Hydroeconomic asymmetries and common-pool overdraft in transboundary aquifers

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Abstract

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Key Points:

\begin{itemize}
\item Economic and hydrogeologic differences between users affect common-pool externalities in shared aquifers
\item Combined asymmetries in energy cost, groundwater profitability and aquifer response can exacerbate overdraft incentives
\item A shift in asymmetries might have facilitated the world’s first distance-based groundwater treaty.
\end{itemize}

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Abstract

The common-pool nature of groundwater resources creates incentives to overpump that contribute to their rapid global depletion. In transboundary aquifers, users are separated by a territorial border and might face substantially different economic and hydrogeologic conditions that can alternatively dampen or amplify incentives to overpump. We develop a theoretical model that couples principles of game theory and groundwater flow to capture the combined effect of well locations and user asymmetries on pumping incentives. We find that user asymmetries in either energy cost, groundwater profitability or aquifer response tend to dampen incentives to overpump. However, combinations of two or more asymmetry types can substantially amplify common-pool overdraft, particularly when the same user simultaneously faces comparatively higher costs (or aquifer response) and profitability. We use this theoretical insight to interpret the emergence of the Disi agreement between Saudi Arabia and Jordan in association with the Disi-Amman water pipeline. By using bounded non-dimensional parameters to encode user asymmetries and groundwater connectivity, the theory provides a tractable generalized framework to understand the premature depletion of shared aquifers.

1 Introduction

Groundwater supplies approximately 40% of global irrigation needs [Siebert et al., 2010] and 50% of urban water consumption [Zektser and Everett, 2004], where it serves as a reliable source of water in times of increasing climate variability [Marchionni et al., 2020; Müller et al., 2021]. Yet global aquifer resources are being mined at an alarming rate. Groundwater extracted from long-term storage contributes nearly 1 mm per year to global sea level rise [Wada et al., 2010] and at least 1.7 billion people are living in areas where groundwater resources are under threat [Gleeson et al., 2012]. Groundwater is also fundamentally a shared resource. Most aquifers have multiple distinct users simultaneously exploiting them, often across territorial boundaries. Nearly 600 aquifers are shared between countries, compared to approximately 270 internationally shared river basins [UNESCO-IHP, 2015].

Pumping cost externalities are an important feature of shared aquifers that accelerates their depletion [Negri, 1989]. They arise because pumping by individual users affects groundwater levels faced by the other users of the aquifer, and hence their pumping costs. Under these conditions, individual users can make a private profit from the extracted groundwater without having to pay for the collective cost of their pumping. This gives users the incentive to pump more than what would maximize the value of the extracted water, leading to a premature depletion of the resource – a phenomenon we refer to as common-pool overdraft [Müller et al., 2017; Penny et al., 2021a]. A variety of institutional arrangements exist to address this issue in shared aquifers, including market-based mechanisms, [e.g., Bruno and Sexton, 2020] or local collective action [e.g., Lopez-Gunn, 2003]. However, formalized cooperative institutions are exceedingly rare when the shared aquifer extends beyond territorial borders. Several river treaties regulate riparian aquifers to the extent that they influence the quantity and quality of transboundary river flows. For example, the 1999 Convention on the Protection of the Rhine (Art. 3(a)) aims to maintain and improve the quality of the Rhine’s waters by preventing, reducing or eliminating as far as possible pollution “including that from groundwater”. However, only a small fraction (14%) of transboundary surface water agreements include a clause pertaining to groundwater, and most (87%) of these treat groundwater as a subsidiary of surface water [Gior dano et al., 2014]. To our knowledge, only six international agreements focus primarily on the management of internationally shared aquifers, and only two of them place specific restrictions on groundwater use [Burchi, 2018]: (i) The Genevese aquifer treaty, signed in 1978 between Switzerland and France, limits annual pumping volumes on both sides of the border [de los Cobos, 2018], and (ii) the Disi Aquifer Agreement, signed in 2014 between Jordan and Saudi Arabia, places restrictions on pumping distances by establishing a
Two important features of transboundary aquifers distinguish them from other shared
aquifers and might shed light on the distinctive nature of common-pool issues in these
aquifers. First, well fields are separated by a political or administrative border, which gives
rise to spatially distinct well fields (i.e., no enclaves). This contrasts with non-transboundary
aquifers, where users can own land and wells that adjoin to, or even enclave into, another
user’s land (Figure 1). The spatial distance between each country’s well fields in trans-
boundary aquifers plays a key role in determining pumping cost externalities and incen-
tives to overpump. This principle is embodied in the Disi agreement, where pumping cost
externalities are kept in check by setting a minimum distance between well-fields [Müller
et al., 2017]. Second, non-transboundary aquifers are often shared by a large number of
users facing comparable unit costs of energy for pumping and market prices of water and
agricultural products. In contrast, transboundary aquifers are typically shared by a small
number of parties – generally the two governments on either side of the shared border.
Although a large number of stakeholders might influence internal water policy and poli-
tics on either side of the border, formal trans-boundary cooperation will ultimately emerge
between the two governments that have sovereignty over the shared water and the jurisdic-
tion to sign an international agreement. The effect of internal politics will be reflected in
the objectives and constraints faced by the two governments and affect their utility func-
tion (see Discussion in Section 4.1). These dynamics, and their underlying hydrogeologic
and economic circumstances, can be substantially different on either side of the border and
lead to strongly asymmetric incentives to use the aquifer (Figure 1). For instance in the
context of the Disi Aquifer, Jordan does not produce a significant amount of oil and faces
unit pumping costs approximately 3 times higher than Saudi Arabia, who has access to a
substantial volume of oil (see Discussion in Section 4.2 and [Müller et al., 2017]). These
differing circumstances contribute to regional power asymmetries that have strong impli-
cations for international water relationships that are extensively discussed elsewhere [Fer-
ragina and Greco, 2008]. It remains unclear how asymmetries might shape transboundary
groundwater policy, both in terms of economic and hydrogeologic differences between two
countries. For instance, a party being aware of the higher energy costs faced by the other
party might increase – or decrease – their incentives to overpump. Consequently, asym-
metries also affect the distance between each party’s well fields that is necessary to mit-
igate these incentives. They also introduce an additional dimension to the common-pool
problem, where in addition to being overused, groundwater can also be misallocated if the
relative distribution of abstracted water across parties does not maximize their joint utility.
To our knowledge, these effects have not been examined within a formal theoretical model.

With this in mind, we develop a game theoretic model that captures economic and
hydrogeologic asymmetries in transboundary groundwater scenarios. Formulating the
problem in this fashion allows a clearer understanding of the controlling variables that
must be managed when designing transboundary treaties. Non cooperative game theory
has been widely used, both to represent conflicting incentives along transboundary rivers
[e.g., Eleftheriadou and Mylopoulos, 2008; Khachatryan and Schoengold, 2019; Dema,
2014], and to capture the effect of pumping cost externalities on groundwater user incen-
tives in shared (non-transboundary) aquifers [e.g., Negri, 1989; Gardner et al., 1997;
Provencher and Burt, 1993]. However, few studies have considered transboundary aquifers,
where the two important characteristics described above – spatial separation and asym-
metric conditions – prevail. The earliest study that we are aware of uses game theory to
simulate groundwater cooperation along the US-Mexico border [Nakao et al., 2002]. In
line with most early models of pumping cost externalities in shared aquifers [e.g., Negri,
1989; Gardner et al., 1997; Provencher and Burt, 1993], the study represents the aquifer
as a homogeneous “bathtub” and neglects the attenuating effect of distance on draw-down.
The spatial nature of pumping cost externalities was later accounted for by Brozović et al.
[2010] using the Theis solution to model the drawdown around individual wells in non-
transboundary aquifers. The Theis solution provides for a mathematically tractable way to
couple transient groundwater behavior with a game theoretical model where users dynam-
ically optimize their pumping across multiple periods. However, it relies on strongly sim-
plifying physical assumptions that might misrepresent the spatial distribution of drawdown
over large distances. Müller et al. [2017] used the principle of superposition to couple a
(static) game theory model with a fully calibrated 2D finite-difference model that accounts
for the complex behavior of real aquifers. The game was applied in a transboundary con-
text to show that an agreement imposing a minimum distance between pumping centers
reduced incentives to over-pump in the Disi-Saq/Ram aquifer shared between Jordan and
Saudi Arabia. More recently, the model was extended to incorporate the effect of mutual
trust on incentives to commit to a formal agreement over internationally shared ground-
water [Penny et al., 2021a]. These studies have shown that pumping cost externalities de-
crease sharply with spatial distance. However, we do not know whether and how this rela-
tionship is affected by the economic and hydrogeologic asymmetries between parties that
often emerge in transboundary aquifers. Bridging this gap has direct practical relevance
in terms of linking the emergence of transboundary groundwater cooperation to domes-
tic policies (such as the development of strategic water infrastructure, see Discussion in
Section 4.2) that might amplify or dampen these asymmetries.

In Section 2, we derive a theoretical model that couples principles of game the-
ory and groundwater flow to capture the combined effect of the spatial distance between
well-fields and the hydro-economic asymmetries between users on pumping incentives.
The non-dimensional nature of the model allows the interaction between hydro-economic
asymmetries, pumping distances and common-pool overdraft to be investigated efficiently
and comprehensively within a generalized theoretical framework. The parameter space of
the model is enumerated in Section 3, where we specify the conditions causing hydroe-
omic asymmetries to (i) dampen or (ii) amplify incentives to overpump, (ii) to cause
water resources to be misallocated (in addition to overpumped), and (iii) to cause pump-
ing cost externalities to increase (not decrease) with spatial distance. Intuition to inter-
pret each of these theoretical insights is given in Section 3.1-3.4. Section 4 discusses the
realism of the model with respect to its key underlying assumption (Section 4.1) and il-
lustrates its application to interpret the emergence of the existing Disi aquifer agreement
(Section 4.2).

2 Theory

2.1 Utility and assumptions

Consider an aquifer shared by two parties during an arbitrary planning horizon \( t^* \)
[\( T \)]. Each party generates a utility \( U \) (e.g., monetary profit) from the total volume of
groundwater that they are able to pump during that period. For party \( i \), assume that the
net present value of this utility is expressed as:

\[
U_i(q_i, q_j) = \alpha_i q_i - \beta_i q_i \cdot d_i(q_i, q_j)
\]

(1)

where \( q_i \) and \( q_j \) [\( L^3 \)] are the volumes of groundwater pumped by each party. Note that
these represent total pumping volumes over the whole period \( t^* \) and across all the wells
within each party’s well field. Parameter \( \alpha_i \) [\$/\( L^3 \)] is the marginal benefit associated with
the pumped water, or (equivalently) its volumetric market price. If there are no formal
markets for water, \( \alpha_i \) can be interpreted as a shadow price reflected in the value of the
produced agricultural product [Müller et al., 2017] and/or the unit cost of the cheapest
alternative water source [Penny et al., 2021a]. The second term on the right-hand-side
of Equation 1 represents pumping costs, which are jointly proportional to the volume of
pumped water \( q_i \) and the average depth \( d_i \) [\( L \)] to groundwater. The proportionality factor
\( \beta_i \) [\$/\( L \cdot L^3 \cdot L^{-1} \cdot L^{-3} \)] can be interpreted as the unit cost of the energy used to pump the wa-
ter. Both \( \alpha_i \) and \( \beta_i \) are assumed constant during the period \( t^* \) but allowed to vary between
the two parties. The function \( d_i(q_i, q_j) \) indicates the depth of the groundwater level below
Figure 1. Schematic representation of the two key differences between transboundary and non-transboundary aquifers considered in this study. Wells in transboundary aquifers are spatially separated by a territorial boundary, on either side of which very different economic and hydrogeologic conditions might prevail.

party i’s well field. This depth is averaged over time and space using the (given) pumping capacity of each well as weight. We assume a two-dimensional confined groundwater flow so that the principle of superposition applies [Reilly et al., 1987]:

\[
d_i(q_i) = d_{0,i} + D_{ii}q_i + D_{ij}q_j,
\]

where \(d_{0,i} [L]\) is the average depth to groundwater under party i’s well field during period \(t^*\) if neither party exploits the aquifer during that period. If the aquifer was previously unexploited, this corresponds to the initial static depth of the groundwater. Parameters \(D_{ii} [L/L^3]\) and \(D_{ij} [L/L^3]\) represent the hydrogeologic response of the aquifer and indicate the effect of a unit volume withdrawal by a party on the average drawdown at their own \((D_{ii})\), and at the other party’s \((D_{ij})\) well fields. As before, the unit withdrawal and ensuing drawdowns are averaged across the relevant party’s well field, with well capacities used as weights. The relative weights of each well within their well field are assumed constant throughout the period \(t^*\). Other important assumptions behind the utility function in Equations 1-2 are discussed in Section 4.2, along with their applicability to transboundary aquifers.

2.2 Hydroeconomic asymmetries

Asymmetries in the transboundary groundwater system described above can arise via any of the hydrogeologic or economic parameters pertaining to parties \(i\) or \(j\). In order to make the problem more tractable and generally applicable, we work in a dimensionless framework [e.g., Bolster et al., 2011] that reduces the number of key variables to the following four non-dimensional parameters.
The hydrogeologic connectivity ratio $k$, given by
\[ k := \frac{D_{ij}}{\sqrt{D_{ii}D_{jj}}} \in [0, 1], \]
provides information about how sensitive the drawdown experienced by each party is to the other party’s pumping. For instance, $k \to 0$ would indicate that the well fields of the two countries are hydraulically distinct and do not affect each other, whereas $k \to 1$ would indicate a perfectly overlapping well fields or a “bathtub” type aquifer.

The hydrogeologic asymmetry ratio $g$ (or its transformed version $r$), given by
\[ g := \frac{\sqrt{D_{ii}}}{\sqrt{D_{ii} + \sqrt{D_{jj}}}} \in [0, 1] \]
\[ r := 1/g - 1 = \frac{D_{ij}}{D_{ii}} \in [k, k^{-1}] \]
captures differences in well drawdown on either side of the border, and indicates whether either country would more rapidly increase groundwater depth at its own wells even if pumping rates were equal. To simplify notation, we substitute the hydrogeologic asymmetry parameters by the transformed parameter $r$ in all derivations below. For spatially distinct well-fields, $r$ is bounded by $k$ and $k^{-1}$ if the effect of each party’s pumping on the other party’s costs does not exceed its effect on their own costs (i.e. $D_{ij} < D_{ii}$ and $D_{ij} < D_{jj}$). Parameters $k$ and $r$ are determined by the material characteristics of the aquifer and the spatial layout of the wells, as described in Appendix A.

Asymmetries in economic conditions faced by both parties are represented by the energy cost ratio $c$ and the intrinsic probability ratio $p$, given by
\[ c := \frac{\beta_i}{\beta_i + \beta_j} \in [0, 1] \]
\[ p := \frac{P_i}{P_i + P_j} = \in [0, 1]. \]
The intrinsic profitability $P_i = \alpha_i - \beta_i d_0$ indicates the profit associated with the first drop of water pumped by each party under autarkic conditions (i.e. if their cost were unaffected by the other party’s pumping). Both $c$ and $p$ vary between 0 and 1 and take a value of 0.5 if both parties face identical conditions.

### 2.3 Nash Equilibrium Pumping

Absent any coordination or cooperative institutions, each party determines their optimal pumping volume $(q^*_i, q^*_j)$ so as to maximize their own utility, knowing that the other party maximizes theirs. This situation where each party optimally responds to the other party’s pumping decision is known as a Nash Equilibrium and satisfies the joint first order conditions [see, e.g., Müller et al., 2017]:
\[ \left\{ \begin{array}{l} \frac{\partial U_i}{\partial q_i} \bigg|_{q^*_i} = 0 \\ \frac{\partial U_j}{\partial q_j} \bigg|_{q^*_j} = 0. \end{array} \right. \]

Applied to the utility function in Equation 1, these conditions imply (see Mathematica notebook at https://www.wolframcloud.com/obj/mmuller1/Published/GW_Assymetry.nb):
\[ \left\{ \begin{array}{l} q^*_i = A \cdot \frac{2c r (1-c) - k r (1-p) c}{p (4-k^2) c (1-c) r^2} \\ q^*_j = A \cdot \frac{-2c (1-p) c - k p (1-c) r^2}{p (4-k^2) c (1-c) r^2} \end{array} \right. \]
with $A = \frac{p}{pr^2(1-c)}$. Because the utility functions $U_i(q_i)$ and $U_j(q_j)$ are quadratic and take the form of inverted parabolas, $q_i^*$ (respectively $q_j^*$) indicates a global maximum of $U_i(q_i)$ (respectively $U_j(q_j)$). We further specify that $q_i^*$ and $q_j^*$ must be non-negative, meaning that neither party can make a profit by injecting water into the aquifer. Instead, if conditions are such that one party does not pump ($q_i^* = 0$ or $q_j^* = 0$), the other party will exploit the aquifer on their own (autarkic conditions) and not be affected by pumping cost externalities. Taking the derivative of $U_i$ (resp. $U_j$) by $q_i$ (resp. $q_j$) while keeping $q_j$ (resp. $q_i$) at zero yields pumping volumes in autarkic conditions:

\begin{align}
q_i,0 &= A \cdot \frac{1}{2c} - \frac{1-p}{2pr^2(1-c)} \\
q_j,0 &= A \cdot \frac{1}{2c} - \frac{1-p}{2pr^2(1-c)}
\end{align}

Combining Equations 9-11, the non-negative Nash Equilibrium pumping for party $i$ can finally be expressed as:

\begin{align}
q_i^{NE} = \begin{cases} 
\min(0, q_i^*) & \text{if } q_j^* > 0 \\
q_{i,0} & \text{otherwise}
\end{cases}
\end{align}

with an equivalent expression for party $j$.

### 2.4 System Optimal Pumping

In contrast, a centralized authority seeking to maximize system-level welfare will assign pumping volumes to each party so as to maximize the summed utility of the system $U = U_i + U_j$. These System-Optimal pumping volumes satisfy the joint first order conditions [see, e.g., Müller et al., 2017]:

\begin{align}
\frac{\partial U_i}{\partial q_i} |_{\hat{q}_i} &= 0 \\
\frac{\partial U_j}{\partial q_j} |_{\hat{q}_j} &= 0
\end{align}

and can be expressed as (see Mathematica notebook at https://www.wolframcloud.com/obj/mmuller1/Published/GW_Assymetry.nb):

\begin{align}
\hat{q}_i &= A \cdot \frac{2pr^2(1-c)-kr(1-p)}{pr^2r(1-c)-kpr} \\
\hat{q}_j &= A \cdot \frac{2pr^2(1-c)-kr(1-p)}{pr^2r(1-c)-kpr}
\end{align}

The pumping allocation $(\hat{q}_i, \hat{q}_j)$ corresponds to a maximum point of $U(q_i, q_j)$ if the determinant of the Hessian matrix of that function is negative. It can be shown that this second-order condition holds if (see Mathematica notebook at https://www.wolframcloud.com/obj/mmuller1/Published/GW_Assymetry.nb):

\begin{align}
k < 2\sqrt{\frac{c}{1-c}}.
\end{align}

Because $U$ is a joint quadratic function of $q_i$ and $q_j$, the obtained pumping volumes correspond to a global maximum. As before, parties are not permitted to inject water into the aquifer. Therefore, if $\hat{q}_i$ or $\hat{q}_j$ reaches zero, the autarkic pumping volumes of the other party represent system-optimal abstractions (they satisfy Equation 13). Non-negative System-Optimal pumping for party $i$ (or, equivalently, for party $j$) are finally expressed as:

\begin{align}
q_i^{SO} = \begin{cases} 
\min(0, \hat{q}_i) & \text{if } \hat{q}_j > 0 \\
q_{i,0} & \text{otherwise}
\end{cases}
\end{align}
2.5 Common-pool overdraft metrics

Our analysis hinges on the idea that asymmetries affect incentives to over-exploit the shared aquifer, leading to both an excessive volume of pumping and a loss in the utility that pumped groundwater allows to generate. The above non-dimensional framework allows both effects to be expressed in generalized relative terms and as functions of strictly non-dimensional parameters.

The pumping ratio \( \rho_q \) describing the relative excess in total groundwater withdrawals under Nash Equilibrium, compared to System Optimal conditions, is defined as

\[
\rho_q := \frac{q_i^{NE} + q_j^{NE}}{q_i^{SO} + q_j^{SO}}
\]

(17)

The utility ratio \( \rho_u \) describing loss in utility associated with common-pool overdraft can similarly be expressed as:

\[
\rho_u := \frac{U(q_i^{NE}, q_j^{NE})}{U(q_i^{SO}, q_j^{SO})}
\]

(18)

where the function \( U(q_i, q_j) = U_i(q_i, q_j) + U_j(q_i, q_j) \) represents the total utility generated by the system for a given pumping allocation. The relative groundwater overdraft \( \rho_q \) is almost always greater than 1, and the relative utility \( \rho_u \) is always less than or equal to one. Ratios that diverge from one can be viewed as undesirable because they denote greater overdraft (\( \rho_q > 1 \)) or less utility (\( \rho_u < 1 \)) than could be achieved with system-optimal pumping.

Per the non-dimensionalization carried out in Section 2.2, all dimensional characteristics can be encompassed into a single dimensional parameter \( A = \frac{P_i}{\sqrt{q_i q_j}} \) \([L^3]\) that is identical for all pumping expressions (Equations 9, 10-11 and 14). This feature allows \( \rho_q \) to be expressed as a combination of non-dimensional parameters. In particular, if Nash Equilibrium and System Optimal pumping volumes are strictly positive for both players, the overdraft ratio can be expressed as (see Mathematica notebook at https://www.wolframcloud.com/obj/mmuller1/Published/GW_Assymetry.nb):

\[
\rho_q = \frac{3c(1 - c) - k^2}{(4 - k^2)c(1 - c)} \left[ 1 + kr \frac{(1 - c) - p + 2cp}{2(1 - c)r^2 p + 2c(1 - p) - kr} \right]
\]

(19)

Substituting the non dimensional parameters in the equation for \( \rho_u \), the joint utility function can be re-expressed as:

\[
U(\tilde{q}_i, \tilde{q}_j) = (P_i + P_j) \cdot A \left[ p\tilde{q}_i + (1 - p)\tilde{q}_j - p \left( c\tilde{q}_i + kr\tilde{q}_j + (1 - c)\tilde{q}_j \right) ight]
\]

(20)

where \( (\tilde{q}_i, \tilde{q}_j) = (q_i/A, q_j/A) \) are non-dimensionalized pumping volumes and, as before, \( A = \frac{P_i}{\sqrt{q_i q_j}} \). This, again, allows us to express \( \rho_u \) as a combination of strictly non-dimensional parameters representing the hydrogeologic connectivity and hydroeconomic asymmetry of the system. In particular, if all pumping volumes are strictly positive the utility ratio yields (see Mathematica notebook at https://www.wolframcloud.com/obj/mmuller1/Published/GW_Assymetry.nb):

\[
\rho_u = \frac{3c(1 - c) - k^2}{(4 - k^2)c(1 - c)(4 - k^2)} \left[ 4 + \frac{k^2}{c(1 - c)} \frac{c^3(1 - p)^2 + r^2 p^2(1 - c)^3}{(1 - p)^2 + r^2 p^2(1 - c) - rkp(1 - p)} \right]
\]

(21)

3 Results

Using the theory, we consider common-pool overdraft for 4913 combinations (i.e. 17 distinct and independent values of 3 different variables resulting in \( 17^3 \) combinations) of hydro-economic differences between two aquifer users in terms of energy costs (c),
Common-pool overdraft for the 4913 considered combinations of economic and hydrogeologic asymmetries. Relative excess pumping ($\rho_q$) and relative utility losses ($\rho_U$) respectively displayed in panels A and B as functions of hydrogeologic connectivity $k$. Combinations with asymmetries along one, two or three dimensions are represented in green, blue or gray, respectively. Common-pool overdraft for completely identical users (full symmetry) is represented in black.

1. In situations with one type of asymmetry (i.e. in either $p$ or $c$ or $g$, in green on Figure 2A), $\rho_q$ remains smaller than for the baseline case of fully identical users. This suggests that single asymmetries always dampen excessive groundwater withdrawals due to common-pool externalities.
2. Despite point 1, $\rho_U$ can still exceed the baseline case of fully identical users (green in Figure 2B). This suggests that asymmetries can amplify the utility losses due to pumping cost externalities without amplifying the aggregate excess of water withdrawn.
3. If two or more types of asymmetries are simultaneously present (blue and grey on Figure 2A), $\rho_q$ can exceed the baseline case of fully identical users. This suggests that multiple types of asymmetries can interact to amplify excessive groundwater withdrawal.
4. Lastly, $\rho_q$ can vary non-monotonically with $k$ and, in particular, decreases with $k$ for sufficiently high values of $k$ (Figure 2A). This contradicts the premise that common-pool overdraft decreases with the physical distance between users, which is a fundamental premise of distance-based agreements.

These four insights are discussed separately in the following paragraphs.
3.1 Overdraft and misallocation

We first address the second insight, where asymmetries can simultaneously decrease relative excess pumping, while increasing the ensuing relative loss of utility. This can be seen on Figure 3A-C, where \( p_q \) (blue) and \( p_U \) (red) can both simultaneously be lower than their fully symmetric counterparts (dotted). This paradox emerges because asymmetries cause water to not only be overpumped but also misallocated between the users, i.e., the proportion of pumped water ultimately used by each party is not optimal. By causing water to be misallocated, Nash-equilibrium pumping decreases the total utility of the system even if the total volume of pumped water across users is not much different from the system-optimal case (e.g., Figure 3A for \( k < 0.5 \)). As expected, a system-optimal allocation assigns more water to user \( i \) if they have a higher aquifer profitability (Figure 3D), a lower energy cost (Figure 3E) or a smaller aquifer response (Figure 3F). In these cases we refer to party \( i \) as the advantaged user. However, Nash equilibrium pumping will cause water to be misallocated by being too evenly distributed between the two users, i.e., the disadvantaged user will pump more then they should. This can be seen on Figure 3D-F, where Nash Equilibrium allocation (red) are closer to an pumping allocation ratio of 0.5 (equal pumping) than what would be System optimal (blue). For some intuition, consider that the water pumped by the disadvantaged user could have generated more profit to the system if it were instead pumped by the advantaged user. Under these conditions, the system-optimal allocation of water will assign less water to the disadvantaged user than what they would have pumped if left to their own devices (Nash Equilibrium).

In the context of this paper, we are predominantly interested in the effect of user asymmetries on the premature depletion of aquifers due to common-pool overdraft. Because of this, the remainder of the discussion focuses on \( p_q \), which describes excess water withdrawals, rather than \( p_U \), which describes utility losses and conflates the effects of overdraft and water misallocation. In a similar vein, our results suggests that for some (extreme) combinations of asymmetries, common-pool externalities can cause users to communally pump less groundwater than system-optimal (see Figure 2B and blue regions in Figure 4). However, because this outcome does not contribute to the premature depletion of aquifers, we focus the discussion on asymmetric situations where common-pool externalities cause an excess in groundwater withdrawal, that is, \( p_q > 1 \).

3.2 Single asymmetries dampen common-pool overdraft

For intuition as to why single asymmetries dampen common-pool overdraft (Insight 1), consider that the disadvantaged user will not pump in the Nash Equilibrium scenario if asymmetries are either too extreme, or insufficiently attenuated by a low hydrogeologic conductivity, for them to make a profit. This happens because the advantaged user pumps enough for groundwater levels to sufficiently drop, leading to excessively high pumping costs for the disadvantaged user. Under these conditions, only one user exploits the aquifer and there can be no common pool overdraft (see Figure 3 A-C for \( k > 0.75 \)). By ‘disadvantaged’ user, we mean the party facing higher costs (\( c > 0.5 \)), lower intrinsic profit (\( p < 0.5 \)) and/or a stronger aquifer drawdown response (\( g > 0.5 \)), and vice versa for the ‘advantaged’ user. At lower levels of connectivity (e.g., \( k \in [0.3, 0.75] \) on Figure 3A and C or \( k \in [0.5, 0.75] \) in Figure 3B) or asymmetry (e.g., compare Figure 3A-C and 3G-I for \( k > 0.75 \)), pumping by the advantaged user drops and the disadvantaged user faces sufficiently low pumping costs to make a profit. Under these conditions, both users exploit the aquifer at the Nash Equilibrium, but the advantaged player is still able to make a substantially larger profit from the pumped water. Because of this, a system-optimal allocation of groundwater that seeks to make the most out of the extracted water would not allow the disadvantaged user to pump. In other words, the disadvantaged user should not pump but nonetheless pumps. This causes the advantaged user to face higher pumping costs and pump less in the Nash Equilibrium scenario compared to a System-Optimal scenario where the disadvantaged player does not pump. In other words, the higher consuming
users pumps less than they should. This reduced pumping by the higher-consuming user yields an overdraft ratio that is lower for highly asymmetric users than for fully identical users with identical pumping rates (compare the solid and dotted $\rho_Q$ lines for $k < 0.75$ on Figure 3BC and, to a smaller extent, 3A).

### 3.3 Non-monotonic relationship between overdraft and connectivity

Regarding the non-monotonic relationship between $\rho_q$ and $k$ (Insight 4), consider that there is no overdraft for excessively low values of $k$ because pumping by either user does not affect the groundwater levels and pumping costs faced by the other. As expected, overdraft increases with $k$ for low values of $k$, as users have an increasing effect on each other’s costs. Overdraft then peaks when $k$ is large enough for the disadvantaged player to have to stop pumping in a system-optimal allocation. Beyond that level and for increasing values of $k$, conditions in the Nash Equilibrium become increasingly unfavorable for (and lead to reduced pumping by) the disadvantaged user. In other words, the difference between Nash Equilibrium and System-optimal pumping (and therefore $\rho_q$) decreases until reaching 0 (and therefore $\rho_q = 1$) when the disadvantaged user stops pumping in the Nash Equilibrium scenario.

### 3.4 Combined Asymmetries can amplify overdraft

In situations with asymmetries along more than one dimension, it becomes possible for $\rho_q$ to exceed the symmetric baseline value (Insight 3). In these situations, user asymmetries interact to amplify common-pool overdraft. This situation is represented by red surfaces in Figure 4 and arises when asymmetries in costs ($c$ and/or $g$) work to compensate for the effect of asymmetries in profitability ($p$). This happens, for example, if user $i$ benefits from more favorable profitability conditions than user $j$ ($p > 0.5$) but also faces higher pumping costs due to higher energy unit costs ($c > 0.5$, Figure 4B), a stronger aquifer drawdown response ($g > 0.5$, Figure 4D) or a combination of both (Figure 4C).

To provide intuition consider the case displayed in Figure 5, where user $i$ benefits from a profitability level that is three times as high as user $j$ ($p = 0.75$) but also faces three times the energy costs ($c = 0.75$). Because the two asymmetries compensate exactly, both users face the same cost-benefit calculus when optimizing their individual pumping. Both users will therefore have identical pumping rate in the Nash Equilibrium scenario (Figure 5A, dashed). However, system-optimal pumping allocations would discourage pumping by user $j$ because (a) his own productivity is lower than user $i$ ($p = 0.75$) and (b) his effect on the pumping costs faced by user $i$ is higher because user $i$ faces higher energy costs. In other words, pumping by user $j$ has larger pumping cost externalities. Drawdown on user $i$ caused by pumping by user $j$ has higher systemic costs than drawdown on $j$ caused by $i$. Compared to fully symmetric baseline conditions (Figure 5B dotted), this causes the volume allocated to user $j$ under system-optimal conditions to decrease (Figure 5A, blue solid) and leads to a sharp increase in $\rho_q$ (Figure 5B, blue).

### 4 Discussion

The model offers practically relevant theoretical results on the nuanced effects of user asymmetries on common-pool overdraft incentives. The theory that we present is mathematically tractable and provides intuition to interpret these results, but it is also a strongly simplified representation of the dynamics and incentives that emerge in shared aquifers. In this section, we argue that despite its simplified nature, the theory provides qualitative insights that are nonetheless helpful to understand real transboundary aquifers. We first review and discuss some of the strongest assumptions of the model (Section 4.1). We then apply the theory to the Disi aquifer shared between Jordan and Saudi Arabia (Section 4.2). We use the model to relate the emergence of the distance-based ground-
Figure 3. Overdraft and misallocation for single asymmetries, with regards to profitability (left), energy costs (middle) and aquifer response (right). Panels A-C: Relative excess water withdrawal (blue) and relative utility loss (red) ratios are displayed against hydrogeologic asymmetries. Corresponding ratios for identical users (full symmetry) are indicated as dotted lines for comparison. Panels D-F: Pumping allocations are represented as the fraction of total water withdrawn allocated to user $i$ in the Nash Equilibrium (dashed) and System-optimal (solid) scenarios. Nash equilibrium allocations are closer to the 0.5 horizontal line of equal allocation (dotted). Panels G-I: Identical graphs as panels A-C but for slightly lower levels of asymmetry in intrinsic profitability (Panel G, $p = 0.65$ instead of $p = 0.75$ in Panel A), energy costs (Panel H, $c = 0.65$ instead of $c = 0.75$ in Panel B) and aquifer response (Panel I, $g = 0.65$ instead of $g = 0.75$ in Panel C).
Figure 4. Effect of combined asymmetries on $\rho_Q$. Panels A-E map $\rho_Q$ values on the profitability asymmetry ($\rho$) vs. hydrogeologic conductivity ($k$) plane for different combinations of asymmetries in costs ($c$) and hydrogeologic response ($g$). Red lines represent equivalues of $\rho_Q$, starting at $\rho_Q = 1$ for $k = 0$ and increasing in increments of 0.5 along the red arrows. Red surfaces represent zones where the pumping overdraft ratio is larger than the symmetric case ($\rho_Q > \rho_Q^*$), blue surfaces represent zones where $\rho_Q < 1$, and grey surfaces represent zones where $\rho_Q$ is undefined because only one user exploits the aquifer in both the Nash Equilibrium and System Optimal scenarios. Solid black lines represent $k$ values beyond which one user stops pumping in the Nash Equilibrium scenario. Dashed black line represent $k$ values beyond which one user should stop pumping in the system-optimal scenario. Panel F maps the $c$ and $g$ asymmetry combinations considered in Panels A-E. Pie charts represent the distribution of $c$-$k$ combinations resulting in $\rho_Q < 1$ (blue), $1 < \rho_Q < \rho_Q^*$ (white) and $\rho_Q > \rho_Q^*$ (red), which correspond to the areas occupied by the corresponding colors in Panels A-E. Configurations that are qualitatively equivalent to Panels B-E (with users $i$ and $j$ swapped) are indicated with a ‘~’ symbol.
Figure 5. Example of user asymmetry configuration amplifying \( \rho_q \). Panel A: Normalized pumping rate \( q/A \) against hydrogeologic connectivity \( k \) for users \( i \) (red) and \( j \) (blue) under the Nash Equilibrium (dash) and System-Optimal Scenarios (solid). Pumping rates are non-dimensional because normalized by \( A = \frac{P_i}{(P_i+P_j)D_{ni}} \) (see Equations 9 and 14). Panel B: Relative pumping excess (\( \rho_q \), blue) and utility loss (\( \rho_U \), brown) ratios against hydrogeologic connectivity \( k \). Equivalent ratios for identical users (full symmetry) are displayed in dotted lines. For both panels, asymmetry parameters are \( c = 0.75, p = 0.75, g = 0.5 \).

water agreement to changing asymmetry conditions associated with the development of the Disi-Amman pipeline.

4.1 Modelling Assumptions

The quadratic utility function in Equations 1 and 2 is an extremely simplified representation of groundwater user incentives that has been used in previous work to parsimoniously capture the dynamics of pumping cost externalities in transboundary aquifers [e.g., Müller et al., 2017; Penny et al., 2021a]. It relies on the following important assumptions that should be kept in mind when interpreting our results.

First, it assumes that the considered dynamics are captured by the contrasting incentives of the two governments that have jurisdiction over the water on either side of the border and the power to engage into formal transboundary agreements. While several aquifers are shared by three countries or more, transboundary interference of pumping on draw-down decreases rapidly with distance [Müller et al., 2017], so the aggregate influence of a third country on local pumping costs are likely negligible. We assume that the utility derived by each government from the aquifer is affine to the total profit realized by the wells on its territory, implying that internal (e.g., domestic politics, taxes and subsidies) and regional (e.g., trade, bilateral cooperation) factors are assumed to either scale or shift the utility function by a constant. Their effect are therefore either embedded in the parameters \( \alpha_i \) and \( \beta_i \) (scale) or they do not affect the pumping that maximizes the utility function (constant shift).

Second, following Loáiciga [2004], the model assumes that the benefits and costs of exploiting the aquifer are respectively proportional to the volume of pumped groundwater, and to the potential energy necessary to obtain it. This allows parameters \( \alpha_i \) and \( \beta_i \) to be respectively interpreted as the unit price of pumped groundwater volume and the unit cost of pumping energy. Because formal markets for water are rare, \( \alpha_i \) is often a shadow price to be determined using proxies, such as the cost of conveyance infrastructure [Müller...
et al., 2017], or the cost of of obtaining water from an alternative source [Penny et al., 2021a]. The linear form of the utility function implies that (i) the cost of drilling and setting up the pumps is small compared to the life-time (energy) costs of operating them, (ii) the shadow price of water is exogenously given and not itself affected by groundwater production [see Dang et al., 2016], (iii) systemic costs of decreased water levels beyond pumping cost externalities (e.g., decreased streamflow production [Sahu and McLaughlin, 2021]) are neglected and (iii) water is the limiting factor of production. This last assumption can be relaxed by adding a pumping threshold to utility function beyond which pumping does not generate additional benefits [Penny et al., 2021a]. Here we assume that this threshold is substantially higher than equilibrium pumping rates, which is appropriate in a water-limited agricultural context [Müller et al., 2017].

Third, we assume that each party is aware of the utility function of the other party and that its parameters are constant throughout the considered planning horizon. Under these conditions, each party can plan their complete pumping schedule in advance, knowing the effect that the pumping by the other party will have on their own costs. In other words, both parties have complete foresight and so do not need to react dynamically to unforeseen changes in the other party’s pumping. The model represents the total utility derived from the total volume of water pumped during the considered exploitation period or planning horizon. As such, it aggregates over the actual schedule of pumping rates by either party, which might vary from year to year. The model can be extended to relax the assumption of full information and incorporate uncertainties on the environment [Müller et al., 2017] or the other on the party’s trustworthiness [Penny et al., 2021a].

Last, we assume that drawdown is expressed as a linear combination of (current and past) pumping from all wells in the shared aquifer. This principle – known as the superposition principle – governs groundwater flows in confined aquifer. A comparable (though less tractable) game theoretical model based on an equivalent expression for (steady state) unconfined aquifers is discussed in [Penny et al., 2021b,a].

4.2 Application to the Disi aquifer

Despite its simplified nature, the theory is helpful to understand how large economic and hydrogeologic disparities on either side of the border can drive incentives to over-exploit transboundary aquifers. For example, such asymmetries might have played an important role in the feasibility of the agreement for the Management and Utilization of the Ground Waters in the Al-Sag /Al-Disi aquifer, signed between Saudi Arabia and Jordan in 2014 and imposing the pumping restriction zones mapped on Figure 6A. Jordan and Saudi Arabia face substantially different unit energy costs (c = 0.77) and aquifer responses (g = 0.71), respectively due to a cheaper access to oil in Saudi Arabia and different well-field layouts between the two countries (see Appendix B). Historically, Saudi Arabia has been exploiting the aquifer extensively for export-oriented irrigated agriculture [Elhadj, 2004], whereas exploitation on the Jordanian side (also mostly for agriculture) was much more limited. However, with the commission of the Disi-Amman pipeline in 2014, Jordan has transitioned to a much more extensive exploitation of the aquifer for the urban water supply of the city of Amman. Based on data from Müller et al. [2017](see Appendix B) we estimate that this transition has caused a shift in the asymmetry in groundwater profitability as groundwater used for urban water supply has a comparatively larger marginal value than irrigation water. Consequently, p increased from 0.32 to 0.83, as groundwater became comparatively more profitable/valuable on the Jordanian side of the border with the commission of the pipeline. Mapping on the k vs. p asymmetry plot for c = 0.77 and g = 0.71 shows that the change in p associated with the Disi pipeline had a dramatic effect on common-pool overdraft (Figure 6B).

Before the pipeline, Saudi Arabia benefited from more favorable conditions according to all three asymmetry parameter and its pumped volume (∼ 1000 million cubic me-
Figure 6. Application to the Disi aquifer shared between Jordan and Saudi Arabia. Panel A Represents the approximate location of agricultural and pipeline well fields, and the approximate locations of the pumping restriction zones imposed by the treaty. Pumping is proscribed within the area delineated by solid red lines. Pumping is limited to municipal (water supply) purpose within the area delineated by the dashed red lines. Panel B represents a p-k plot similar to Figure 4 for the prevailing energy cost ($c = 0.77$) and hydrogeologic ($g = 0.71$) asymmetry conditions, with values of $p$ before and after the pipeline represented in blue and orange, respectively. The zone with $p_Q$ larger than the corresponding symmetric value is represented in grey. Under pre-pipeline conditions, the disadvantaged user (Jordan) would stop pumping for $k > 0.11$. Panels C-D. Relative excess pumping and relative utility losses for asymmetry conditions before (blue) and after (orange) the pipeline. Ensuring that $p_Q < 1.3$ and $p\Delta U > 0.95$ under after the pipeline implies keeping $k$ below 0.35. Panel D represents the relationship between $k$ and the distance between well fields as derived in Appendix A. The inter-wellfield distances corresponding to $k = 0.11$ and $k = 0.35$ are 55km and 51km, respectively.
ters per year, $MCM/yr$ was an order of magnitude larger than Jordan ($\approx 100 MCM/yr$) [Müller et al., 2017]. In fact, our model suggests that Jordan would have stopped exploiting the aquifer altogether if hydrogeologic connectivity would have reached $k = 0.11$ (Figure 6B, blue). Using the simplified groundwater model in Appendix (represented on Figure 6E), we estimate that this would have happened if the centers of mass of both countries’ nearest well fields were located within 55 kilometers from each other (they are currently at approximately 150 kilometers).

However, by shifting $p$ in favor of Jordan, the pipeline dramatically increased common pool overdraft which now exceed the baseline symmetric case (grey zone in Figure 6B). Depending on the hydrogeologic connectivity, pumping cost externalities can cause an excess water withdrawal of up to 400% (Fig 6C) and lead to a decrease in utility by up to 20% compared to system optimal levels (Fig 6D). Examining the $\rho_p$ vs. $k$ relation in Figure 6C shows a non-monotonic relationship between overdraft and connectivity. However, $\rho_p(k)$ peaks at very high levels of connectivity $k$, which can be avoided with the Disi agreement by establishing a minimum distance between pumping wells. Our model suggests that, in the context of the Disi aquifer, maintaining common-pool utility losses of less than 5% or, equivalently, a common-pool overdraft of less than 30%, requires hydrogeologic connectivity to be capped at approximately $k = 0.35$ (Figure 6C and D). This corresponds to a distance of approximately 50 km between well field centers of mass (Figure 6E), which is roughly comparable to the buffer distance of 20 to 50 km imposed by the Disi agreement (Figure 6A).

5 Conclusion

Transboundary groundwater management represents an emerging concern due to the increasing reliance on groundwater resources and extensive overdraft in aquifers worldwide. Transboundary groundwater policy remains in its early stages and there is a need to synthesize across aquifers in order to more effectively and equitably generate policy approaches. In order to explore the role of hydrogeologic and economic asymmetries in transboundary scenarios, and identify opportunities for cooperation, we developed a tractable theoretical model to characterize the combined effect of hydrogeologic connectivity and hydroeconomic user asymmetry on pumping cost externalities and the associated incentive to over-exploit shared aquifers. The theory suggests that asymmetries introduce second perverse effect of pumping cost externalities, where water is not only over-exploited but also misallocated across users. It also reveals that, counterintuitively, incentives to overpump can sometimes increase with increasing distance between users, if these users face sufficiently asymmetric hydroeconomic conditions. Lastly, although single asymmetries (in either energy costs, intrinsic profitability or aquifer response) tend to decrease common-pool overdraft, combinations of asymmetries along multiple dimensions can substantially amplify incentives to overpump. This phenomenon might have taken place in the Disi aquifer, where the introduction of the Disi pipeline on the Jordanian side allowed the asymmetry in intrinsic profitability of the aquifer to counterbalance opposite asymmetries in the energy costs and aquifer response. This, according to the theory, might have allowed common-pool overdraft to increase dramatically. We surmise that this shift in the symmetry landscape caused by a domestic policy in Jordan (the construction of the Disi pipeline) has increased the feasibility of the world’s first known distance-based transboundary groundwater agreement.

A: Spatial Considerations

We examine the relationship between parameters $k$ and $r$ and the spatial layout of the wells exploiting the aquifer. Consider an aquifer with two-dimensional confined groundwater flow and fully penetrating wells, and assume that the planing horizon $r'$ is long enough for groundwater flows to reach a steady state equilibrium. Under these condi-
tions, Thiem’s solution can be used to describe the steady state drawdown around individual wells. For a single well, we have $D_{ii} [L/L^3]$:

$$D_{ii}^{\text{(Single Well)}} = \frac{t^*}{2\pi T} \ln \frac{R_p}{r_0} \tag{A.1}$$

where $r_0 [L]$ is the well radius, $T [L^2/T]$ is the transmissivity of the aquifer and $R_p [L]$ the radius of influence of the well, which determines the distance beyond which steady state abstraction from the well has a negligible effect on groundwater heads. This distance is determined by the storativity $S [-]$ of the aquifer and its transmissivity $T$, as well as the considered time scale $t^*$ of exploitation and can be approximated as $R_p = 1.5 \sqrt{T \cdot t^* / S}$ [Cooper Jr and Jacob, 1946]. The superposition principle [Reilly et al., 1987] can be invoked to obtain the steady state average drawdown caused by a unit abstraction across $N$ wells:

$$D_{ii} = \frac{1}{N} \sum_{l} \left( \frac{1}{N} \frac{1}{2\pi T} \ln \frac{R_p}{r_0} + \sum_{k,l} \frac{1}{N} \frac{1}{2\pi T} \ln \frac{R_p}{||r_{kl}||} \right)$$

Drawdown at well $l$

$$= \frac{1}{2\pi T} \ln \frac{R_p}{\bar{R}} \tag{A.2}$$

with $r_{kl}$ the distance vector linking well $k$ and $l$. The variable $\bar{R} = r_0^{1/N} \mu_G(N)^{1-1/N}$ is an equivalent radius of a fictitious single well that the same average groundwater response $D_{ii}$ as the entire well field. The function $\mu_G(N)$ designates the geometric mean of the non-zero distances between all the wells of the well field. For an hexagonally packed rectangular well field, this function can be approximated as (Figure A.6):

$$\mu_G(N) \approx (0.391 + 0.035 \cdot a) \sqrt{N} \cdot M \tag{A.3}$$

where $a \geq 1$ is the shape factor of the rectangular well field and $M$ is the minimum distance between its wells (e.g., due to a pivot irrigation arm).

The equivalent expression for $D_{ij}$ yields:

$$D_{ij} = \frac{1}{N} \sum_{l} \left( \frac{1}{N} \frac{1}{2\pi T} \ln \frac{R_p}{L} + \sum_{k,l} \frac{1}{N} \frac{1}{2\pi T} \ln \frac{R_p}{||r_{kl} + L||} \right)$$

$$= \frac{1}{2\pi T} \ln \frac{R_p}{L} - \frac{1}{N^2} \frac{1}{2\pi T} \sum_{l} \sum_{k,l} \left( \ln \frac{||r_{kl} + L||}{||L||} \right) \tag{A.4}$$

where $L$ is a vector linking the centers of gravity of the two well fields. The distance vectors $r_{kl}$ would link individual wells from different well fields, if the well fields were positioned such that their centers of gravity overlapped. Assuming that well fields are spatially distinct, such that $\mu_G(||r_{kl}||) << ||L||$, $D_{ij}$ can be approximated as:

$$D_{ij} \approx \frac{1}{2\pi T} \ln \frac{R_p}{L} \tag{A.5}$$

The error on associated with the above approximation decreases sharply with $L$ and remains below 5% for spatially distinct well fields where $L > \mu_G(||r_{kl}||)$ (Figure A.7).

Finally, parameters $r$ and $k$ can then be expressed as a function of parameters representing the geometric layout of the wells and the hydrogeologic characteristics of the
Figure A.1. Panel B: Empirical approximation of the geometric mean of the distances between wells in a rectangular hexagonally packed well field (y-axis). Measured mean distances in synthetically generated well fields (dots) and the corresponding empirical approximations (lines) are plotted against the square root of the number of wells (x-axis) for different well-field shape factors (colors). Panel B: Error on $k$ simulated for rectangular and hexagonally packed well fields. Parameters for the two well fields were independently drawn from the sets \{10, 50, 100\} and \{0.1, 0.5, 1\} for the number of wells and the shape factor (respectively) of each well field. Distance $L$ between the well field centers of gravity was varied between 0 and 1000 units. In the simulation, we used a well radius of 0.5 units, a well radius of influence of 100,000 units and a minimum distance between hexagonally packed wells of 100 units.

The economic parameters for the Disi aquifer were taken from Müller et al. [2017] (Table 3). Pumping returns are $\alpha_i = 0.039$ and $\alpha_j = 0.056$ or $\alpha_j = 0.24$ USD per cubic meter of water for Saudi Arabia and for Jordan without and with the Disi-Amman pipeline, respectively. Pumping energy costs for Saudi Arabia and Jordan are $V_8 = 0.011$ and $V_9 = 0.037$ USD per cubic meter water per 100 meter lift. The estimated average static lifts are $h_{0,i} = 13$ and $h_{0,j} = 140$ meters for Saudi Arabia and Jordan. These parameters yield intrinsic profitability values of $P_i = 0.03757$ and $P_j = 0.01789$ or $P_j =$...
0.1882 USD per cubic meter of water for Saudi Arabia and for Jordan without and with the pipeline. Together these yield the economic asymmetry parameters $c = 0.77$ and $p = 0.32$ (without pipeline) or $p = 0.83$ (with pipeline). Aquifer response parameters obtained from Müller et al. [2017] (Table S2) and were $D_{ij} = 0.17 \cdot 10^{-6}$ and $D_{jj} = 0.42 \cdot 10^{-6}$ meters drawdown per cubic meters pumping per year, yielding $r = 0.41$ or $g = 0.71$.

To relate hydrogeologic connectivity to the spatial distance between well fields (Figure 6D), we used a representative radius $R_i = 44$ km. This radius was obtained using Equation A.2 using $D_{ii} = 0.17 \cdot 10^{-6} m^3/(m^3/yr)$ (see above) and an average transmissivity $T = 1275 m^2/d$ obtained from Table S3 in Müller et al., 2017. The radius of well influence $R_p = 56 km$ was obtained using a storativity $S = 0.01$ (Müller et al., 2017, Table S3) and assuming a planning horizon of $t^* = 30 y$ [Müller et al., 2017]. These considerations yield a non-dimensional $R_i^*$ value of 0.73, which approximately corresponds to the blue line on Figure 6D.

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