are one standard deviation above/below the average. (d) The error distribution of the amplitude.
Adjoint-state waveform inversion using the S-net system for tsunami source imaging and early warning

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Key Points:

• We use an adjoint-state tsunami inversion method that images tsunami sources directly from ocean bottom pressure gauge (OBPG) recordings.
• The OBPG recordings of S-net provide sufficiently uniform azimuthal coverage within a short collection time for accurate adjoint inversions.
• Our case study of the 2016 Fukushima earthquake shows seafloor deformation consistent with shallow slip on a steep normal fault.

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We explore the potential of the adjoint-state full waveform tsunami inversion method for tsunami early warning using S-net, an array of off-shore ocean bottom pressure gauges. Compared to finite-fault tsunami source inversions, the method we use does not require as densely gridded Green’s functions to obtain a high resolution result, thus reducing computation time. What it does require is a dense instrument network with good azimuthal coverage. The density and coverage of the S-net pressure gauges fulfill this requirement and reduce the data collection time, thus making it possible to invert the recordings for the tsunami source and issue a timely tsunami warning. We apply our method to synthetic waveforms of the 2011 $M_W$ 9.0 Tohoku earthquake and tsunami with triggered secondary sources as well as the data from the 2016 $M_W$ 6.9 Fukushima earthquake. The results of the synthetic tests of the $M_W$ 9.0 earthquake show that using the first 5 minutes of the waveforms, the adjoint-state inversion method achieves good performance with an average accuracy score of 93%, with the largest error of predicted wave amplitudes ranging between -5.6 to 1.9 m. The secondary sources are clearly resolved using the first 20 mins of the waveforms. Our application to the 2016 Fukushima earthquake shows the required waveform duration to achieve accurate adjoint inversion for smaller events is longer than that of a larger event. However, using the first 25 minutes of the waveforms, the inversion yields a tsunami source that is sufficient for making accurate predictions of arrival times and amplitudes. Assuming a uniformly distributed fault slip, we estimated a stress drop for the latter event to be 4.6 MPa, which is in line with estimations from recent studies.

## Plain Language Summary

Issuing timely tsunami warnings is essential in allowing at-risk populations to have adequate evacuation time. However, traditional frameworks for issuing warnings rely on accurate models for the earthquake that caused the tsunami. These models can be inaccurate in the immediate aftermath of an earthquake and can lead to underestimated wave heights and late wave arrival time predictions. To address this issue, we have developed a method that can yield the tsunami source without the need for an earthquake model, simply relying on the wave height data recorded by pressure sensors at the bottom of the ocean. This can also be useful when tsunami-causing events other than earthquakes (e.g. volcanic eruptions) occur. We test this method on recorded data from the 2016 Fukushima earthquake’s tsunami as well as synthetic waveforms from the 2011 Tohoku tsunami. Our results for the former show accurate predictions of wave arrival times and heights when we use the first 25 minutes of the wave height data. For the latter, predictions are accurate when we use the first 5 minutes of the data.

## 1 Introduction

Fast and accurate estimation of tsunami source is essential for tsunami early warning. Compared to the methods of tsunami source estimation based on seismic waves, the methods that utilize tsunami waves are more accurate and are particularly useful in scenarios such as submarine volcanic eruptions and landslides. However, the low density of tsunami stations limits the application of the tsunami observation for tsunami early warning.

After the 2011 Tohoku earthquake, an offshore deep-ocean observation network, S-net (Seafloor Observation Network for Earthquakes and Tsunami) was constructed, which covers the major potential regions of tsunami sources along the Japan Trench. The high density and large coverage of the S-net pressure gauges shorten the data collecting time, which makes it possible to invert the recordings for the tsunami source and issue a tsunami warning. Several methods based on tsunami recordings have been developed and tested using synthetic waveforms and recordings of S-net pressure gauges (e.g. Aoi
et al. (2019); Inoue et al. (2019); Mulia and Satake (2021); Tanioka (2020); Tsushima and Yamamoto (2020); Y. Wang and Satake (2021); Yamamoto, Aoi, et al. (2016), Yamamoto, Hirata, et al. (2016)).

The issue with using traditional finite-fault tsunami source inversion methods is that to attain a high resolution, one needs densely-gridded Green’s functions, which is not always possible to implement in the immediate aftermath of a tsunami. Pre-calculated Green’s functions can curtail this issue. However, covering a large expanse of the ocean requires a coarse grid or a large amount of spacing within the Green’s functions, leading to a heavy computational burden. For example, if we mesh the entire north-eastern Pacific Ocean from the Aleutian Islands to the coast of the State of Washington with a 1 arc-minute spacing, the total number of grids will be about 2000000, which requires several months or even years to calculate the Green’s functions with hundreds of CPU cores. On the other hand, the adjoint-state tsunami inversion is suitable for real-time application because it recovers accurate initial water elevation in only a few iterations (Zhou et al. (2019)). So, the adjoint-state full-waveform inversion of the tsunami source has two major benefits compared to the traditional finite-fault tsunami source inversion method: high computational efficiency and high resolution. This allows the tsunami source image to be used quickly and directly in forward tsunami simulation, producing accurate forecasts of tsunami height and arrival time.

However, an ideal adjoint state inversion needs dense near-source stations with uniform azimuthal coverage, which makes it difficult to be applied to early warning in most subduction zones. S-net fits the requirement of the adjoint state inversion method very well and provides an opportunity to test the performance of the method to be used for early warning. The large scale and near-equal spacing of the S-net stations guarantees the uniform azimuthal coverage within a short time (within ~ 30 min). Moreover, the offshore ocean bottom pressure gauges (OBPGs) are preferred for the adjoint state method compared to tide gauges for three main reasons: (1) ocean-bottom pressure gauges do not get affected by wave reflections off the coast as much as tide gauges (2) wave speeds are not as affected by the resolution of bathymetry data and mesh size for computation as tide gauges, (3) the nonlinear effect of propagating water waves is negligible since the OBPGs are located at large depths.

In this study, we applied the adjoint-state inversion method to synthetic waveforms of the 2011 $M_w$ 9.0 Tohoku earthquake and tsunami in addition to the S-net data of the 2016 $M_w$ 6.9 Off Fukushima earthquake, the largest tsunami event in the near field so far. S-net is designed to monitor the tsunami generated by megathrust earthquakes like the Tohoku earthquake. Here we applied our adjoint state method to synthetic waveforms to demonstrate its effectiveness using the OBPGs network for rapid source imaging and early warning purposes. We also add multiple secondary sources to the Tohoku earthquake to test its resolvability for possible triggered submarine landslides. We briefly describe our method in section 2 and the pre-processing of our data in section 3. In section 4, we report the results of applying our adjoint method to synthetic waveforms of the 2011 $M_w$ 9.0 Tohoku earthquake and recorded tsunami waveforms of the 2016 $M_w$ 6.9 Fukushima earthquake. In section 5, we discuss what our results tell us about the accuracy of the adjoint inversion method for large and small tsunamigenic earthquakes as well as its potential in tsunami early warning systems. We also show the results of our inversion for the source parameters of the 2016 event and compare them to that of other studies.

2 Method

The adjoint-state tsunami source inversion method is a numerical technique, developed by Zhou et al. (2019), citation hereafter referred to by Z19, to solve for the tsunami source (initial water elevation) by minimizing the difference (misfit) between the observed
A key component of the adjoint-state method is the use of time-reversal imaging to obtain an initial model for the tsunami initial water elevation (IWE). Time-reversal imaging of the tsunami source was first done by Hossen et al. (2015) and involves reversing observed waveforms (how wave heights vary in time at a particular location) in time and back-propagating them using station locations as point sources. The idea is that the resulting wavefields (how wave height varies with space) at the end of the reversed time series will constructively interfere at the location of the original source, with some amplitude and locational distortion depending on station density and azimuthal coverage.

Following the workflow from Z19, time-reversal imaging is first applied to our observed waveforms to obtain the initial model of the tsunami source. The source is then forward propagated to station locations and subtracted from observed waveforms to obtain the data residue. The misfit between observations and predictions is then calculated using the L2-norm misfit functional (equation 5 in Z19). Next, we minimize this misfit with each iteration as much as possible. Plessix (2006), Tarantola (1986) and Tromp et al. (2004) showed that the gradient of the misfit function (equation 6 in Z19) with respect to source parameters is equivalent to the adjoint wavefield, obtained by back-propagating the data residue at each station to the original source via time-reversal imaging. The gradient is then used to update the source model using a conjugate direction scaled by a certain step length, as done in the conjugate gradient method (equation 9 in Z19). The conjugate method is a numerical method of gradient descent that minimizes the difference between observed and predicted waveforms. The size of the step length is itself scaled (by empirically obtained values of 2 or 0.5) at each iteration depending on which value results in smaller data residue after propagating the new source. With a new source obtained, the algorithm is repeated for the specified number of iterations, usually until convergence of the residue occurs.

For the time-reversal and the forward simulations, we use the Parallel Cornell Multi-grid Coupled Tsunami modeling package (PCOMCOT; Liu et al. (1998); An et al. (2014); X. Wang and Liu (2006); Zhu et al. (2021)) in the spherical coordinate system. This package uses finite difference methods to solve the shallow water equations in Cartesian or spherical coordinate systems. The calculation region (138°E to 148°E, 32°N to 45°N) covers the entire region of S-net and is divided into 400 by 520 grids. We use a calculation grid size of 90 arcseconds, which shortens the calculation time to less than 5 s for 1 forward simulation with 16 CPU cores and yields relatively accurate results in the open sea, which is less sensitive to the bathymetries compared to coastal regions.

### 3 Data Processing

#### 3.1 Observed and synthetic recordings of S-net

The S-net is a seafloor observation network which records both water pressure and seismic signals. It was installed starting in 2016 in response to the devastation caused by the 2011 Tohoku tsunami. These sensors measure the absolute water pressure and are manufactured by Paroscientific, Inc. (e.g. Pulster et al. (2009); Watts and Kontoyianis (1990)). The water pressure is transferred to the pressure sensors which are sealed in a metal house via a diaphragm made of hard rubber. The observed pressure is then time-stamped based on GPS information and transmitted to a landing station for con-
version and final transmission to the data center (Aoi et al. (2020)). The S-net is capable of observing tsunamis with amplitudes less than 1 cm (Kubota et al. (2018)).

At the time this study was conducted, the largest recorded tsunami event close to the network was the 2016 $M_w$ 6.9 Fukushima earthquake. The locations of all currently installed stations that are used for the synthetic test are shown in Figure 1. The stations located at the outer-rise region were not yet installed in 2016, so we only use data from 125 stations for the analysis of the 2016 $M_w$ 6.9 Off Fukushima earthquake. To understand the performance of the adjoint approach using S-net for future events on the scale of the 2011 Tohoku tsunami, we generated the synthetic tsunami waves for this event at the locations of all currently installed S-net stations. In our first synthetic test, we adopt a slip model of the 2011 $M_w$ 9.0 Tohoku earthquake (Figure 2) inferred from tsunami data (Fujii et al. (2011)). In the second synthetic test, in addition to the signal of the Tohoku earthquake, we added multiple secondary sources representing triggered submarine landslides assuming they emerge simultaneously with the mainshock. Each secondary source is a binary square source with an uplift and a subsidence of 20 meters on each side. The side length of the square is 20 km (Figure 2). We use bathymetry with a resolution of 15 arcsec, which is finer than the 90 arcsec resolution used in the adjoint inversion. This difference in resolution is designed to test the effect of bathymetry error on the accuracy of the adjoint inversion.

### 3.2 Waveform window selection

We previously proposed to select a window which contains only the first complete peak and trough for the adjoint inversion to avoid artifacts caused by the reflected wave at the tide gauges near the coast (Z19). However, since the S-net OBPGs are located in the open sea, there are no reflected waves within 10 min of the tsunami origin time at most of the stations. We confirmed this by running inversions on our synthetic data with and without the peak-and-trough window selection method. The results of this confirmation (Figure S1) show that the method of choice makes little difference in misfit and IWE results. Consequently, we opted to use all the available waveforms (locations shown in Figure S2) from the origin time to $t_0$, where $t_0$ is the time when the tsunami warning is issued. We discuss the shortest $t_0$ that achieves an accurate warning in section 5. Using all the available waveforms is also a simpler strategy for automated implementation since we do not need to consider a specific window for each station.

### 3.3 Filtering and artifacts removing

One issue with the recordings of S-net ocean bottom pressure gauges is that the initial waveforms of the stations close to the source are contaminated by tsunami-irrelevant steps (Kubota et al. (2018); Kubota et al. (2021); Nakata et al. (2019)). These tsunami-irrelevant steps can be caused by tilt due to ground shaking or long-term mechanical drift. Moreover, the tsunami signal mixes with seismic waves and Earth’s tides. Therefore, after resampling the waveforms to 2 Hz, we low-pass filter (100 s) them to remove the seismic signal following Kubota et al. (2018). Then, the tsunami-irrelevant steps are removed if the time derivative at a particular time is greater than 0.0005 m/s. We show examples of this correction in Figure S3c. Finally, a high-pass filter (1000 s) was applied to remove tidal effects. Details about this processing can be found in the supplementary information (Text S1, Figure S3).

### 3.4 The estimation of the coseismic displacement at the stations

Tsunami waves are a type of surface gravity wave. The backward propagations in the adjoint state inversion process require the histories of the water surface elevation at each station. As shown in Figure 3, the water elevation (black lines) is the sum of the water column height (blue lines) and the vertical seafloor displacement. However, the
OBPGs measure the in-situ pressure which can be converted to the height of the water column above a station. We assume the water elevation and water column height are equal when the seafloor displacement is small compared with the tsunami signal. Since the coseismic seafloor displacement attenuates rapidly over distance, this assumption is valid for distant stations (station c in Figure 3). In the case of the Tohoku earthquake, for the stations that are more than 50 km away from the principal-slip region (Slip > 5 m), the coseismic displacement is relatively small (< 1 m) compared to the tsunami height (>4 m). However, for the stations within the principal-slip zone, we need to add the coseismic displacement to the measured water column height to obtain the water elevation (stations a and b in Figure 3). We assume the IWE at the beginning of the tsunami propagation is equivalent to the vertical coseismic displacement at each station. The coseismic deformation can be estimated with the permanent change of equilibrium pressures (Figure 2b) before and after the earthquake. When a long enough recording after the earthquake is available, this is not difficult to estimate as shown by Tsushima et al. (2012) and Inoue et al. (2019). However, for warning purposes, we need to estimate the coseismic deformation in a short time. For the stations within the principal-slip region, since the primary tsunami signal passes by quickly (in the first 5 to 10 min), we can assume the equilibrium point is reached at the end of the available recordings and estimate the coseismic displacement. Assuming the available window is $t_0$ min long, we take the average tsunami amplitudes within the $t_0$-1 to $t_0$ min time window as the new equilibrium water elevation at a station. We test different window lengths in the later sections to find the shortest window that is accurate enough for warning purposes. It is important to note that stations that are not within the principal-slip region do not need such correction since the local coseismic displacement is small. Even if such correction is desired, it can not be carried out since the wave height may not have reached the assumed equilibrium point in the time window of interest (e.g. station c in Figure 3).

Figure 3b demonstrates the effectiveness of our correction method when we manually select stations located within the principal-slip region and correct coseismic deformation only for those stations. However, for early warning purposes, it is necessary to automatically identify stations within the principal-slip region. Our analysis revealed that when the peak of the first 5 min waveform is within the first $t_0$-1 minutes, we can successfully identify the station as being within the principal-slip region. This empirical criterion yields satisfactory correction results for waveforms of various lengths and can be easily automated. Figure S4 shows the results for a 5-minute waveform case, and we utilized this criterion in the Tohoku and Fukushima event applications. In future work, we could further enhance the accuracy of our approach by combining our method with that of Inoue et al. (2019), who proposed a more reliable method to identify the uplifting area according to systematic studies of the patterns of pressure histories recorded by S-net.

We note this procedure based on wave-height equilibrium for estimating coseismic deformation is only a first-order approximation for conducting time reversal imaging to obtain the starting model of the adjoint inversion. In each adjoint iteration, we actually use the inverted IWE from the previous iteration as the seafloor displacement at each station. We update the coseismic deformation iteratively according to the inversion result of the IWE. We correct the pressure recording and calculate data residual in the $i$th iteration based on the inverted IWE of the iteration $i$-1:

$$\delta d_i = d - (d_{obs} + c_{i-1})$$

where $\delta d_i$ is the data residue at the $i$th iteration for a station, $d$ is the predicted waveform, which contains the coseismic deformation. $d_{obs}$ is the observed waveform, which does not contain the coseismic deformation. $c_{i-1}$ is the coseismic deformation of the $(i-1)$th iteration at the station. It is considered the same as the inverted IWE of the $(i-1)$th iteration at the location of this station, which is updated according to the adjoint wave-
field as described in the method section. $c_0$ is estimated from the last minute of the available waveforms, as we introduced. With further iterations, the IWE, and consequently the coseismic deformation, will converge, reducing potential errors from the initial estimation.

4 Results

We test the performance of the adjoint state inversion method on synthetic and observed data of large tsunami events near the Japan Trench. We choose to adopt synthetic data of the 2011 Tohoku earthquake due to it being the largest tsunamigenic earthquake ever recorded in Japan. We also applied our method to data from the 2016 Fukushima earthquake, which generated the largest tsunami recorded by S-net to date. In order to understand the method’s capability of delivering rapid warnings, we evaluate the accuracy of the IWE and waveform predictions using different window lengths and iteration numbers. Quantitatively, the accuracy can be described using the variance reduction (VR) between predicted and observed waveforms for each test. This formula has been used in various forms in seismic (Shao and Ji (2012)) and tsunami (Kubota et al. (2020)) studies to assess the fit between observations and predictions. The formula we use to evaluate waveform prediction accuracy is shown in equation 2 and is the explicit form of the formula in Kubota et al. (2020):

$$VR_{waveform} = \left(1 - \frac{\sum_{i=1}^{N_s} \sum_{j=1}^{N_t} (x_{obs,i,j} - x_{pred,i,j})^2}{\sum_{i=1}^{N_s} \sum_{j=1}^{N_t} (x_{obs,i,j})^2}\right) \times 100(\%)$$

where $x_{obs,i,j}$ and $x_{pred,i,j}$ are the observed and predicted water elevation values at the $j$th data sample of the $i$th station, respectively. $N_s$ is the total number of stations and $N_t$ is the total number of data samples at a particular station.

The formula we use to evaluate the accuracy of our IWE predictions in the synthetic test is a modified version of equation 2:

$$VR_{IWE} = \left(1 - \frac{\sum_{i=1}^{N_i} (x_{obs,i} - x_{pred,i})^2}{\sum_{i=1}^{N_i} (x_{obs,i})^2}\right) \times 100(\%)$$

where $x_{obs,i}$ and $x_{pred,i}$ are the synthetic and predicted IWE values at the $i$th location in the simulation region, respectively. $N_i$ is the total number of IWE values. Notice that in both equations (2) and (3), the variance between observations and predictions is normalized by the sum of the squares of observations at a station or the synthetic IWEs. We do not apply equation 3 to our obtained IWE values from the 2016 Fukushima event because there is no ground truth for that application. We will, however, compare our results to those obtained by Kubota et al. (2021) and the Japanese Meteorological Agency (2017) (JMA).

4.1 Application to synthetic waveforms of the 2011 $M_w$ 9.0 Tohoku earthquake

Figure 4 shows the IWE results produced when we applied the adjoint technique to our synthetic data of the 2011 $M_w$ 9.0 Tohoku earthquake and tsunami. We notice that the pattern and amplitudes of the tsunami sources resolved using waveforms of different lengths are very similar, all of which reproduce the input model with a waveform VR ($VR_{waveform}$) greater than 0.85 and a model VR ($VR_{IWE}$) greater than 0.7 after 5 iterations (Figure 7(a)). The most obvious difference between the input and resolved IWE is the presence of artifacts at the outerise region ($\sim 1^\circ$ east of the Japan trench), $\sim 2$ m uplift is seen in Figures 4F-M (longer-length waveforms) and $\sim 2$ m subsidence is
seen in Figures 5C-D (shorter-length waveforms). These artifacts are not seen in the input model (Figure 4A). Since the adjoint technique’s success relies on high station density with good azimuthal coverage, we attribute these artifacts to the lack of stations to the east of the Japan trench.

Figure 4 reveals that our estimated tsunami source results change significantly in the first 5 iterations of our inversion. The smearing of the input model $<0.2^\circ$ east of the Japan trench in the initial guess (first column from the left) decreased substantially in the first 5 iterations. The decreases are most apparent when using waveforms of length 6 or 8 min, where we see that the $>2$ m of uplift decreases to $<1$ m. The $(VR)_{\text{waveform}}$ (Figure 7a) also shows that the similarity between the synthetic and predicted waveforms increases rapidly in the first 4 iterations and then plateaus. So, we opted to use the result after 5 iterations as our final result for tsunami early warning. However, to model the tsunami source accurately for purposes of source parameter inversions, we would have to conduct at least 10 iterations of our inversion (Figure 7c) to see the convergence of $(VR)_{\text{IWE}}$.

Figure 4 also shows how the inversion results change with increasing iterations for various time window lengths. We notice that the residual (Figure 4 rightmost column) between the synthetic (input) data and our results is minimal as we increase the length of waveforms used in the inversion. For example, when using 4-min waveforms, the area of $>2$ m residual (Figure 4E) is very large compared to that at 8-mins (Figure 4M). Also, generally, the longer the waveforms we used, the higher both VR values converge to (Figures 7a and 7c). This is because longer waveforms carry more information and reach more sensors, which improves the coverage of stations used in the inversion. Comparisons between observed and predicted waveforms are shown in Figure S5.

4.2 Application to synthetic waveforms with secondary sources added

We then test the resolvability of the adjoint method for possible triggered submarine landslides. Secondary sources are important to model since they can increase the run-up of the initial tsunami waves. Moreover, since they can lead to later-arriving tsunami waves, having the capability of resolving these sources can improve a warning system’s tsunami forecasts. We model these landslides as secondary, binary sources with uplift and subsidence of 20 m on each side (Figure 2). Since the landslides are much smaller in area than the uplift/subsidence region of the primary source, they need more time to propagate to a sufficient number of stations for accurate resolvability. Given this, we apply the inversion to longer length waveforms (15, 20, and 25 mins). We use 6 sources to test the resolvability at different locations surrounding the principal-slip region.

Figure 5 shows that 4 out of 6 sources are clearly resolved when 20 min waveforms are available. The two sources near the trench are not resolved very well, possibly due to the reflection effect of the trench. The minimum and maximum amplitudes are -16.32 and 16.37 m after 5 iterations, which are reasonably accurate compared to the maximum amplitude of the input model (20 m). When using 15 min waveforms, the minimum and maximum amplitudes obtained are -14.66 and 12.85 m after 5 iterations, with only 2 input sources being resolved relatively clearly. In contrast, using 20-min waveforms resolves the uplift and subsidence of 3 sources and the subsidence of a fourth source relatively clearly. Using 25-min waveforms improves the resolvability of the fourth source, with the two sources near the trench still not being resolved very well, possibly due to reflection effects of the trench. Overall, using 25-min waveforms resolves the secondary sources best despite having very similar results to the result using 20-min waveforms. These results show that 15-min waveforms are not long enough to resolve most secondary sources we modeled but a minimum waveform length of 20-min waveforms is sufficient for that purpose.
4.3 Application to S-net data of the 2016 $M_w$ 6.9 Fukushima earthquake

In order to understand the performance of the adjoint method on real data, we applied it to S-net data of the 2016 $M_w$ 6.9 Fukushima earthquake. Figure 6 shows that using the waveforms of the first 15 min, the inversion results of the Fukushima earthquake show much smaller amplitudes compared to the result when we use longer waveforms. This is because only three stations contain the first peak when only 15 min of data is available (Figure S6a). This means that the inversion is not fed enough information to obtain an accurate enough IWE. Examining Figure S6, we see that the data fitting using only the first 15 min is much worse than that of 25 min, where more than 8 stations with complete first peaks with amplitude larger than 0.05 m are available. This, combined with the results from Figure 6 indicate that when we use waveforms that are 25-60 min long, our data fitting improves immensely compared to using 15-min waveforms.

Further examining Figure 7, we notice that the initial guess is improved greatly after 5 iterations (using 25-60 min waveforms) of our inversion but does not change that much after 5 iterations. This means that the inversion converged after 5 iterations, which is consistent with our application to the synthetic data of the 2011 Tohoku event. As a result, we use the 5-iteration result as our best output for tsunami warning. Moreover, our subsidence region and our boundary between uplift and subsidence match those obtained by Kubota et al. (2021)) and JMA, respectively (Figure 6 - 40-min and 60-min waveform results).

5 Discussion

5.1 The required waveform duration for large and small earthquakes

One important finding in our study is that the required waveform duration to achieve accurate adjoint inversion for larger events is shorter than that of a smaller event. For instance, the $M_w$9.0 earthquake requires a time window of 5 min to achieve an accurate result while the smaller $M_w$6.9 Fukushima needs a window of at least 25 min long. A number of factors can be used to explain this difference. The first factor involves the rupture size. For self-similar earthquakes, the rupture area scales with $M_w^{2/3}$. This means that the corresponding area of seafloor deformation in a larger magnitude earthquake is usually bigger compared to that of a smaller magnitude event. Thus the number of near-field stations (located directly above the rupture zone of the earthquake) is also greater for larger events. Since the rupture propagation ($\sim$2-3 km/s) is significantly faster than tsunami waves ($\sim$0.2-0.3 km/s), the near-field stations receive the tsunami signal faster and directly capture the coseismic displacement. A greater number of the near-field stations allow the major pattern of the tsunami source to be constrained faster and more accurately. With the Tohoku earthquake application, >25 stations recorded tsunami wave arrivals in the first 5 mins after the origin time. Examining the IWE results in Figure S7, we notice that the inversion of 5-min data for Tohoku resulted in an IWE that resembles the synthetic data quite well ($VR_{IWE}$ converging to $\sim$83%, Figure 7c). In contrast, only 3 stations recorded arrivals in the first 15 mins of the Fukushima application, which is insufficient to capture the major pattern of the IWE given its much smaller scale and higher resolution is needed. So we observe that the IWE does not match the expected uplift-subsidence pattern obtained by other studies until at least 25-mins waveforms are used (Figure 6). For the $M_w$6.9 Fukushima, the source size is expected to be smaller than 50 km, which is close to the interstation distance of the S-net array. As a result, very few near-field stations are directly above the peak slip point (distance of closest station to the epicenter is $\sim$30 km). This phenomenon of larger earthquakes being forecasted more accurately than the smaller ones is also reported by Mulia and Satake (2021) and can also be used to explain why the synthetic test with secondary sources requires waveforms of at least the first $\sim$20 minutes to clearly resolve those synthetic landslides (Fig-
The second factor involves the ocean depth at the location of rupture. Compared to the Tohoku earthquake, the 2016 Fukushima earthquake is located in a region with a shallower ocean depth (∼226 compared with ∼4400 m), so the speed of the tsunami wave propagation is ∼47 m/s (compared to ∼209 m/s). This means that the arrival time of the tsunami wave at the closest station is about 10 min. So, we have to wait longer for the tsunami wave to reach enough stations in a smaller earthquake scenario when it is located in a region with a shallow ocean depth.

Another factor is that the smaller-earthquake application’s waveforms have a lower signal to noise ratio. Figure 6 shows that using the waveforms of the first 15 min, the inversion results of the 2016 Fukushima earthquake show much smaller amplitudes and are very scattered compared to the result when we use longer waveforms. Figure S6 (a) shows that only three stations show the first peak clearly within the first 15 min. In addition to the longer tsunami propagation time (due to shallower depth) and absence of sufficient near field stations, this is also because the signals of these stations are noisier due to the incomplete removal of the tsunami-irrelevant steps (e.g. settlement of the instrument after deformation). In contrast, the data fitting using the first 25 min is much better, with 8 stations having complete first peaks and fewer stations being affected by the contamination of the steps. Although the two earthquake applications differed in the time window length required for an accurate result, the number of iterations (5) until variance reduction convergence is the same (see Figure 7).

5.2 Potential for tsunami early warning

An effective tsunami early warning system is dependent on time management. So, one needs to ensure that the minimum amount of time is spent on each step (data processing, inversion, prediction and information dissemination) in order to allow for as much time as possible for evacuation. Assuming the data processing including filtering, detiding and removal of tsunami-irrelevant steps is done instantaneously after the data is collected, the total tsunami warning time would be the sum of the data collection and computation times for the adjoint inversion and final forward simulation of the obtained IWE. Since the data collection time is much longer than the computation time, it is critical to use the minimum collection time that results in a reliable IWE estimate (5 and 25 min for Tohoku and Fukushima, respectively). For real-time application, the inversion using the waveforms within 5, 6, 7, 8, 10, 20, 25, and 30 min could be calculated as soon as the data is collected, which yields consistently-updating results that could be integrated into current tsunami warning systems. Since the computation time increases with the number of iterations in our inversion, we seek to minimize the number of iterations used to obtain a reliable IWE estimate. In most of our inversions, we find the variance reduction between observed and predicted waveforms plateaus after ∼5 iterations (Figure 7), with each iteration taking ∼16 seconds to complete. Although some cases require up to 10 iterations for convergence, for a preliminary estimation of the IWE for early warning purposes, we propose to use the result after 5 iterations to shorten the calculation time. This would then be updated with more accurate inversion results as longer length waveforms become available and more time is allowed for further iterations to run. The inversion itself takes about 1 min and 20 s to run using 16 CPU cores (AMD 3990 X @ 2.9 GHz) if we use the waveforms within the first 25 min from all of the S-net stations and terminate the inversion after 5 iterations using a calculation grid resolution of 90 arcsec. If we use the first 5 mins of waveform, the inversion takes 45 s. These processing times can potentially be reduced with the addition of more computational power. Assuming the final forward simulation that is used to predict the arrival times and amplitudes takes about 40 seconds, the total time for issuing the warning is about 7 min for
large earthquakes like the 2011 $M_w$ 9.0 Tohoku earthquake and about 27 min for smaller earthquakes located in shallow regions with a smaller tsunami wave speed. For the 2011 Tohoku earthquake, the first coastal arrival based on eyewitness accounts and clocks stopped by tsunami inundation was at about 23 min (Figure 1 in MUHARI et al. (2012)). This means that a hypothetical warning issued after using our inversion technique will be absent from a blind zone assuming a similar scenario were to occur. When applying the adjoint method to tsunami warning in the real world, we propose to invert the waveforms for the source and then issue an update to the source model every 5 min.

To further evaluate the accuracy of the inversion results, we run a forward simulation of tsunami waves using the inverted model after 5 iterations and compare the amplitude and time of the first peak of the predicted wave with the recordings at coastal tide gauges and GPS buoys (Figure 8, Figure S8). Note that for the Tohoku event, we utilize tide gauge recordings based on forward modeling the synthetic data shown in Figure 4A. This is to ensure that most inaccuracies we see in our inversion result’s forward modeling are due to errors in our inverted source model as opposed to differences between the synthetic and true source models. For the synthetic test of the Tohoku earthquake, if we use the model inverted from the waveforms available within 5 min, the errors of the amplitudes range between $-5.55$ and $1.93$ m. The largest discrepancy, with a magnitude of $-5.55$ m, corresponds to station Soma. The predicted amplitude at this station is 8.01 m, while the actual peak amplitude is 13.56 m (Figure S9a). It’s likely that this underestimation of the amplitude is due to insufficient constraints for smaller scale features when only 5 min waveforms are available. The use of 8 min waveforms probably provides more accurate resolution of small scale features, including small patches with high IWE, leading to a more accurate prediction of the peak amplitude at station Soma of 12.34 m (Figure S9b). Other potential sources of error in the waveform modeling may include artifacts resulting from reflections off the Japan Trench or inaccuracies in the bathymetry data. Following Tsushima et al. (2009)), we estimate the average accuracy score of the amplitude. This score is defined similarly to equation (3) in this paper, except $z_{\text{mod}}$ and $z_{\text{pred}}$, are replaced by the $A_{\text{obs}}$, and $A_{\text{pred}}$, respectively. $A_{\text{obs}}$, is defined as the maximum positive amplitude of the first tsunami wave observation at the ith coastal tide station and $A_{\text{pred}}$, is the maximum positive amplitude of the predicted waveform. When using the first 5-min and 8-min waveforms, the average accuracy score of the amplitudes is 93% and 98% respectively. For the 8-min waveforms, the amplitude errors range between $-1.92$ and 1.35 m. For the Fukushima earthquake, the errors of the amplitudes range between $-0.20$ and 0.22 m using 25-min waveforms, with an average accuracy score of 78%. The errors of the arrival times range between -9.11 and 14.61 min when using the first 5 min for the Tohoku event and 25 min for the Fukushima earthquake. Since $\sim 85\%$ of our predicted arrival times are $\pm 4$ mins of the true arrival time, they are accurate enough for warning purposes.

5.3 Comparison of the $M_w$ 6.9 earthquake result with previous results

The USGS-derived focal mechanism indicates that the Fukushima earthquake occurred on a normal fault. This indicates that the subsidence region in our IWE result (Figure 6) corresponds to the hanging wall moving down (and the uplift shows the foot wall moving up) on a North-east to South-west trending fault trace. Our obtained tsunami source distribution of the $M_w$ 6.9 Fukushima earthquake (using 60-min waveforms after 10 iterations) shows a major subsidence region with a peak of $\sim 1.3$ m and a region of uplift with a peak of $\sim 0.65$ m. The location and range of the subsidence is consistent with the IWE inferred from S-net data by Kubota et al. (2021), which resolved the subsidence region with a peak of $\sim 2.0$ m. Such subsident deformation is expected for the normal-faulting Fukushima event and corresponds to the hanging wall moving down (and the uplift shows the foot wall moving up) on a North-east to South-west trending fault trace. However, the associated uplift in the IWE caused by the foot wall moving up is not necessarily expected and is dependent on the estimated depth of the slip. Al-
though the region of uplift is not clear in Kubota et al. (2021)’s result, a finite fault slip model obtained by Japanese Meterological Agency (2017) inverted from seismic waves predicts a region of uplift (also shown in Figure 3 of Nakata et al. (2019)). The sharp boundary between the subsidence region (with a peak of $\sim 2.4$ m) and an uplifted region (with a peak of $\sim 0.5$ m) in JMA’s result is very close to the location and strike of the boundary line resolved by our method (Figure 6 green dashed line). However, none of the results in the tsunami-based inversions we compare to in Table S1 predict this uplifting region. The existence of a discernable uplifting region is related to the dip angle of the fault and the depth distribution of the slip: If the dip angle changes from 49 to 59º, the maximum of predicted uplift will increase from 0.13 to 0.28 m. A shallower slip distribution would also cause more clearly seen uplifting, as well as a sharper boundary between the uplifting and subsidence region. Thus the uplifting region we resolve suggests the existence of a minor slip patch at shallow depth. The dip angle of this patch is probably steeper than the average dip angle of the entire fault. This steepness could be explained by dynamic triggering on a splay fault which has been shown to occur in tsunamigenic earthquakes (Wendt et al. (2009); van Zelst et al. (2022)).

Our resolved IWE also allows us to model the rupture extent and location. Assuming a rectangular fault with uniform slip distribution with the fault orientation fixed to the USGS W-phase solution (strike=42º, dip angle=49º, rake=-101º), we searched for the fault parameters based on the IWE achieved using 60-min waveforms after 10 iterations. We performed a grid search for 5 parameters: the longitude, latitude and the depth of the center of the fault, the slip, and the fault width. Assuming a moment magnitude of $2.484\times10^{19}$ N.m (from USGS W-phase inversion) and a rigidity of 30 GPa, we estimate the length of the fault and build a uniform slip model for each combination of parameters. The best fitting model to our IWE is located at 141.537º E, 37.243º N, with a centroid depth of 8 km. The average slip for a model at this location is 3 m, with a width of 12 km. The fault length was then estimated to be 23 km. The comparison between our results and those obtained by other authors (including the Geospatial Information Authority (2017)) are summarized in Table S1. Compared to the average of each parameter of the other results, the center of our obtained fault is located $\sim 5.4$ km to the southeast and is $\sim 1$ km shallower (Figure S10b, Table S1). Table S1 shows that the width and length of our rectangular fault model are $\sim 0.6$ km and 3.7 km shorter than the average, respectively. Most of our obtained fault area overlaps with the average obtained by other studies (Figure S10a). Finally, our slip is $\sim 0.78$ m more than the average of the others. These differences are small and are close to the size of the grid we use ($\sim 5$ km). We show the seafloor displacement predicted from our best uniform fault model in Figure S10c. This shows that, compared to our obtained seafloor displacement in Figure 6, the seafloor displacement derived from our best fault model is shifted slightly south. Our grid search demonstrates that the IWE obtained with the adjoint inversion method can be used to characterize the earthquake source relatively accurately.

With these obtained parameters, the stress drop was estimated to be 4.6 MPa using:

$$\Delta \sigma = \frac{cM_0}{(LW)^{1.5}}$$

where $\Delta \sigma$ is the stress drop, $c$ is a constant ($=8/3$, when Poisson’s ratio is 0.25, Kanamori and Anderson (1975)), and $L$ and $W$ are the lengths and widths of our rectangular fault, respectively. Our estimation of the stress drop is very close to Kubota et al. (2021)’s estimation (2021) of 4.2 MPa. According to those authors, the shear stress change caused by the 2011 Tohoku earthquake (a thrust fault earthquake) at the epicenter of the 2016 Fukushima event is $\sim 2$ MPa. This is smaller than our estimated stress drop of the 2016 Fukushima event (a normal fault earthquake). They explained this stress discrepancy by suggesting that the Tohoku earthquake amplified already existing horizontal extensional stresses in the shallow (<15 km) portion of the bending continental
6 Conclusions

In this study, we evaluated the potential of the application of the adjoint-state full wavefield tsunami inversion method for tsunami early warning using S-net OBPGs. We use a time-derivative-based algorithm to remove the tsunami-irrelevant step functions in OBPG recordings and an automatic procedure to extract the coseismic deformation for stations close to the source. We apply the adjoint inversion method to synthetic waveforms of the 2011 $M_{w}$ 9.0 Tohoku earthquake and recordings of the 2016 $M_{w}$ 6.9 Fukushima earthquake. Using all available S-net stations, our result shows that the tsunamis associated with the $M_{w}$ 6.9 and $M_{w}$ 9.0 earthquakes can be accurately predicted 27 min and 7 mins, respectively, after they occurred. Triggered secondary sources of the $M_{w}$ 9.0 event were accurately resolved using 20-min waveforms. These results benefit from the fast estimation of the coseismic deformation beneath the closest stations to the epicenter. Moreover, we showed that tsunamis caused by $M_{w} < 7$ earthquakes tend to require longer waveforms to capture the major pattern (which has a much smaller scale) and achieve an accurate result than larger events. One plausible explanation is that the smaller rupture area leads to a longer data collection time. This is because tsunami waves need to travel a further distance to arrive at enough stations for an accurate inversion result to be obtained. Another explanation is that the Fukushima earthquake has low signal-to-noise ratio at OBPG recordings near the principal-slip region due to the tsunami irrelevant contamination. This means that waves need to travel a further distance such that this low SNR effect is minimized. Our calculated accuracy scores for the 25-min and 5-min waveform windows for Fukushima and Tohoku were 78 and 93%, respectively. The predicted amplitude and timing of first arriving waves at most of our selected coastal tide gauges and GPS buoys for both applications had errors of $\sim$2 to 1 m and $\sim$±4 mins, which are small enough for warning purposes. Our estimation of the fault parameters and stress drop associated with the 2016 event are generally consistent with those obtained by other studies. However, we resolve an uplifting region in our IWE that is possibly explained by shallow slip on a steeper fault plane. Overall, our study reveals that tsunami sources and triggered secondary sources can be accurately resolved using the adjoint inversion of OBPGs which thus has the potential to be integrated into a tsunami warning system.

Data and code availability

We downloaded the S-net OBPG data (https://www.seafloor.bosai.go.jp) from Japan’s National Research Institute for Earth Science and Disaster Prevention (NIED) website (https://doi.org/10.17598/NIED.0007). The waveforms of GPS buoy stations that are used to evaluate the performance of tsunami prediction are digitized from Nakata et al. (2019) for the Fukushima earthquake. For the Tohoku event, the location of the stations are from the Sea Level Station Monitoring Facility and Intergovernmental Oceanographic Commission of UNESCO (www.ioc-sealevelmonitoring.org). COMCOT software package is downloaded from https://github.com/AndybnACT/comcot-gfortran. The General Bathymetric Chart of the Oceans (www.gebco.net) provided our high resolution bathymetry data. The adjoint inversion code is available on github (https://github.com/xieyqgeo/adjoint_tsunami_inversion).
Acknowledgments

This work is supported by the NSF CAREER grant EAR-1848486 and the Leon and Joanne V.C. Knopoff Fund. We thank Chao An for valuable discussions about tsunami wave propagation simulation. We thank Hiroaki Tsushima for sharing their understanding and processing method about the tsunami-irrelevant steps.

References


Figure 1. Map showing the S-net stations (yellow circles) and the Japan Meteorological Agency epicenters (red stars) for the synthetic test (larger star) and the 2016 Fukushima earthquake (smaller star).
Figure 2. (a) The IWE predicted from the slip model of the 2011 Mw 9.0 Tohoku earthquake (Fujii et al. (2011)) with secondary sources added. Red star denotes JMA epicenter of the 2011 event. The horizontal and vertical axes correspond to degrees of longitude and latitude respectively. The contours indicate different uplift/subsidence levels due to the Tohoku earthquake. The green circles indicate S-net stations, with the named ones being used in (b). (b) Examples of the waveforms from labeled stations in (a) and the correction for coseismic deformation for the synthetic test without the secondary sources.
Figure 3. Illustration of coseismic effect at stations in the principal-slip region. Modified from Inoue et al. (2019). A) shows water (blue) and ocean bottom (orange) behavior at 6 time steps with ocean bottom pressure gauges indicated by the yellow triangles and labeled a, b and c. Stations a and b are located within the principal slip region while station c is located far away from it. The blue and black arrows indicate pressure-derived water column height (blue line in B) and water elevation (black line in B) with respect to the location of the ocean floor prior to an earthquake, respectively. B) shows the water height recordings at each station at each time step. Red-dashed line in both A) and B) corresponds to the height of the water column prior to the earthquake.
Figure 4. The result of the synthetic test using the stations available within 4, 6, and 8 min from the second row to the third row, respectively. The first row shows the input model (synthetic data) from Figure 2a with the secondary sources removed. In the second row and beyond: the first, second, third and fourth columns are the initial model, the results after 5 iterations, the results after 50 iterations, and the misfit between the inverted and input model (inversion result IWE minus input model IWE) after 5 iterations respectively. Each plot is labeled with a letter in the top right for reference. The contour lines and colors represent the uplift or subsidence of the water surface. The horizontal and vertical axes correspond to degrees of longitude and latitude respectively.
Figure 5. The result of the synthetic test with multiple secondary sources using the stations available within 15 (b), 20 (c) and 25 min (d) after 5 iterations. Input model shown in (a). The horizontal and vertical axes correspond to degrees of longitude and latitude respectively.
Figure 6. The result of the $M_w$ 6.9 Fukushima earthquake using the stations available within 15, 25, 40 and 60 min from the first row to the fourth row. The left, middle and right columns are the initial model, the results after 5 iterations and the results after 10 iterations. Contour lines show uplift or subsidence at intervals of 0.5 m. The red dashed lines indicate the major subsidence region with amplitude larger than 0.2m, resolved by Kubota et al. (2021) using S-net data. The green dashed lines indicate the boundary between subsided and uplifted regions of the IWE predicted from the finite fault slip model of JMA (JMA, 2017b) inverted from seismic waves. The horizontal and vertical axes correspond to degrees of longitude and latitude, respectively.
Figure 7. The change of variance reduction with each iteration. The variance reduction of waveforms of (a) the synthetic test of the $M_w$ 9.0 Tohoku earthquake and (b) the Fukushima earthquake. (c) The variance reduction of the IWE model of the synthetic test.
Figure 8. The accuracy of the predicted arrival time and amplitudes at tide gauges and GPS buoys using the waveform from the first 5 min of the synthetic test for Tohoku event and 25 min of the Fukushima event. (a) Correlation between predicted and recorded arrival times of the first peak for the two applications. (b) Correlation between the predicted and recorded amplitude of the first peak for the two applications. (c) The error distribution of the arrival time. The red curve is the fitted Gaussian distribution. The dashed lines are one standard deviation above/below the average. (d) The error distribution of the amplitude.
Adjoint-state waveform inversion using the S-net system for tsunami source imaging and early warning
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Introduction

This file contains all the supplementary information for aforementioned study. Note that some figures are on multiple pages. This was done to keep subplots readable. In those cases, captions of each figure can be found after the last subplot is shown.
Text S1. Remove the tsunami-irrelevant steps.

We first calculate the time derivative of the low pass filtered waveform (Figure S3a). We pick the peak of the absolute value of the time derivative to get the time of the step, $t_3$. If the peak is larger than 0.0005 m/s, we remove the step. To remove the step, we first estimate the average value of a small segment before and after the steps, $h_{before}$ and $h_{after}$ (Figure S3b). Then we shift the whole waveform after the step by $d_h = h_{after} - h_{before}$. Finally we perform a linear interpolation during the step (from $t_2$ to $t_4$) using the waveform segment between $t_1$ and $t_2$ and the shifted waveform between $t_4$ and $t_5$. The length of the segments ($w_2$) and the duration of the step ($w_1$) are determined by trial and error. The best lengths are 2 min for $w_1$ and 1 min for $w_2$, which results in stable and accurate results. A longer $w_1$ or $w_2$ is biased by the long term trend of the waveforms. If we use a shorter $w_1$, the window from $t_1$ to $t_2$ and $t_3$ to $t_4$ (for estimating $h_{before}$ and $h_{after}$) may overlap with the step. A shorter $w_2$ cannot produce stable estimation of $h_{before}$ and $h_{after}$ when the data is noisy.

References


## Table S1. Comparison between different rectangular fault models.

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Figures begin on next page
Figure S1. Results and misfit decrease of the synthetic tests using different methods. Method 1 (left) uses picked segments of the waveforms of the closest stations, which contain complete peaks and troughs. Method 2 (middle) uses the whole waveforms (8 min) of the closest stations. Method 3 (right) uses the whole waveforms (8 min) of all of the stations. Note the colorbar color scheme is slightly different from that found in the main text. The horizontal and vertical axes correspond to degrees of longitude and latitude respectively.
Figure S2. All the available stations for (a) the synthetic test and (b) the 2016 Mw 6.9 Fukushima earthquake. The stars are the JMA epicenters. The background is the initial water elevation in meters obtained by the adjoint state inversion after 5 iterations using the available waveforms within 5 min and 25 min, respectively. Red line shows a rough trace of the Japan Trench. The horizontal and vertical axes correspond to degrees of longitude and latitude respectively.
March 30, 2023, 9:40pm
Figure S3.  (a) Time derivative of the recording. Horizontal axis corresponds to time in seconds with respect to the earthquake origin time. Vertical axis corresponds to the time derivative in meters per second.  (b) An example of the correction for the tsunami-irrelevant steps at the station S2N14.  (c) Examples of the correction for the tsunami-irrelevant steps on S-net data for the 2016 Fukushima event. The blue curves are the waveforms after filtering with a passband of > 100 s to remove seismic signals. The red lines are the waveform after removing the tsunami-irrelevant steps around the origin time. Horizontal axis corresponds to time in seconds with respect to the earthquake origin time. Vertical axis corresponds to the amplitude in meters.
Figure S4. Examples of the waveforms from labeled stations in Figure 2(a) and the automatic correction for coseismic deformation for the synthetic test without the secondary sources. Horizontal axis corresponds to time in minutes with respect to the earthquake origin time. Vertical axis corresponds to the tsunami wave amplitudes in meters.
Figure S5. Data fitting of the synthetic tests at example stations using available waveforms within (a) 4 min, (b) 5 min, (c) 6 min, (d) 7 min and (e) 8 min (f) 10 min. Horizontal axis corresponds to time in seconds with respect to the earthquake origin time. Vertical axis corresponds to the amplitude in meters.
March 30, 2023, 9:40pm
Figure S6. Data fitting of the 2016 Mw 6.9 Fukushima earthquake at example stations using available waveforms within (a) 15 min, (b) 25 min, (c) 40 min and (d) 60 min.
Figure S7. The result of the synthetic test using the stations available within 5 min. The first, second, third and fourth columns are the initial model, the results after 5 iterations, the results after 50 iterations, and the misfit between the inverted and input model (inversion result IWE minus input model IWE) after 5 iterations respectively. The contour lines and colors represent the uplift or subsidence of the water surface. The horizontal and vertical axes correspond to degrees of longitude and latitude, respectively.
Figure S8. The station distribution of tide gauges and GPS buoys that were used to evaluate the accuracy of tsunami prediction (Figure 8) for (a) the synthetic test and (b) the 2016 $M_w$ 6.9 Fukushima earthquake. The horizontal and vertical axes correspond to degrees of longitude and latitude respectively.
Figure S9. The comparison between the synthetic and predicted waveforms at the tide gauge station Soma when using the waveforms of the first 5 min (a) and 8 min (b). The synthetic waveform is the result of forward simulation using the input slip model as shown in Figure 5A in the main text. The predicted waveform is simulated from the inverted initial water elevation. Horizontal axis corresponds to time in seconds with respect to the earthquake origin time. Vertical axis corresponds to the tsunami wave amplitudes in meters.
Figure S10. Results of our grid search to find the best fitting fault model to our initial water elevation for the 2016 Fukushima event. (a) shows our best fitting rectangular fault (brown) with our obtained fault center given by the brown star. The purple rectangle corresponds to the average fault dimensions of other studies shown in Table S1. (b) shows our fault center compared with fault centers obtained by other studies. (c) shows the initial water elevation associated with our best fitting fault model. The horizontal and vertical axes correspond to degrees of longitude and latitude respectively.