Discrete State-Space Current Control of PWM Converters: Robust Tracking Control with Additional Dynamics

Rodrigo Guzman Iturra¹ and Peter Thiemann²

¹South Westphalia University of Applied Sciences, Campus Soest
²South Westphalia Univ Appl Sci FH SWF

March 14, 2023

Abstract

Proportional-resonant current controllers are the stationary frame counterparts of current proportional-integral controllers that operate in the rotating reference frame. Recently, proportional-resonant current controllers have been extensively investigated to reduce the complexity produced by the coordinates transformation and to achieve zero steady error in the case of multi-frequency reference signals. This paper introduces an alternative discrete approach to the state-of-the-art continuous proportional-resonant current controller. In particular, the direct discrete design of a state-space current controller with additional dynamics with multi-frequency robust tracking capabilities is presented in this contribution. Moreover, this paper discusses the state-space formulation, the design, and the stability analysis of such a state-space controller and compares its performance with the state-of-the-art continuous proportional-resonant current controller with delay compensation. It was found that the maximum frequency that the proposed controller can target falls within the limit of switching frequency divided by five in contrast to the limit of switching frequency divided by seven of the continuous proportional-resonant controller. Furthermore, the effectiveness of the proposed state-space controller is demonstrated through hardware-in-loop (HiL) tests and experimental measurements carried out in a one kVA active power filter based on voltage detection.
Discrete State-Space Current Control of PWM Converters: Robust Tracking Control with Additional Dynamics

Rodrigo Guzman Iturra1∗, Peter Thiemann1

1 Electrical Machines, Drives and Power Electronics Laboratory, South Westphalia University of Applied Sciences, Lübecker Ring 2, Soest, Germany
∗ E-mail: rodrigo.guzman@ieee.org

Abstract: Proportional-resonant current controllers are the stationary frame counterparts of current proportional-integral controllers that operate in the rotating reference frame. Recently, proportional-resonant current controllers have been extensively investigated to reduce the complexity produced by the coordinates transformation and to achieve zero steady error in the case of multi-frequency reference signals. This paper introduces an alternative discrete approach to the state-of-the-art continuous proportional-resonant current controller. In particular, the direct discrete design of a state-space current controller with additional dynamics with multi-frequency robust tracking capabilities is presented in this contribution. Moreover, this paper discusses the state-space formulation, the design, and the stability analysis of such a state-space controller and compares its performance with the state-of-the-art continuous proportional-resonant current controller with delay compensation. It was found that the maximum frequency that the proposed controller can target falls within the limit of switching frequency divided by five in contrast to the limit of switching frequency divided by seven of the continuous proportional-resonant controller. Furthermore, the effectiveness of the proposed state-space controller is demonstrated through hardware-in-loop (HiL) tests and experimental measurements carried out in a one kVA active power filter based on voltage detection.

1 Introduction

A vast number of power converters applications such as grid-feeding photovoltaic inverters, generator-side/grid-side converters for wind turbine systems, and industrial active rectifiers are commanded by a cascade or two loop control scheme [1,2]. For example, consider the voltage driven shunt active power filter (VSAPF) depicted in Fig. 1. The VSAPF, as many other grid-tie power converters, is typically driven by an inner current control and an outer DC-Bus voltage control loop [3]. On the one hand, on each phase, a current controller ensures that the phase current matches the commanded reference. To this end, the current controller adjusts the converter’s phase voltage considering in the calculation process the actual grid voltage [2,4]. On the other hand, the outer DC-Bus voltage control loop defines the current references at the fundamental frequency. These references are adjusted in function of the amount of active power (P_{dc}) that the converter must exchange with the grid in order to maintain the DC-Bus voltage constant [1,3]. Furthermore, in case of the VSAPF, an additional outer loop is placed in parallel to the DC-Bus voltage controller [2,4]. The augmented controller is the harmonics controller that sets the current reference at harmonic frequencies. Indeed, a VSAPF is meant to emulate an ohmic resistance of R_{x} (Ω) at harmonic frequencies with the aim to damp resonances in power systems [2]. In simple terms, the VSAPF injects harmonic currents (i_{x}) proportional to the harmonic voltages (v_{PCCx}) present in the grid according to control law i_{x} = \frac{P_{dc}}{Q_{ref}^{x}} x_{x} where x stands for any phase r,s or t. Besides, the VSAPF can be also used for voltage support through the injection of reactive power (Q_{ref}) into the power system. Certainly, through the action of the VSAPF’s injected currents, the harmonic voltage distortion in the power grid can be reduced and harmonic resonances can be suppressed. Clearly, in the VSAPF and many other applications, an excellent operation of the power-electronics converter depends heavily on the performance of its inner current control loop. Although nowadays most of the current controllers are implemented in discrete form within a digital control system (e.g. microcontroller, DSP, FPGA, etc.), the most common approach is to carry out their design in the continuous domain [2]. Subsequently, the digital implementation is achieved by discretization of the continuous current controllers in a process known as digital design by emulation [5,6]. For example, the discretization of continuous current controllers can be carried out using the Forward-Euler, Tustin and Zero-Pole Matching (MPZ) approximations [5]. However, the current controllers’ performance can be pushed forward using direct discrete design and digital control techniques [6]. In fact, recent work [7,8] dealt with the direct design of discrete proportional- integral (PI) controllers in the rotating reference frame or d-q frame. As three phase sinusoidal signals in stationary frame are portrayed in the rotating reference frame as two DC quantities, PI controllers implemented in the rotating reference frame are enough to achieve zero state error (i.e. robust tracking). Similarly, discrete state-space based current controllers in d-q frame are reported in [9-12]. On those studies, the integral of the current error is added as additional state in the state-feedback controller with the objective to achieve robust tracking. However, if the current reference involves many different frequencies (as in the VSAPF case) or if the current control should be carried out without coordinates transformation (as in semiconductor lithography [13]), a different control strategy is needed. Indeed, at the present, all state-of-the-art current controllers that exist to tackle the robust tracking problem without coordinate transformation are continuous controllers. Specifically, the continuous proportional-resonant current controller with delay compensation [14] and the modified continuous state-space current controller [15,16]. Furthermore, if the switching frequency of the power converter is label as F_{PWM}, Hans et al. in [17] reported that for a systematic controller design, the maximum frequency that can be targeted by a proportional-resonant controller with delay compensation is F_{PWM}/10. On the other hand, the maximum frequency that can be targeted by a modified continuous state-space controller was reported in [16] and lies below F_{PWM}/15. The latter is due to the fact that the continuous state-space current controller lacks of counter-measurements against the computational delay inserted by the digital pulse width modulator (DPWM) in a digital control system. On this basis, to the best of
the authors knowledge, a discrete state-space current controller with robust tracking capability against sinusoidal signals that does not resort to coordinate transformation has not been reported in literature yet. Thus, this is the humble contribution brought by this paper. In particular, the aim of this scientific paper is to illustrate the direct design of a stationary frame discrete state-space current controller with additional dynamics that has multi-frequency robust tracking capabilities. In contrast with the modified continuous state-space controller, the discrete state-space controller with additional dynamics takes into account the computational delay due to the digital implementation and compensates for it though a state-feedback gain. Additionally, the proposed state-space controller is capable to track reference sinusoidal signals up to the \( F_{PWM}/5 \) limit. This is beyond the \( F_{PWM}/10 \) limit of the proportional-resonant controller reported in [17] and beyond the limit \( F_{PWM}/7 \) reported in [18]. Although the limit reached by the proposed controller is smaller than the \( F_{PWM}/2 \) achieved by the sensitivities-based proportional-resonant controller reported in [19], the controller design explained in this paper is much simpler than the proportional-resonant controller design based on sensitivity functions. As consequence the design complexity of the proposed controller is significantly lower than the one exhibit by the sensitivities-based resonant controller. The latter makes the discrete state-space controller with additional dynamics more amenable for automated control design. Moreover, this paper is organized as follows: Section 2 describes the physics-based model of the power converter connected to the grid and delves into the formulation of the discrete state-space equations used in this research work. Section 3, presents the discrete state-space controller design and the construction of the additional dynamics. Besides, this section also covers the calculation of the states feedback gains. Furthermore, results obtained using a Hardware-in-the-Loop (HiL) test-bench are presented in section 4. Within the HiL test-bench, a power converter interfaced to a strong grid is emulated in a PLECS RT Box, and the power converter is commanded by the discrete state-space controller implemented in a real microcontroller. Section 5 presents experimental results obtained through VSAPF of 1 kVA connected to a weak grid. Finally, section 7 performs the main conclusions.

2 Mathematical modelling

Fig. 1 shows the power circuit of the PWM converter analyzed in this paper. The power converter consist of three phase legs, with each leg connected to one phase of the power mains through a coupling inductor \( L \). It should be noticed that each coupling inductor exhibits an inherent resistance \( R \) that takes into account the non-zero resistance of the inductor’s windings. Also, the connection point between the coupling inductors and the power grid is labeled as point of common coupling (PCC). Furthermore, each leg consists of semiconductors arranged in a half-bridge bridge configuration operating with a switching frequency \( F_{PWM} \). In consequence the sampling period turns out to be \( T_s = \frac{1}{F_{PWM}} \) for single update mode and \( T_s = \frac{1}{2 F_{PWM}} \) for double update mode [20]. Lastly, the DC-Bus is formed by two capacitors connected in series that sustain together the entire DC-Bus voltage \( v_{dc} \).

2.1 Continuous State-Space Model of PWM Converter

If the average voltage provided by the three inverter legs sum to zero \( \dot{v} + \dot{v} + \dot{v} = 0 \) and if the grid voltages are balanced \( v_{PCC} = v_{PCC} + v_{PCC} = 0 \), each current phase of the power converter can be controlled independently [21]. With this in mind, the following functional differential equation is valid for phase 4 [4,22]:

\[
\frac{di_r(t)}{dt} = -\frac{R}{L}i_r(t) + \frac{1}{2L}u_r(t - T_s) - \frac{1}{L}v_{PCCs}(t) \quad (1)
\]

where \( u_r(t) \) is the inverter’s gain and \( u_r(t) \) is the duty cycle that is fed to the DPWM that governs the inverter leg phase 4. Moreover, the time-delayed input \( u_r(t - T_s) \) that appears in (1) is due to the DPWM. In effect, when a DPWM is used, sample and zero order holds (ZOHs) are triggered and synchronized with the carrier. The ZOHs are used to obtain discrete samples of the reference current, the inductor current and the grid voltage as regular time intervals as can be seen in Fig. 2 [21]. Of course, a finite time is needed to compute the next duty cycle after the ZOHs provide new values in the digital control system. The latter is illustrated at the bottom part of Fig. 2. For this reason, the duty cycle needs to considered as a delayed input \( u_r(t - T_s) \) in the state-space model presented in (1). It is due to this delayed input that the continuous system in (1) becomes an infinite dimensional system. The latter contains an infinite number of poles and therefore it is difficult to handle for control purposes [23]. It is granted that the equations for currents in phases \( s \) and \( t \) are analogous to the one presented in (1). Thus, in the incoming discussion, the modelling and control will be generalized to any converter.
phase and the phase subscript is going to be dropped. Furthermore, we can write a compact state-space model of (1) by defining the inductor’s current as the system’s output, namely $y(t) = i(t)$. Thus, from (1), it follows that:

$$i(t) = A_i(t) + b u(t - T_s) + e v_{DC} \quad (2)$$

$$y(t) = e i(t)$$

where:

$$A = \begin{bmatrix} \frac{-R}{L} & 0 \\ \frac{1}{L} & 0 \end{bmatrix}; \quad b = \frac{v_{DC}}{2}; \quad e = \frac{-1}{L}; \quad c = 1 \quad (3)$$

For the discretization process in the next section, we will focus only on the effect of the converter voltage over the inductor current by setting $v_{DC} = 0$. This can be justified because the grid voltage is considered as a disturbance signal. If the latter is measured, it can be neutralized by a disturbance feed-forward compensator [22]. Otherwise, if the grid voltage is not measured, the disturbance signal will be compensated anyway by an appropriate design of the additional dynamics.

### 2.2 Discrete State-Space of PWM Converter Model

There are many methods to discretize the continuous system described by (2) and (3). For example, the Forward Euler transformation, the Tustin with prewarping transformation, and the ZOH transformation just to mention a few [5, 6]. Among these many possibilities, it is rather intuitive to choose the ZOH transformation as discretization method due to the nature of the DPWM illustrated in Fig. 2. As matter of fact, Petit et al. in [20] proved mathematically that the most accurate physics-based discrete model of the PWM power is obtained through the ZOH transformation. Truly, the ZOH-based discrete model of the PWM converter provides an exact representation of the behavior of the inductor current $i(t)$ at the sampling instants. In addition, if the ZOH transformation is used over (2) and if a new additional state that stores the past values of the input is defined, the discrete equivalent of (2) becomes a finite dimensional system given that the duty cycle changes in a piecewise-constant fashion [24]. Of course, this is very advantageous for the control engineer because this finite dimensional model is much easier to handle. Additionally, through pole placement and manipulation of the additional state, the computational delay of the PWM digital implementation can be managed much better than with continuous control methods. Even, this delay can be compensated using state-space control techniques as it will be shown in the next section. On this basis, let us name the additional discrete state to handle the delayed input as $d[k] = u[k - 1]$. With this in mind, the ZOH discrete state-space representation of the PWM power converter including the computational delay (for one phase) is as follows [6, 24]:

$$\frac{y[k]}{i[k + 1]} = \frac{\Phi}{d[k + 1]} = \begin{bmatrix} \Phi_1 \\ \Phi \end{bmatrix} \begin{bmatrix} i[k] \\ d[k] \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u[k] \quad (4)$$

$$y[k] = \left[ \begin{array}{c} 1 \\ 0 \end{array} \right] \cdot \begin{bmatrix} i[k] \\ d[k] \end{bmatrix} \quad (5)$$

where:

$$\Phi = e^{A T_s} = e^{-\frac{R T_s}{L}} \quad (6)$$

$$\Gamma_1 = \int_0^{T_s} e^{A t} \cdot b \, dt = \frac{v_{DC}}{2 R} \cdot \left( 1 - e^{-\frac{R T_s}{L}} \right) \quad (7)$$

In effect, when (6) and (7) are used to portray the continuous model described by (2) and (3) to the discrete domain, the physical variables defined as states in the continuous model are preserved in the discrete model. Furthermore, (4) and (5) can be written in a compact form:

$$\begin{bmatrix} i[k + 1] \\ d[k + 1] \end{bmatrix} = F \cdot \begin{bmatrix} i[k] \\ d[k] \end{bmatrix} + G \cdot u[k] \quad (8)$$

$$y[k] = H \cdot \begin{bmatrix} i[k] \\ d[k] \end{bmatrix} \quad (9)$$

defining the following matrices:

$$F = \begin{bmatrix} \Phi & \Gamma_1 \\ 0 & 0 \end{bmatrix}; \quad G = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \quad H = \left[ \begin{array}{c} 1 \\ 0 \end{array} \right] \quad (10)$$

### 3 Discrete State-space controller with additional dynamics

#### 3.1 State-space current controller with additional dynamics

The block diagram of the discrete state-space controller with additional dynamics proposed in this paper can be seen in Fig. 3. In particular, the grid voltage and the inductor current are obtained in the discrete control system through analog to digital (A/D) converters. Those are triggered by the PWM unit at the peaks of the triangular carrier. Moreover, if robust tracking is to be achieved by a control system, the control problem has to be mathematically formulated in an error space that satisfy differential equations with the same nature as the exogenous signals [6]. Put it simple, if the dynamics assigned to handle the error are designed considering a model of the reference or the disturbance exogenous signals, the controller will perform perfect reference tracking and perfect disturbance rejection. In practice, the latter statement means that the additional dynamics state-space representation should contain poles at the frequencies were the reference inputs and disturbances are expected. Therefore if the aim is to track without steady state error a set of $n$ sinusoidal signals, where each individual signal exhibits a frequency $w_i = 2 \pi f_i$, the additional dynamics polynomial should be constructed as follows [24]:

$$\delta(z) = \prod_{i=1}^{n} \left( z - e^{-j w_i T_s} \right) \cdot \left( z - e^{-j w_i T_s} \right) = \delta(z) \quad (11)$$
Based on this polynomial, the additional dynamic matrices can be built as:

$$\Phi_a = \begin{bmatrix} -\delta_1 & 1 & 0 & \ldots & 0 \\ -\delta_2 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\delta_{2n-1} & 0 & 0 & \ldots & 1 \\ -\delta_{2n} & 0 & 0 & \ldots & 0 \end{bmatrix}$$  \hspace{1cm} (12)

$$\Gamma_a = \begin{bmatrix} -\delta_1 \\ \vdots \\ -\delta_{2n-1} \\ -\delta_{2n} \end{bmatrix}$$  \hspace{1cm} (13)

Given (12) and (13), what is missing to finalize the state-space discrete controller presented in Fig. 3 is the calculation of the feedback vectors gains $L_1$ and $L_2$ and the feed-forward gain $g$. Actually, the gain $g$ can be adjusted manually during commissioning with the aim to improve the response to a setpoint change. On the other hand, the vector gains $L_1$ and $L_2$ can be expanded with the additional dynamics matrices (12) and (13) by adding new states to the system. The new states are added to the system through an additional vector $x_a[k]$ which contains $2n$ rows, namely it represents $2n$ new states. With these new states in mind, an augmented state-space system with states $\begin{bmatrix} i[k] \\ d[k] \\ x_a[k] \end{bmatrix}$ can be formulated as follows [24]:

$$\begin{bmatrix} i[k+1] \\ d[k+1] \\ x_a[k+1] \end{bmatrix} = \begin{bmatrix} \Phi & \Gamma_1 & 0 \\ 0 & 0 & 0 \\ \Gamma_a & 0 & \Phi_a \end{bmatrix} \begin{bmatrix} i[k] \\ d[k] \\ x_a[k] \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u[k]$$  \hspace{1cm} (14)

for sake of convenience and compactness in the notation, we label the augmented state-space matrices as follows:

$$\Phi_d = \begin{bmatrix} \Phi \\ \Gamma_a \end{bmatrix}, \quad \Gamma_d = \begin{bmatrix} \Gamma_1 \\ \Phi_a \end{bmatrix}$$  \hspace{1cm} (15)

Finally, lets us define a new feedback vector gain $L = [L_1 \ L_2]$. The augmented feedback vector $L$ places the eigenvalues or poles of ($\Phi_d - \Gamma_d L$). Thus the gain vector $L$ can be solved through pole placement algorithms such as the Ackermann’s formula, and Robust Pole Assignment for any desired poles locations given that the pair of matrices ($\Phi_d$, $\Gamma_d$) are controllable [6]. The challenge is the selection of the appropriate poles.

### 3.2 Closed-Loop Poles Selection

Indeed, there are several methods to select the pole locations when full-feedback state-space techniques are used for control design. Neacsu in [25] mentions the Bessel polynomials, N-th order matching polynomials, linear quadratic regulators (LQRs), and Hilbert transform as pole placement techniques typically used in power electronics applications. Certainly, the best technique to carry out the poles selection for the discrete state-space controller in (14) still needs to be investigated. In this paper, we resort to Bessel polynomials. Because, this a very simple technique that provides prototype systems that have an acceptable reference change response and practically no overshoot. With this in mind, the roots of normalized Bessel polynomials scaled to 1 second settling time can be seen in Table 1. Recall that the original discrete-state space model is defined by (4),(6) and (7). Clearly, it can be seen that the original system has two states. Therefore, in order to select the first pair of target poles, we can select a Bessel Polynomial of second order from Table 1 and scale it by a desired settling time $T_{sett,0}$ as follows:

$$s_{p,q} = \frac{-4.0530}{T_{sett,0}} \pm \frac{j2.34}{T_{sett,0}}$$  \hspace{1cm} (16)

On the other hand, the poles related with the additional dynamics are related with pure sinusoidal signals as can be seen in (11). Indeed, those poles are entirely determined by the vector gain $L_2$. In any case, it is advisable to add damping to these pure complex poles. Accordingly, the target poles for the additional dynamics are chosen as the combination of a first order Bessel polynomial from Table 1, scaled with an appropriated settling time $T_{sett,1}$, and the complex poles due to the sinusoidal. This implies that for each frequency $w_i$ considered in the additional dynamics, the following target poles are defined as:

$$s_i = \frac{-4.62}{T_{sett,1}} \pm jw_i$$  \hspace{1cm} (17)

$$s_{i+1} = \frac{-4.62}{T_{sett,1}} - jw_i$$  \hspace{1cm} (18)

### Table 1 Roots of normalized Bessel Polynomials scaled to 1 second settling time. Based on [25].

<table>
<thead>
<tr>
<th>Order</th>
<th>Pole Locations of Bessel Polynomials</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-4.62</td>
</tr>
<tr>
<td>2</td>
<td>-4.0530 ± j2.34</td>
</tr>
<tr>
<td>3</td>
<td>5.0093 - j3.9668 ± j0.7845</td>
</tr>
<tr>
<td>4</td>
<td>4.0156 ± j0.7023 - j5.5281 ± j1.6553</td>
</tr>
<tr>
<td>5</td>
<td>-6.4480 - j6.1104 ± j6.3142 - j5.9268 ± j3.0813</td>
</tr>
<tr>
<td>6</td>
<td>-4.2169 ± j7.5300 - j6.2613 ± j4.4018 - j7.1205 ± j1.4540</td>
</tr>
</tbody>
</table>

The letters α, β, γ, and η refer to current references and switching control signals for the converter.
As final step, the desired continuous poles defined by (16), (17) and (18) are transformed to discrete domain using zero-pole matching:

\[ z_p = e^{\alpha T_s}; \quad z_q = e^{\beta T_s}; \quad z_i = e^{\gamma T_s}; \quad z_{i+1} = e^{\delta T_s}. \]

The results of the last equation and (15) are used as inputs to the pole placement algorithm. Thereafter, the pole placement algorithm outputs the vector gains \( L_1 \) and \( L_2 \) and the control design is finalized. Notice that the selection of the settling times \( T_{\text{Sett}L0} \) and \( T_{\text{Sett}L1} \) are the degrees of freedom for the control design. However, the settling times should be selected based on the real capabilities of the power electronics system. Thus, we follow a simple rule stated in [6] that says that a discrete controller should have at least five samples during the rising time of the controlled variable. Hence, for the discrete state-space current controller it follows that:

\[ T_{\text{Sett}L0} = 5 \cdot T_s \]  

Moreover, from our experience, the settling time of the additional dynamics should be larger than the settling time of the original system. A factor 5 to 10 times is recommended, consequently:

\[ T_{\text{Sett}L1} = M \cdot T_{\text{Sett}L0}; \quad M \in [5, 10] \]  

Nevertheless, if multiple frequencies are targeted by the additional dynamics, it is possible that the pole placement algorithm can not allocate all the poles considered in the additional dynamics with the same settling time. In such a case, a different settling time for each of the frequencies needs to be specified. A good starting point is to select a distance factor of 3 to 5 between the settling times in the poles related with the additional dynamics.

### 3.3 Stability Analysis

Undeniably, the interaction of the state-space controller and the plant should be tested in terms of stability and robustness. In this paper, we resort to the very well known method of of gain and phase margins to assess the controlled system’s robustness [5]. Of course, the stability of the closed loop presented in Fig. 3 can be investigated by analyzing its open loop dynamics. For this purpose, let us redraw a simplified equivalent discrete model of the block diagram presented in Fig. 3 as in Fig. 4. Notice that all the feed-forward gains are dropped. This is justified because the feed-forward terms do not affect the stability of the closed loop and therefore they can be neglected for the stability analysis [26]. In addition, the open loop block diagram is obtained in Fig. 4 by breaking the closed loop presented in Fig. 3 at the input of the plant [24]. From Fig. 4, the mathematical model that describes the open loop dynamics can be obtained by setting \( u[k] \) as the input to the open loop dynamics and \( w[k] \) as the output of the open loop dynamics. Specifically:

\[ \begin{bmatrix} i[k+1] \\ d[k+1] \\ x_a[k+1] \end{bmatrix} = \Phi_d \cdot \begin{bmatrix} i[k] \\ d[k] \\ x_a[k] \end{bmatrix} + \Gamma_d \cdot u[k] \]  

\[ w[k] = [L_1 \quad L_2] \cdot \begin{bmatrix} i[k] \\ d[k] \\ x_a[k] \end{bmatrix} \]  

Furthermore, equations (22) and (23) can be used to define a dynamical system in any scientific computing software. For example, in MATLAB we can resort to the command \( ss \). Once, the system is defined in the in scientific computing software, transfer functions in the z-domain or Bode diagrams can be generated directly from the state-space dynamics of the open loop dynamics. In the latter case, the gain and phase margins can be read directly from the Bode diagram.

### 3.4 Duty Cycle Saturation

From the block diagram in Fig. 3, it can be seen that four terms contribute to the duty cycle \( u[k] \). First, the full-state feedback contributions with the term \( \alpha[k] \) to the duty cycle through the vector gain \( L_1 \). Second, the additional dynamics, block that processes and handles the error space dynamics contributes the term \( \beta[k] \). Granted, that these two are the contribution of the feedback control to the manipulated variable \( u[k] \). Third, the feed-forward gain that takes care for the disturbance rejection of the measured disturbance contributes the term \( \gamma[k] \). Finally, the feed-forward gain that can be used to improve the setpoint change transient contributes the term \( \eta[k] \). On this basis, the complete duty cycle is calculated as:

\[ u[k] = \alpha[k] + \beta[k] + \gamma[k] + \eta[k] \]  

Furthermore, in order to maintain the converter within the linear modulation region, it is necessary to limit the signal \( u[k] \) within the range \([-1, 1]\) [22]. Of course, it is possible that for certain conditions, the duty cycle reaches its maximum possible value +1 or its minimum possible value -1 for several sampling periods (actuator saturation). At this point, it is not wise to process further the error space with the additional dynamics because the converter is already working at its limits. Therefore, the approach followed in this paper is similar to the conditional integration reported in [6, 24]. Put it simple, this means that whenever the calculation of \( u[k] \) reaches +1 or -1 according to the calculation shown in (24), the additional dynamics block is not computed and the additional states not updated. In brief, it means that the internal states of the additional dynamic block are keep constant until the actuator abandons its saturation state.

### 4 Hardware-in-Loop (HIL) verification

#### 4.1 HIL Setup

HIL is selected as first real-time method to assess the performance of the discrete state-space controller with additional dynamics. Indeed, this method allows us to investigate the proposed control algorithm in a high fidelity scenario with a high degree of safety. In particular, a PLECS RT Box 1 is used as HIL platform to emulate one phase of a grid-tie power converter. Additionally, a ripple filter is considered as well within the power circuit. Furthermore, the emulator converter is governed by a Texas instruments microcontroller TMS320F28069M, embedded in an LAU/CHXL-F28069M board interfaced with RT Box 1. On the one hand, the emulated power electronics in the RT Box 1 can be conceived as “digital twin” of a real power converter [27]. Because all the power electronics and power system components are virtual components. On the other hand, the state-space control algorithm is implemented in a real microcontroller. Therefore, it can be concluded that the control system is realistically tested and verified. Moreover, Table 2 summaries the parameters of the scenario considered for the HIL tests.

In addition, a picture of the HIL setup can be seen in Fig. 5. The host PC displays in real-time the signals as seen by the microcontroller for control and evaluation purposes. Moreover, in the same figure, it can be seen that the signals that correspond to the power circuit are outputted by the RT Box 1 and captured by an oscilloscope. In this regard, the power circuit emulated in the RT Box

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power System</td>
<td>230 V (L-N), 50 Hz</td>
</tr>
<tr>
<td>Distribution Transformer</td>
<td>1 MVA, 5% X_{pu} and ( \Delta_f = 20 )</td>
</tr>
<tr>
<td>Coupling Inductor</td>
<td>L = 2.3 mH, R = 2.5mΩ</td>
</tr>
<tr>
<td>Ripple Filter (parallel to PWM Converter)</td>
<td>( C_F = 300 μF, F_{PWM} = 45mF )</td>
</tr>
<tr>
<td>Switching Frequency</td>
<td>10 kHz</td>
</tr>
<tr>
<td>Sampling Time ( T_s )</td>
<td>100 μs (single update mode)</td>
</tr>
<tr>
<td>( V_{OC} )</td>
<td>700 V</td>
</tr>
</tbody>
</table>
can be seen in Fig. 6. In essence, the power grid consist on a voltage source in series with a resistance and an inductor. These two are the elements of the equivalent impedance of the power system, namely the impedance of the supply feeder and/or distribution transformer referred to the secondary side of the transformer. Also, notice that the reference currents feed to the microcontroller are generated in the LAUNCHXL-F28069M board. Equally important, it is the control algorithm deployed on the microcontroller which can be seen in Fig. 7. The current controller is based on the state-space controller algorithm is implemented in a microcontroller TMS320F28069M.

4.2 State-space controller design for 50 Hz robust tracking

In order to illustrate the design process of the discrete state-space controller with additional dynamics, let us consider the power converter depicted in Fig. 1 with the parameters stated in Table 2. With these parameters in mind, (4) and (5) become:

\[
\begin{bmatrix}
  i[k + 1] \\
  d[k + 1]
\end{bmatrix}
= \begin{bmatrix}
  0.9999 & 9.333 \\
  0 & 0
\end{bmatrix}
\begin{bmatrix}
  i[k] \\
  d[k]
\end{bmatrix}
+ \begin{bmatrix}
  0 \\
  1
\end{bmatrix}
\cdot u[k] 
\]

(25)

Furthermore, if the reference and disturbance signals are expected at \( f_1 = 50 \, Hz \), the additional dynamics matrices (12) and (13) become:

\[
\Phi_a = \begin{bmatrix}
  1.9990 & 1 \\
  -1 & 0
\end{bmatrix}; \quad \Gamma_a = \begin{bmatrix}
  1.9990 \\
  -1.000
\end{bmatrix} 
\]

(27)

If follows that the augmented state-space matrices for the pole placement and defined by (14) and (15) become:

\[
\Phi_d = \begin{bmatrix}
  0.9999 & 9.333 & 0 & 0 \\
  0 & 0 & 0 & 0 \\
  1.9990 & 0 & 1.9990 & 1 \\
  -1 & -1 & 0 & 0
\end{bmatrix}; \quad \Gamma_d = \begin{bmatrix}
  0 \\
  0 \\
  0 \\
  0
\end{bmatrix} 
\]

(28)

With the setting times defined as \( T_{\text{SetL0}} = 500 \, \mu s \) and \( T_{\text{SetL1}} = 3 \, ms \), the desired poles can be calculated as follows:

\[
s_{p,q} = \frac{-4.0530}{500\mu s} \pm \frac{j2.3400}{500\mu s} = -8.106e03 \pm j4.68e03 
\]

(29)

\[
s_{1,2} = \frac{-4.6200}{3ms} \pm j314.16 = -1.54e03 \pm j314.16 
\]

(30)

Moreover, the poles can be portrayed to the discrete domain using zero-pole matching as follows:

\[
z_{p,q} = e^{p_{q}T_s} = 0.3968 \pm j0.2006 
\]

(31)

\[
z_{1,2} = e^{1.2T_s} = 0.8568 \pm j0.0269 
\]

(32)

Using (28),(31) and (32) as inputs to the Ackermann’s formula leads to:

\[
L_1 = [0.0823 \, 0.4917]; \quad L_2 = [0.0141 \, 0.0132] 
\]

(33)

Using all the latter results, the open loop dynamics characterized by (22) and (23) can be used in MATLAB to obtain the bode diagram depicted in Fig. 8. The gain and phase margins can be easily seen from this plot. It follows that the overall controlled system exhibits a gain margin of 14.88 dB and a phase margin of 44.62°. From the figure, a very high gain of almost 200 dB can be seen at the frequency \( \omega_1 = 2 \cdot \pi \cdot 50Hz = 314 \, rad/s \). Obviously, this very high gain leads to robust tracking for reference sinusoidal signals of 50 Hz and a robust disturbance rejection for disturbance signals with a frequency of 50 Hz also.

4.3 Comparative evaluation for 50 Hz tracking

Using the HiL setup we have performed three real-time simulations. Actually, in the first two simulations, we set as current reference a sinusoidal signal with a single frequency of \( f_1 = 50 \, Hz \). In addition the amplitude of the single frequency signal changes every 0.1 seconds, at a rate of 10 Hz, from 7.5 A to 15 A. This behavior was programmed within the reference generation block on the RT Box 1. Furthermore, the first simulation is carried out with the state-space...
controller with additional dynamics designed in section 4.2 (with feedforward gain $g = 4e-02$). The HiL results of this first simulation are depicted in Fig. 9. Notice that the amplitude and frequency of the voltage at the $V_{PCC}$ and the PWM voltage at the output of the converter agree with the parameters stated in Table 2. Granted, that the average output voltage $V_{Avg}$ correspond to the output of the state-space controller scaled by the inverter gain $v_{DC}$. Moreover, we have compared the proposed controller with the state of the art proportional-resonant controller with delay compensation simulated in a second HiL test. In particular, the resonant controller used in the
comparison has the following form [14]:

\[ C(s) = K_p + K_I \frac{\cos(2\pi f_1 T_s) - (2\pi f_1)\sin(2\pi f_1 T_s)}{s^2 + (2\pi f_1)^2} \]  \hspace{1cm} (34)

where \(K_p=0.0374\) V/A and \(K_I=13.0550\) V/As are calculated for a 60° phase margin according to the tuning rules suggested in [21]. According to [5], the most accurate discrete implementation of the continuous resonant controller defined by (34) is achieved through the Tustin with pre-warping approximation. The latter transformation holds the infinite gain, the resonance frequency and zeros location. Thus, we have found the discrete equivalent of (34), that is needed in the microcontroller for the second HiL test, using the Tustin transformation with a pre-warping frequency of 50 Hz. Moreover, Fig. 10 shows the comparison of the HiL results obtained in these two simulations. Actually, the figure shows the sampled variables at the microcontroller side. In particular, the results on the left side correspond to the proportional-resonant controller and the results on the right side correspond to the state-space controller. For an easier comparison, Fig. 11 shows the sampled current and the calculated error for the reference change at \(t = 0.2\) s for both simulations on the same plot. From Fig. 11, it can be seen that the current settles faster to zero steady state error with the state-space controller than with the proportional-resonant controller. In matter of fact, the proportional-resonant controller settles in 6.4 ms and a RMS error of 1.673 A is measured for the transient during the reference change. In case of the state-space controller, it settles in 4.8 ms and the RMS error measured during this transition reaches the value of 1.95 A. Nonetheless, the state-space current controller exhibits more oscillations than the proportional-resonant controller. The latter can be explained because the phase margin achieved by the state-space current controller of 44.62° is smaller than the phase margin of 60° achieved by the proportional-resonant current controller. Nevertheless, it can be concluded that the performance of the state-space controller is comparable with the performance of the state-of-the-art proportional resonant controller with delay compensation. Actually, the state-space controller with additional dynamics settles 25% faster than the proportional resonant controller with delay compensation.

4.4 State-space controller design for multi-frequency tracking

The strength of the state-space controller with additional dynamics is that it targets several frequencies beyond the common limits of resonant controllers. For instance, Hans et al. in [17] reported the limit \(F_{PWM}/10\) and Leascu et al. in [18] reported the limit \(F_{PWM}/7\). It follows that according to the mentioned research work, for a switching frequency of 10 kHz, the highest harmonic frequency that can be tracked without steady state error is found to be the 28th harmonic (1428 Hz when \(F_{PWM}/7\) is chosen). The direct discrete design and the additional dynamics can help to overcome this limit. Thus, in order to test the multi-frequency tracking capability of the proposed controller, a third simulation was carried out using the HiL setup. As for the latter two simulations, the parameters for this third simulation are also given by Table 2. However, there is a minor difference. The DC-Bus voltage is increased to 800 V to handle the harmonics and the higher \(\frac{di}{dt}\)s in the reference signal. Specifically, the current reference in the third simulation is composed of four signals. Each of them with a different frequency and a different amplitude. In fact, the components of the reference current can be seen in Table 3. Moreover, the same harmonic frequencies were added as background distortion to the voltage of the grid (see Table 4). It is granted, that the amplitudes and phases in the current reference and in the background voltage were arbitrary selected. However, the frequencies were chosen so that three components lie below the maximum limit of the resonant controller. These three first components have frequencies 50 Hz, 650 Hz (13th harmonic) and 1150 Hz (23th harmonic). Additionally, the four component is selected beyond the limit of the resonant controller, the 37th harmonic at 1850 Hz. Moreover, following the design guidelines written in section 3, a discrete state-space controller was designed considering the frequencies 50 Hz, 650 Hz, 1150 Hz and 1850 Hz in the formulation of the additional dynamics. Furthermore the settling times, necessary for the poles calculation, were selected as 500 µs for the main poles, and the settling times for the additional dynamics related with the 50 Hz, 650 Hz, 1150 Hz and 1850 Hz were set as \(T_{Sent1}=5\) ms , \(T_{Sent2}=30\) ms , \(T_{Sent23}=35\) ms and \(T_{Sent37}=40\) ms respectively. Based on these, the gains \(L_1\) and \(L_2\) are found using the pole
Fig. 11: Sampled current and calculated error on the microcontroller side during the HiL test for the reference change at $t = 0.2s$. The proportional-resonant controller settles to zero steady state error in 6.4 ms with an RMS error during the transition of 1.67 A. The state space controller settles in 4.8 ms and the RMS error during the transitions is 1.95 A.

Table 3 Components of the current reference for the multi-frequency tracking

<table>
<thead>
<tr>
<th>Harmonic Number</th>
<th>Frequency (Hz)</th>
<th>Amplitude (A)</th>
<th>Phase (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>15</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>650</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>23</td>
<td>1150</td>
<td>1.5</td>
<td>0</td>
</tr>
<tr>
<td>37</td>
<td>1850</td>
<td>0.15</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4 Components of the grid voltage (with background distortion) for Multi-frequency tracking HiL test

<table>
<thead>
<tr>
<th>Harmonic Number</th>
<th>Frequency (Hz)</th>
<th>Amplitude (V)</th>
<th>Phase (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>325.27</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>650</td>
<td>16.26</td>
<td>0</td>
</tr>
<tr>
<td>23</td>
<td>1150</td>
<td>8.76</td>
<td>0</td>
</tr>
<tr>
<td>37</td>
<td>1850</td>
<td>3.25</td>
<td>0</td>
</tr>
</tbody>
</table>

Placement algorithm place in MATLAB. Specifically:

$$L_1 = [0.0369, 0.4396]$$  \hspace{1cm} (35)

$$L_2 = [0.0037, 0.0049, 0.0060, 0.0058, 0.0042, 0.0026, 0.0020, 0.0024]$$  \hspace{1cm} (36)

with the additional dynamics matrices as:

$$\Phi_a = \begin{bmatrix} 6.1290 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -17.6592 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 31.3755 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -37.6907 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 31.3755 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -17.6592 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} ; \Gamma_a = \begin{bmatrix} 6.1290 \\ -17.6592 \\ 31.3755 \\ -37.6907 \\ 31.3755 \\ -17.6592 \\ -1 \end{bmatrix}$$  \hspace{1cm} (37)

Furthermore, a stability analysis was carried out over the multi-frequency design using MATLAB. In this regard, a gain margin and a phase margin of 10.3 dB and 42.6° were found respectively.

Moreover, Fig. 12 shows the results obtained on the HiL setup. At the beginning of the HiL test, the amplitude of all the reference components stated in Table 3 are set to the half of their value and after 80 ms are returned back to their full value. This was done with the aim to force a reference change in all frequencies during the experiment. In contrast with the previous figures, Channel 3 this time shows the reference current $i^*$ in color blue. Also, notice that $V_{PCC}$ is not pure sinusoidal anymore. Instead it contains an important share of harmonics as can be seen in Table 4. Additionally, the inductor current $i$ in yellow and the reference $i^*$ in blue are superimposed in the oscilloscope picture. From the figure, it is possible to see that the output current follows very closely the reference command despite of the voltage harmonic disturbances coming from the PCC. Moreover, Fig. 13 shows the current reference, the sampled current and the calculated error seen by the microcontroller in this third simulation. In Fig. 13, the tracking error in steady state is practically zero and settles very fast after the setpoint change at $t = 0.33s$. Furthermore, a FFT of the signals contained in Fig. 13 is performed for the first 4 cycles. The FFT is performed using the PLECS scope and the results are depicted in Fig. 14. On the upper part of Fig. 14, it can be seen that the amplitude of reference and sampled current are practically the same for the frequencies 50 Hz, 650 Hz, 1150 Hz and 1850 Hz. Furthermore, on the bottom part of Fig. 14, the FFT of the current error can be seen. Several components different from 50 Hz, 650 Hz, 1150 Hz and 1850 Hz can be observed in the error signal. This can be explained due to the unavoidable quantization that arises during the sampling process [4]. Nevertheless, it can be seen that the error amplitude reaches minimum values, below 0.029 A, for the frequencies 50 Hz, 650 Hz, 1150 Hz and 1850 Hz. Those frequencies are exactly the ones targeted by the additional dynamics. Altogether, this third HiL simulation shows the capability of the state-space controller to perform multi-frequency tracking up to the $F_{PWM}/5$ limit.

5 Experimental Results

5.1 STATCOM: State-space controller for 50 Hz robust tracking

A scaled down laboratory prototype is used to evaluate experimentally the state-space controller with additional dynamics. Actually, the laboratory prototype can be seen in Fig. 16. The secondary of a RUHSTRAT variable transformer is chosen as PCC for the experiment. We have set a RMS voltage of 30 V (L-N) at this node...
by adjusting the variable transformer. Furthermore, a three-phase ripple filter consisting of only capacitors (150 µF per phase) is connected to the experiment’s PCC. A two level power converter, as the one depicted in Fig. 1, in the form of a SEMIKRON IGBT Stack (SEMITEACH B6U+E1CIF+B6CI) is utilized as power converter with 100 V of DC-Bus voltage. Each leg of the SEMIKRON stack is interfaced to the PCC through an inductor with nominal value 1.544 mH. Lastly, the power converter is controlled by a dSPACE MicroLabBox with processor DS1202 where the control system shown in Fig. 1 was implemented. In particular, the DS1202 processor runs the discrete state-space controllers with additional dynamics as current control algorithm for each phase. Furthermore, for this first experiment, the state-space current controllers were designed according to the guidelines described in section 3 targeting only the 50 Hz component. Furthermore, the MicroLabBox obtains the voltages at the PCC through three ELDITEST GE8115 differential probes and the inductor currents through three AC/DC Fluke current clamps i30s. On the other hand, the DC-Bus is supplied by a DC power supply ElektroAutomatik EA-PS 8720-15. Although it is possible to have an entirely self-supported DC-Bus without the DC power supply, the DC source is used because we wanted to avoid large DC-Bus charging currents on the anti-parallel diodes of the SEMIKRON Stack on every experiment trial. In any case, a DC-Bus voltage controller was implemented and it took care to maintain the DC-Bus voltage constant releasing active power to the grid during transients. Moreover, and based on the setpoints $P_{Ref}$ and $Q_{Ref}$, the reference currents for the inner current control loop are calculated in alpha and beta coordinates using PQ theory equations [22]:

\[
i_{\alpha Ref}(t) = \frac{2}{3} \frac{v_{PCC\alpha}}{v_{PCC\alpha}^2 + v_{PCC\beta}^2} P_{Ref} + \frac{2}{3} \frac{v_{PCC\beta}}{v_{PCC\alpha}^2 + v_{PCC\beta}^2} Q_{Ref}
\]

(38)

\[
i_{\beta Ref}(t) = \frac{2}{3} \frac{v_{PCC\beta}}{v_{PCC\alpha}^2 + v_{PCC\beta}^2} P_{Ref} - \frac{2}{3} \frac{v_{Go}}{v_{PCC\alpha}^2 + v_{PCC\beta}^2} Q_{Ref}
\]

(39)

Once the power converter was connected to the grid, we set 400 VAR as $Q_{Ref}$ and $P_{Ref}$ was given by the DC-Bus controller to maintain 100 V DC-Bus voltage. In order to make an impression of the performance improvement brought by the state-space current controller, in the first part of the experiment, between $t = 0.19$ s and

Fig. 13: Current reference, sampled current and calculated error on the microcontroller side during the multifrequency HiL test. State space controller with additional dynamics with poles at 50 Hz, 650 Hz, 1150 Hz and 1850 Hz.

Fig. 14: FFT of the current reference and sampled current signals on the microcontroller side depicted in Fig. 13

Fig. 15: FFT of the signals on the microcontroller side: Current reference and sampled current depicted in Fig. 13

Fig. 16: Experimental setup with 12 kVA adjustable transformer and SEMIKRON IGBT Stack
\( t = 0.26 \text{ s} \) in the incoming figures, we used as feedback current controller just a proportional gain and disturbance feed-forward gain as in [21]. Then, we switched to the state-space discrete current controller with additional dynamics approximately at \( t = 0.26 \text{ s} \). Fig. 17 shows the voltages at the PCC and Fig. 18 shows the sampled inductor currents for the three phases. All this information is recovered from the dSPACE Control Desk application. Notice that the voltages at the PCC have an important share of harmonic content. In effect, the THD measured for one phase reaches the 7\% threshold, with significant components at the 5th and 7th harmonics. As consequence a high distortion can be seen at the positive and negative peaks of the \( V_{\text{PCC}} \) voltages. On the other hand, the tracking performance of phase \( r \) can be seen in Fig. 19. Indeed, the figure shows the current reference in red, the sampled current in blue, and the calculated error in green. Also, from the figure, it can be seen that between \( t = 0.19 \text{ s} \) and \( t = 0.26 \text{ s} \), the phase currents differ from the references and non-zero steady error can be observed. Of course that is not unexpected because as at this stage we are using only a proportional controller [22]. Additionally, the powers \( P \) and \( Q \) exchanged with the grid are measured using a SIMULINK block deployed on the MicroLabBox and the results are depicted in Fig. 20. Due to the errors on the current tracking, the reactive power \( Q \) measured between \( t = 0.22 \text{ s} \) and \( t = 0.26 \text{ s} \) reaches only the value of 300 VAR, that is a deviation of 100 VAR from the setpoint \( Q_{\text{Ref}} \). It turns out, that after the state-space controller is activated at approximately \( t = 0.26 \text{ s} \), the reference tracking error in the three-phases decreases to practically zero as can be seen in Fig. 19. In addition the measured reactive power \( Q \) matches the reference \( Q_{\text{Ref}} \) as can be verified in Fig. 20 for any point on time beyond \( t = 0.28 \text{ s} \). The converter does not exchange active power \( P \) with the grid, because the DC power supply provides the nominal 100 V in the DC-Bus. However due to the transient at \( t = 0.26 \text{ s} \), caused by the switch in the current controller algorithm, there is an increase in the DC-Bus voltage. The latter is compensated by the DC-Bus controller that demands a negative \( P \) to balance the DC bus voltage again to 100 V. Moreover, a setpoint change is performed in the reactive power exchange by increasing \( Q_{\text{Ref}} \) from 400 VAR to 800 VAR. This is done with the idea in mind to test the tracking capability of the state-space controller to significant setpoint changes. Fig. 21 shows the inductor currents measured during the change of \( Q_{\text{Ref}} \) from 400 VAR to 800 VAR. From the oscilloscope capture, it is possible to see that the currents injected by the power converter maintain the 50 Hz shape also during the setpoint change. Similarly to the activation test of the state-space controller, Fig. 22 displays in the same plot the current reference, the sampled current and the calculated error for phase \( r \). From the figure, it is possible to see that the calculated error settles to approximately zero and the sampled current matches the reference current without steady state error. Moreover, Fig 23 shows the measurements for \( P \) and \( Q \) during the setpoint change on \( Q_{\text{Ref}} \). It can be seen from Fig. 23 that after the transient, the measured \( Q \) matches the reference \( Q_{\text{Ref}} \) of 800 VAR.

5.2 VSAPF: State-space controller for multifrequency robust tracking

In order to evaluate the multi-frequency tracking capability of the state-space controller with additional dynamics, the power converter showed in Fig. 16 was operated as STATCOM and VSAPF simultaneously using the control system depicted in Fig. 1. In particular, the block diagram of the control system in Fig. 1 illustrates in detail how the references for the VSAPF’s currents are generated. First, the VSAPF measures the voltages of the grid at PCC. Second, the fundamental component (e.g., 50 Hz) is eliminated from the grid voltages through a notch filter with a center frequency of 50 Hz and quality factor \( Q \) equal to 3.15. It follows that the output of the notch filter contains only the harmonic voltage distortion present in the grid. Third, the harmonic voltage distortion present in the grid...
is scaled by the factor $1/R_s$ to generate the VSAPF’s current reference at harmonic frequencies. The VSAPF is commanded to emulate a harmonic resistance of $R_s \Omega$. Finally, the reference for the fundamental component of the VSAPF’s currents is calculated based on the active power required (PRef) and reactive power required (QRef) using equations (38) and (39). On the one hand, and similar to the first experiment explained in subsection 5.1, we used only proportional gain and disturbance feedforward gain at the beginning of the second experiment. Then, we switched to the state-space discrete current controller with additional dynamics approximately at $t = 0.26 \text{s}$. After the activation of the state-space controller, the measured Q matches the reference of 400 VAR.

![Fig. 20: Active power and reactive power exchange with the grid measured during the experimental test: state-space controller activation. Activation of the state-space controller with additional dynamics at $t = 0.26 \text{s}$. After the activation of the state-space controller, the measured Q matches the reference of 400 VAR.](image)

Fig. 20: Active power and reactive power exchange with the grid measured during the experimental test: state-space controller activation. Activation of the state-space controller with additional dynamics at $t = 0.26 \text{s}$. After the activation of the state-space controller, the measured Q matches the reference of 400 VAR.

![Fig. 21: Experimental measurements of the inductor currents during the set reactive power setpoint change. Waveforms captured at the setpoint change in reactive power from 400 VAR to 800 VAR. Signal descriptions Channel 1: PCC voltage phase r (Yellow), Channel 2: Inductor Current phase r (Green), Channel 3: Inductor Current phase s (blue), Channel 4: Inductor Current phase t.](image)

Fig. 21: Experimental measurements of the inductor currents during the set reactive power setpoint change. Waveforms captured at the setpoint change in reactive power from 400 VAR to 800 VAR. Signal descriptions Channel 1: PCC voltage phase r (Yellow), Channel 2: Inductor Current phase r (Green), Channel 3: Inductor Current phase s (blue), Channel 4: Inductor Current phase t.

The results presented in Fig. 24 to Fig. 26 show the transient during the activation of the state-space current controller with additional dynamics with 50, 250, and 350 Hz frequencies. The latter figure shows that the voltage distortion at the fifth harmonic (250 Hz) is reduced to 772.64 mV due to the enhanced harmonics current tracking brought by the state-space current controller. Likewise, the voltage distortion at the seventh (350 Hz) is reduced from 826 mV to 695 mV due to the action of the state-space current controller within the VSAPF control system. Finally, the current reference, sampled measured current and current error for phase r, gathered by the dSPACE system during the state-space current controller activation, are depicted in Fig. 26. From the figure, it can be seen that the reference current is not a pure sinusoidal signal. Instead, the reference current contains the 250 and 350 Hz components besides of a fundamental component of the VSAPF, measured Q matches the new reference of 800 VAR.

![Fig. 22: Current reference, sampled inductor current and calculated error for phase r measured during the reactive power setpoint change.](image)

Fig. 22: Current reference, sampled inductor current and calculated error for phase r measured during the reactive power setpoint change.

![Fig. 23: Active and reactive powers exchange with the grid measured during the reactive power setpoint change experiment. The discrete state-space current controllers govern the power converter during this experiment. After the reactive power setpoint change, the measured Q matches the new reference of 800 VAR.](image)

Fig. 23: Active and reactive powers exchange with the grid measured during the reactive power setpoint change experiment. The discrete state-space current controllers govern the power converter during this experiment. After the reactive power setpoint change, the measured Q matches the new reference of 800 VAR.

harmonic (350 Hz) is 826 mV. Moreover, Fig. 25 shows the transient during the activation of the state-space current controller with additional dynamics with 50, 250, and 350 Hz frequencies. The latter figure shows that the voltage distortion at the fifth harmonic (250 Hz) is reduced to 772.64 mV due to the enhanced harmonics current tracking brought by the state-space current controller. Likewise, the voltage distortion at the seventh (350 Hz) is reduced from 826 mV to 695 mV due to the action of the state-space current controller within the VSAPF control system. Finally, the current reference, sampled measured current and current error for phase r, gathered by the dSPACE system during the state-space current controller activation, are depicted in Fig. 26. From the figure, it can be seen that the reference current is not a pure sinusoidal signal. Instead, the reference current contains the 250 and 350 Hz components besides of a 50 Hz components, as expected for a VSAPF operation. Moreover, prior to $t = 155.55 \text{s}$, it can be seen that the current error reaches 2 A peak-to-peak amplitude. After $t = 155.55 \text{s}$, when the state-space current controller is activated, the current tracking is improved, as can be inferred from the current error signal that is reduced practically to zero. The results presented in Fig. 24 to Fig. 26 show the experimental verification of the state-space current controller with additional dynamics for the multi-frequency case.
Fig. 24: Experimental measurements, injection of 100 V Ar and VSAPF set to behave as a 1 Ω harmonic resistance. Waveforms captured during steady state before the activation of the state-space current controller. Signal descriptions Channel 1: PCC voltage phase r (Yellow), Channel 2: Inductor Current phase r (Green), Channel 3: Inductor Current phase s (blue), Channel 4: Inductor Current phase t, Channel M1: FFT of PCC’s voltage r.

Fig. 25: Experimental measurements, injection of 100 V Ar and VSAPF activated with command to emulate a resistance of Rs = 1 Ω. Waveforms captured during the activation of the state-space current controller. Signal descriptions Channel 1: PCC voltage phase r (Yellow), Channel 2: Inductor Current phase r (Green), Channel 3: Inductor Current phase s (blue), Channel 4: Inductor Current phase t, Channel M1: FFT of PCC’s voltage r.

**6 Conclusion**

This paper walked through the design of a discrete state-space controller with additional dynamics. The state space controller is capable to track reference signals that contain several frequencies without steady state error and in defiance of disturbance signals at the same frequencies. The latter is achieved if the additional dynamics of the state-space controller are constructed with poles at the frequencies were the reference and disturbance signals are expected. Actually, the tracking capabilities lies a bit beyond the limit of state of the art continuous proportional-resonant controllers with delay compensation. Furthermore, this paper has shown in detail how the physics-based discrete state-space model is derived from the continuous state-space model using the ZOH transformation. State feedback gains are used to locate the poles of original discrete system, namely the inductor current and the artificial state that represents the computational delay. Through pole placement and manipulation of the computation delay state, the computational delay of the PWM digital implementation can be managed and compensated using state-space control techniques. Moreover, this paper illustrated the procedure to construct the inputs to the pole placement algorithm and explained the process to select the target close-loop poles using Bessel polynomial roots. In addition, the procedure to perform the stability analysis of the current discrete state-space controller with additional dynamics was also described. Furthermore, the performance of the single frequency discrete state-space controller was found to be comparable with the proportional-resonant controller with delay compensation in a comparison evaluation carried out using the HiL setup. Finally, the proposed discrete state-space controller was tested on a scaled down laboratory prototype. The power converter in the laboratory’s prototype was operated first as STATCOM and was used to inject reactive power to the mains. The experimental results have shown that the discrete state-space controller with additional dynamics was able to produce very nice 50 Hz sinusoidal currents despite of the fact that the power grid exhibited an important share of harmonic content in the STATCOM experiment. Afterwards, the prototype in the laboratory was used as voltage driven active power filter. In this second experiment, the state-space with additional dynamics was capable to reduce the voltage distortion at the fifth and seventh harmonics by improving the tracking performance of the current controller at 250 and 350 Hz. Altogether, the effectiveness of the proposed state-space controller to track sinusoidal signals with almost no steady state error was verified in HiL tests and in real experiments.

**7 Acknowledgments**

This work was supported by the Zentrales Innovationprogramm Mittelstand (ZIM) from the German Federal Ministry for Economic Affairs and Energy (BWMi). Besides, the article is funded by the Open Access Publication Fund of South Westphalia University of Applied Sciences. In addition, the authors would like to express their most sincere gratitude to Mr. Marvin Cruse for his support during the experimental evaluation.
References


