An upper bound for extreme temperatures over midlatitude land

Yi Zhang\textsuperscript{1} and William Boos\textsuperscript{2}

\textsuperscript{1}University of California, Berkeley
\textsuperscript{2}University of California

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Abstract

Heatwaves damage societies globally and are intensifying with global warming. Several mechanistic drivers of heatwaves, such as atmospheric blocking and soil moisture-atmosphere feedback, are well-known for their ability to raise surface air temperature. However, what limits the maximum surface air temperature in heatwaves remains unknown; this became evident during recent Northern Hemisphere heatwaves which achieved temperatures far beyond the upper tail of the observed statistical distribution. Here, we present the hypothesis, with corroborating evidence, that convective instability limits annual maximum surface air temperatures (TXx) over midlatitude land. We provide a theory for the upper bound of midlatitude temperatures, which accurately describes the observed relationship between temperatures at the surface and in the mid-troposphere. Known heatwave drivers shift the position of the atmospheric state in the phase space described by the theory, changing its proximity to the upper bound. Our theory suggests that the upper bound for midlatitude TXx should increase 1.9 times as fast as 500-hPa temperatures. Using empirical 500-hPa warming, we project that the upper bound of TXx over Northern Hemisphere midlatitude land (40°N–65°N) will increase about twice as fast as global mean surface air temperature, and TXx will increase faster than this bound over regions that dry on the hottest days.
An upper bound for extreme temperatures over midlatitude land

Yi Zhang\textsuperscript{1,2}, William R. Boos \textsuperscript{1,3}

\textsuperscript{1} Department of Earth and Planetary Science, University of California, Berkeley
\textsuperscript{2} Miller Institute for Basic Research in Science, University of California, Berkeley
\textsuperscript{3} Climate and Ecosystem Sciences Division, Lawrence Berkeley National Laboratory

\*Correspondence to: y-zhang@berkeley.edu
Abstract

Heatwaves damage societies globally and are intensifying with global warming. Several mechanistic drivers of heatwaves, such as atmospheric blocking and soil moisture-atmosphere feedback, are well-known for their ability to raise surface air temperature. However, what limits the maximum surface air temperature in heatwaves remains unknown; this became evident during recent Northern Hemisphere heatwaves which achieved temperatures far beyond the upper tail of the observed statistical distribution. Here, we present the hypothesis, with corroborating evidence, that convective instability limits annual maximum surface air temperatures (TXx) over midlatitude land. We provide a theory for the upper bound of midlatitude temperatures, which accurately describes the observed relationship between temperatures at the surface and in the mid-troposphere. Known heatwave drivers shift the position of the atmospheric state in the phase space described by the theory, changing its proximity to the upper bound. Our theory suggests that the upper bound for midlatitude TXx should increase 1.9 times as fast as 500-hPa temperatures. Using empirical 500-hPa warming, we project that the upper bound of TXx over Northern Hemisphere midlatitude land (40°N-65°N) will increase about twice as fast as global mean surface air temperature, and TXx will increase faster than this bound over regions that dry on the hottest days.

1 Introduction

Recent mega-heatwaves—the 2010 Russian heatwave,\(^7\) the 2019 European heatwave,\(^8\) and the 2021 Western North America heatwave\(^9\)—set temperature records more than three standard deviations beyond the local long-term mean of annually hottest daily maximum temperatures
The 2010 Russian heatwave (Fig. 1d), accompanied by severe drought and wildfires, caused thousands of deaths, while the 2019 European heatwave (Fig. 1c) exceeded the memorable 2003 heatwave, setting records in Western Europe. The 2021 Western North America heatwave (Fig. 1b), arguably the most anomalous heatwave recorded, exceeded the previous record by 5°C. Moreover, temperatures in this event broke from the upper tail of the distribution of recorded extreme temperatures, preventing a reliable statistical assessment of its likelihood and calling for a revised physical understanding of heatwaves.

Previous studies identified multiple physical processes involved in midlatitude heatwaves. A prerequisite is an atmospheric anticyclone, with clockwise flow (in the Northern Hemisphere) around a high-pressure center. Subsiding air within anticyclones warms through compression, prohibiting clouds and allowing sunlight to heat the surface; poleward flow in the anticyclone can also transport hotter air into the heatwave. Anticyclones usually drift eastward following midlatitude westerly winds, but can stall over a region in a phenomenon known as blocking, which is especially favorable to heatwaves. Natural modes of variability that modulate the occurrence and movement of anticyclones thus affect heatwaves. Beneath anticyclones, land-atmosphere feedbacks can enhance heatwaves, with warmer air drying soils, which in turn limit surface evaporative cooling and warm surface air more. Processes that affect soil moisture, such as antecedent precipitation and evapotranspiration, therefore affect heatwave severity.

Different heatwaves have been attributed to different processes, and we lack a general theory for midlatitude heatwave intensity. This lack of general understanding is exemplified by the inability to explain the extreme nature of the 2021 Western North America heatwave in terms of the aforementioned processes. Furthermore, we do not know whether different
processes can interact nonlinearly to amplify heatwaves. All of these facts impede accurate future projections.25

2 Physical mechanism and theory

We first present a hypothesis, and associated evidence, for the mechanism that limits surface air temperatures over midlatitude land. Specifically, we hypothesize that convective instability halts heatwave development. Surface air temperature cannot increase indefinitely during heatwaves, but can only rise till the atmospheric temperature profile becomes unstable to convection, which with any associated precipitation would cool the land surface. This hypothesis requires the free-tropospheric temperature profile to be at least episodically near neutral to moist convection, which is an accurate assumption for the tropical atmosphere in general26,27 and tropical heat extremes in particular.28,29 Moist convective neutrality also holds for midlatitude land in summer,30,31 however the implications of this neutrality for heatwaves has not been studied.

We examine this hypothesis using a composite analysis of all annual hottest daily maximum temperatures (TXx) over land between 40°N and 65°N in 2010 (choosing other years does not affect results). We take the time series of a climate variable over a 21-day window centered on the day of TXx for each location, then average the time series of all locations. The resulting composites (Fig. 2) thus show the structural characteristics of many heat events. Supporting the convective instability hypothesis, convective available potential energy (CAPE), which is a measure of convective instability, peaks on the annual hottest day (day 0). Consequently, precipitation increases on day 0, then surface air temperature drops
as precipitation peaks on day 1. The drop of surface air temperature occurs faster than its build-up, consistent with the hypothesis that the fast processes of convection and precipitation rapidly cool the land surface. These composites identify precipitating convection as a common conclusion of heat events over midlatitude land, motivating application of theories for moist convective stability.

Convective instability can be estimated by comparing surface air moist static energy (MSE) to the free-tropospheric saturation MSE, with the difference between these quantities near zero in the event of convection. MSE depends on temperature \( T \), specific humidity \( q \), and geopotential height \( z \):

\[
\text{MSE} = c_p T + L_v q + g z
\]

(1)

where \( c_p \) is the specific heat of air at constant pressure, \( L_v \) is the latent heat of vaporization, and \( g \) is the gravitational acceleration. Surface air temperature can build in a stable column where surface air MSE (MSE\(_s\)) does not exceed free-tropospheric saturation MSE (MSE\(_a^*\)):

\[
\text{MSE}_s \leq \text{MSE}_a^*.
\]

(2)

Using the 500-hPa level to represent the free troposphere (Methods), we find that midlatitude TXx events satisfy equation (2), with MSE\(_s\) only high enough to reach MSE\(_a^*\)\(_{500}\) on the hottest day (Fig. 2). Combining equations (1) and (2), and thermodynamic relations, we obtain an upper bound of surface air temperature (\( T_s \); see Methods for derivation):

\[
T_s \leq T_{500} + \frac{L_v}{c_p} q_{\text{sat}}(T_{500}) + \frac{g z_{500}}{c_p T_{500}} T_{500} - \frac{g}{c_p} z_s,
\]

(3)

where \( T_{500} \) is 500-hPa temperature, \( q_{\text{sat}}(T_{500}) \) is 500-hPa saturation specific humidity, \( T_{500} \) and \( z_{500} \) are 500-hPa constant climatological values (see Methods), and \( z_s \) is surface elevation.
Equation (3) states that the highest possible $T_s$ is determined by $T_{500}$, offset by $z_s$. The $T_s$ upper bound is achieved when the energy in MSE$_s$ is entirely allocated to temperature and surface air specific humidity is zero.

3 Observational evidence

We now assess the consistency of observations with the upper bound expressed by equation (3), examining $T_s + \frac{g}{c_p}z_s$ instead of $T_s$ so that locations with different surface elevations can be readily compared. We show the joint distribution of $T_s + \frac{g}{c_p}z_s$ and $T_{500}$ over land between $40^\circ$N and $65^\circ$N (Fig. 3a; Methods). The theory accurately delineates the highest observed $T_s + \frac{g}{c_p}z_s$ for each $T_{500}$ (Fig. 3a). Few data points fall above the $T_s$ upper bound, where $(T_s, T_{500})$ pairs would produce convective instability. This analysis only includes the Northern Hemisphere because the same latitudes in the Southern Hemisphere are mostly covered by ocean. The agreement between theory and observations (Fig. 3a) suggests $T_{500}$ as the limiting factor of $T_s$, providing new insight on midlatitude heatwaves.

We argued for a top-down control on $T_s$ by $T_{500}$, but causation is not apparent from Fig. 3a. To rule out the alternative possibility that $T_s$ controls $T_{500}$ through convective heating, we examine the time series of heat events. The 500-hPa saturation MSE ($\text{MSE}_{500}^*$), which strongly depends on $T_{500}$, peaks the day before TXx and remains at a similar level on the day of TXx (Fig. 2). If $T_s$ controlled $T_{500}$ through convective heating, $\text{MSE}_{500}^*$ would peak after $T_s$ and the onset of precipitation. Furthermore, individual heatwaves highlighted in Fig. 1b-d were preceded by warm anomalies confined to the atmospheric layer between 300 hPa - 700 hPa, and are succeeded by precipitation (Extended Data Fig. 1). These time
series support the hypothesis that $T_{500}$ controls $T_s$ in midlatitude heat extremes, not the other way around.

4 Connection to well-known heatwave drivers

We demonstrate how the convective-instability mechanism can be used to understand the influence of anticyclones and soil moisture on heatwaves. We use 500-hPa relative vorticity from reanalysis as a proxy for anticyclone strength, with negative values being anticyclonic in the Northern Hemisphere. As expected, 500-hPa relative vorticity is anti-correlated with $T_{500}$ (Fig. 3b), because when warmer air moves poleward conserving potential vorticity, its relative vorticity becomes more negative as planetary vorticity increases. In the $T_s$-$T_{500}$ phase space, anticyclones make warmer $T_s$ possible by moving the atmospheric state to larger $T_{500}$. However, the actual $T_s$ achieved in an anticyclone ranges from the upper bound to tens of degrees Celsius below that bound, indicating that strong anticyclones are necessary but insufficient for high $T_s$.

To investigate the role of soil moisture, we examine daily mean volumetric surface (0-7 cm) soil water content from reanalysis averaged over the antecedent 30 days (the reanalysis used here,\cite{assimilates_soil_moisture},\cite{assimilates_soil_moisture} assimilates soil moisture observations and represents soil moisture better than previous reanalyses; for shallow soil moisture it has comparable skill to the dynamically downscaled land product ERA5-Land\cite{ERA5-Land,ERA5-Land}). Antecedent surface soil water content at a given $T_{500}$ is anti-correlated with $T_s$, with a gradient in $T_s$-$T_{500}$ space that is nearly orthogonal to that of relative vorticity (Fig. 3c). In our convective-instability framework, the role of soil moisture is that dryer soil leads to lower surface air specific humidity ($q_s$) and a
partitioning of MSE_s towards temperature, consistent with the soil moisture-atmosphere feedback; \(^4\text{-}^6,^{16}\text{-}^{18}\) since the \(T_s\) upper bound is only met at zero \(q_s\) (Methods), lowering \(q_s\) moves the actual \(T_s\) toward the upper bound.

To summarize, free-tropospheric anticyclones allow access to larger values of \(T_s\) by increasing \(T_{500}\) (rightward movement in the \(T_s\)-\(T_{500}\) phase space), while low antecedent soil moisture allows the actual \(T_s\) to approach the upper bound by lowering \(q_s\) (upward movement in the phase space). Variations in anticyclone strength and soil moisture align with nearly orthogonal dimensions in the \(T_s\)-\(T_{500}\) phase space; neither factor alone ensures a heatwave, while neither factor has to be extreme to result in an extreme heatwave.

5 Insight into recent heatwaves

The theory can be applied to the three recent mega-heatwaves in Western North America, Western Europe, and Western Russia (Fig. 3d-f). These regions have moderately humid summers, therefore the joint \(T_s\)-\(T_{500}\) distributions are offset below the upper bound (which assumes zero \(q_s\)). If we lower the upper bound by the lowest \(q_s\) achieved over 1979-2021 for each region, the maximum \(T_s\) then better tracks the adjusted upper bound (Extended Data Fig. 2).

Our theory explains the extreme nature of the 2021 Pacific Northwest heatwave, where the highest \(T_s\) (29 June 2021) broke the previous record (22 July 2006) by 5 K even though anomalies of commonly analyzed quantities (500-hPa geopotential height, antecedent precipitation) were mild.\(^9\) For this event, \(T_{500}\) on 29 June 2021 reached 268.2 K, exceeding the 22 July 2006 value by 2.2 K (Fig. 3d), which amounts to a 4.5-K increase in the \(T_s\) upper bound.
by equation (3). Therefore, the $T_{500}$ anomaly alone explains most of the 5-K $T_s$ anomaly, and antecedent soil moisture plays a minor role (Extended Data Fig. 2a).

For the 2019 Western Europe heatwave (Fig. 3e), $T_{500}$ was 1.3 K higher than the hottest day during the 2003 European heatwave, translating to a 2.5 K increase in the $T_s$ upper bound. The actual $T_s$ only broke the 2003 record by 1.5 K, consistent with the fact that $q_s$ was higher in the 2019 heatwave. Neither $T_{500}$ nor soil moisture (Extended Data Fig. 2b) broke previous records; $T_{500}$ for this event ranked at the top 1.5% and soil water content ranked at the bottom 2% for this region in summer months. This heatwave thus exemplifies the aforementioned near-orthogonal interaction between anticyclone strength and soil moisture in the $T_s$-$T_{500}$ phase space.

The 2010 Russian heatwave (Fig. 3f) was driven by desiccated soil (Extended Data Fig. 2c) after prolonged blocking. Antecedent soil water content for the hottest days of this heatwave was 36% less than the summer average and 26% less than the summer minimum of other years for the same region, while $T_{500}$ only ranked at the 93rd percentile of summer daily $T_{500}$ for the region. Compared to the hottest summer day in 2021, the excess $T_{500}$ in 2010 only translates to 2.5 K of increase in the $T_s$ upper bound, but the actual $T_s$ was higher in 2010 by 3.9 K due to desiccated soil; movement in the $T_s$-$T_{500}$ phase space was mainly upward relative to the historical distribution.
6 Trends of annual hottest daily maximum temperatures

We now examine the consistency of historical temperature trends with our theory. The increase of the $T_s$ upper bound ($T_{s,max}$) per unit warming of $T_{500}$ can be obtained by differentiating equation (3):

$$\frac{dT_{s,max}}{dT_{500}} = 1 + \frac{L_v}{c_p} \frac{dq_{sat}(T_{500})}{dT_{500}} + \frac{gz_{500}}{c_p T_{500}}. \tag{4}$$

(Magnitude : 1 0.39 ∼ 1.11 0.21)

Equation (4) is nonlinear in $T_{500}$ due to the near-exponential dependence of $q_{sat}$ on temperature, so the sensitivity of $T_{s,max}$ to $T_{500}$ is larger at warmer temperatures (Fig. 3). The increase in $T_{s,max}$ induced by $T_{500}$ warming is always larger than the $T_{500}$ warming itself, due to contributions from Clausius-Clapeyron (second term on the right hand side of equation (4)) and the geopotential (third term). The Clausius-Clapeyron term ranges from 0.39 to 1.11 in the present climate. The 500-hPa geopotential anomaly, though frequently analyzed for heatwaves, plays a minor role, contributing about one-fifth that of temperature (first term) and about one-fifth to half that of the Clausius-Clapeyron term. Taking $T_{500}$ as 262 K, which is the most common $T_{500}$ value on the annual hottest days over midlatitude land in the present climate, we find the increase in the $T_s$ upper bound per unit $T_{500}$ warming $(\frac{dT_{s,max}}{dT_{500}})$ to be 1.86.

We compare this theoretical ratio with observations and reanalysis. From 1979 to 2021, the warming of TXx averaged over land between 40°N and 65°N is 1.9 times that of $T_{500}$
on such days, from ERA5 reanalysis, with $T_{500}$ increasing at 0.19±0.06 K/decade and TXx
0.36±0.06 K/decade (Fig. 4a, b). TXx from HadEX3 gridded station observations in-
creased by 0.32±0.06 K/decade from 1979 to 2018, and $T_{500}$ from ERA5 for the same period
increased 0.18±0.06 K/decade, with the ratio of the two being 1.8. These similar ratios
show that Northern Hemisphere midlatitude TXx increased over recent decades at a rate
that agrees strongly with equation (4).

In addition, the spatial pattern of TXx trends resembles that of the $T_s$ upper bound
calculated by multiplying the local trend of $T_{500}$ on annual hottest days with the local value
of $\frac{dT_{s,\text{max}}}{dT_{500}}$ from equation (4). The negative trends of TXx over the Eastern United States
and Central Asia correspond to the cooling of $T_{500}$ on the hottest days over those regions
(Fig. 4c, d).

The similar warming trends of TXx and the upper bound of $T_s$ suggest that changes
in surface air specific humidity ($q_s$) on the annual hottest days played a minor role in the
trend of TXx. Drying or moistening of the hottest days should deviate increases in TXx
from the prediction by equation (4). Consistently, the hottest days over most Northern
Hemispheric midlatitude land have not seen significant moistening or drying over recent
decades (Extended Data Fig. 4a, c), despite the robust increase in annual mean $q_s$ (Extended
Data Fig. 4b, d). Though there is uncertainty in $q_s$ data, this result is in line with recent
work finding that $q_s$ on the hottest days has a muted increase\textsuperscript{35} and has even decreased over
certain regions.\textsuperscript{36}
7 Discussion and Implication

We presented evidence from multiple observational sources supporting the hypothesis that convective instability limits peak surface air temperatures over midlatitude land, and we developed a theory that explains the observed relationship between the peak surface air temperature ($T_s$) and 500-hPa temperature ($T_{500}$). This mechanism, focusing on the termination of heatwaves, complements previous descriptions of processes active in the developing phase of heatwaves, providing an upper-bound for heatwaves that is a curve in $T_s$-$T_{500}$ space. The direction of causality between $T_s$ and $T_{500}$ is important; $T_{500}$ warms while convection is suppressed before $T_s$ peaks, then precipitation begins when surface air MSE becomes large enough to satisfy a simple criterion for convective instability ($\text{MSE}_s \geq \text{MSE}_{a}^*$).

Several caveats exist. First, our theory assumes convective plumes have no exchange with the environment, but entrainment of environmental air could affect convective onset. This may not change the first-order picture for all midlatitude land (Fig. 3a), but could alter behavior for certain regions. Second, though most locations receive considerable rainfall following heat events, precipitation following heatwaves is much less over dry regions, such as in Central Asia and the Midwestern United States. The absence of notable precipitation, which could be due to evaporation of falling condensate, does not necessarily contradict the convective-instability mechanism, but further investigation is merited in arid regions. Third, our mechanism does not address the frequency of extreme temperatures; extensions of our theory may provide new insight on heatwave frequency.

A natural next step is to estimate how the upper bound of $T_s$ will increase with future global warming. On the annual hottest days in recent decades, $T_{500}$ (from ERA5) has
warmed at a similar rate as both annual mean $T_{500}$ over Northern Hemispheric midlatitude land and global mean surface air temperature (the latter values were drawn from multiple observational datasets,\textsuperscript{39,40,43} Extended Data Table 1 and Fig. 4b). Given this, the $T_s$ upper bound over midlatitude land should on average increase around twice as fast as global mean surface air temperature. Regional increases of the $T_s$ upper bound depend on the base-state values and warming patterns of $T_{500}$. Regions of warmer $T_{500}$ in the base climate should expect more increase in the $T_s$ upper bound given the same $T_{500}$ warming (Fig. 3), due to the Clausius-Clapeyron nonlinearity in equation (4).

A related question is how TXx will change relative to the upper bound of $T_s$, and the answer depends on $q_s$. Regions that dry on the hottest days should expect a faster increase in TXx than the upper bound. Our results therefore identify two factors that must be constrained for accurate projection of midlatitude extreme temperatures: i) the amount of midlatitude free-tropospheric warming, and ii) surface air specific humidity changes on the hottest days. Understanding the physical processes controlling these factors should be priority in future research on midlatitude extreme temperatures.
Methods

**Derivation of the upper bound of surface air temperature.** Combining equation (2) and equation (1), we have

\[ c_p T_s + L_v q_s + g z_s \leq c_p T_{500} + L_v q_{sat}(T_{500}) + g z_{500}, \]  

(5)

where \( T_s, q_s, \) and \( z_s \) are temperature, specific humidity, and elevation at the surface, \( T_{500}, \) \( q_{sat}(T_{500}), \) and \( z_{500} \) are temperature, saturation specific humidity, and height at the 500-hPa pressure surface, \( c_p \) of 1004.7090 J/kg/K is the specific heat capacity of air at constant pressure, \( L_v \) of \( 2.5008 \times 10^6 \) J/kg is the latent heat of vaporization, and \( g \) is gravity which equals 9.81 m/s\(^2\).

We then write \( q_{sat}(T_{500}) \) and \( z_{500} \) as functions of \( T_{500} \), namely

\[ q_{sat}(T_{500}) \approx \frac{\epsilon e_{sat}(T_{500})}{500 \text{ hPa}}, \]

(6)

where \( \epsilon \) is the molar ratio between water vapor and dry air, \( e_{sat} \) is the saturation vapor pressure given by the Clausius-Clapeyron equation, and

\[ z_{500} = \frac{z_{500}}{T_{500}} T_{500}, \]

(7)

where \( z_{500} \) and \( T_{500} \) are climatological geopotential height and temperature at 500 hPa, taking the values of 5.682 km and 258.8 K, respectively.

While equation (6) is apparent, equation (7) requires some elaboration. Combining hydrostatic balance \( \frac{dp}{dz} = -\rho g \) and the ideal gas law \( p = \rho RT \), we have

\[ d \ln p = -\frac{g}{RT} dz, \]

(8)
where $p$ is pressure, $R$ is the ideal gas constant of dry air, with a value of 287.058 J/kg/K.

We integrate equation (8) from the surface (using a nominal value of 1000 hPa) to 500 hPa,

yielding

$$\ln \frac{1000 \text{ hPa}}{500 \text{ hPa}} = \ln 2 = \frac{g}{R} \int_0^{z_{500}} \frac{dz}{T}. \number(9)$$

We approximate the atmospheric temperature structure as having a constant lapse rate $\Gamma$, i.e.,

$$T = -\Gamma(z - z_{500}) + T_{500}, \number(10)$$

and thus we can integrate equation (8) to get

$$\ln 2 = \frac{g}{RT} \ln \frac{T_{500}}{\Gamma z_{500} + T_{500}}. \number(11)$$

The climatological values $T_{500}$ and $z_{500}$ should also satisfy equation (9):

$$\ln 2 = \frac{g}{RT} \ln \frac{T_{500}}{\Gamma z_{500} + T_{500}}. \number(12)$$

Equation (11) and (12) together give equation (7).

Substituting equation (7) into equation (5), we have

$$c_p T_s + g z_s \leq c_p T_{500} + L_v q_{\text{sat}}(T_{500}) + \frac{g z_{500}}{T_{500}} T_{500} - L_v q_s. \number(13)$$

We take the maximum of the right hand side of equation (13) by setting $q_s$ to zero and thus obtain the upper bound of $T_s$:

$$T_s + \frac{g}{c_p} z_s \leq T_{500} + \frac{L_v}{c_p} q_{\text{sat}}(T_{500}) + \frac{g z_{500}}{c_p T_{500}} T_{500}. \number(14)$$

**Choice of the 500-hPa pressure level.** The pressure level we choose to represent the free troposphere in the theory should be between the planetary boundary layer (PBL) top
and the level of neutral buoyancy (LNB). This level should be far enough from the PBL to not be affected by the surface air temperature, otherwise our theory assuming free-tropospheric control on surface air temperature would not stand; this level should also be frequently coupled to the surface through convection and should be reached by most convective events in summer. The daily-maximum PBL height between 40°N and 65°N on the annual hottest days is around 2 km and could be 5 km over dry areas (based on ERA5), which translates to a PBL top between 550 hPa and 800 hPa. The LNB (calculated from ERA5 hourly data) for summer months between 40°N and 65°N mostly ranges from 250 hPa to 500 hPa. Figures in ref.\textsuperscript{30} also show that convective neutrality extends to the midtroposphere for a substantial fraction of time over Northern Hemispheric land in summer. Therefore, we choose the 500-hPa pressure level to represent the free troposphere in Eq. (1), as it satisfies the two aforementioned requirements.

**Ground observations.** The HadGHCND dataset provides the anomalies of daily maximum temperatures (TX) on a 2.5° × 3.75° spatial grid relative to the 1961-1990 climatology. We create a daily TX climatology using ERA5 data interpolated to the coarser grid of HadGHCND.

**Data Availability**

The ERA5 hourly data on pressure levels and single levels from 1979 to present were downloaded from the Copernicus Climate Change Service Climate Data Store (https://cds.climate.copernicus.eu). GPM data were downloaded from the NASA Goddard Earth Sciences Data and Information Services Center (https://disc.gsfc.nasa.gov/datasets/
GPM_3IMERGDF_06/summary). HadCRUT5 data were provided by Met Office Hadley Centre and downloaded from https://www.metoffice.gov.uk/hadobs/hadcrut5/data/current/download.html. HadEX3 data were provided by Met Office Hadley Centre and downloaded from https://www.metoffice.gov.uk/hadobs/hadex3/. HadGHCND gridded daily temperatures were provided by Met Office Hadley Centre and downloaded from https://www.metoffice.gov.uk/hadobs/hadghcnd/. IUKv2 radiosonde data were provided by Steven Sherwood. MSU/AMSU data produced by Remote Sensing Systems were downloaded from https://www.remss.com/measurements/upper-air-temperature/.

**Code Availability**

The computer code used in this paper is available from the corresponding author.

**References**


**Acknowledgements**

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Figure 1: **Temperatures of recent mega-heatwaves.** 

a, Probability density distribution of normalized annual hottest daily maximum temperatures (TXx) over land between 40°N and 65°N for the period 1979-2021, and the maximum TXx during three mega-heatwaves as labelled. Normalized TXx is calculated location-wise by subtracting the average TXx from TXx then dividing the difference by the standard deviation of TXx for 1979-2021. 

b-d, Surface air temperature anomalies (color shading) and 500-hPa temperature anomalies (contours) during three mega-heatwaves, namely the 2021 Western North America heatwave (b), the 2019 European heatwave (c), and the 2010 Russian heatwave (d). Green boxes highlight the affected regions. Data are based on European Centre for Medium-Range Weather Forecasts Reanalysis 5 (ERA5) hourly data on a 0.25° × 0.25° grid.
Figure 2: Composite time series centered at annual hottest daily maximum temperatures (TXx). Surface air temperature, convective available potential energy (CAPE), 2-meter moist static energy (MSE$_{\text{surface}}$), the saturation moist static energy at 500-hPa (MSE$_{500}^*$) are from hourly reanalysis of ERA5. Precipitation is from GPM daily observations. All time series shown are land averages between 40°N and 65°N of 2010.
Figure 3: Theory for the upper bound of surface air temperatures and evidence from observations and reanalysis data. a, Theory for the upper bound of $T_s$ (black dashed line) and joint histograms of daily-maximum temperatures ($T_s$) and daily-mean 500-hPa temperatures ($T_{500}$) over land between 40°N and 65°N for 2001-2021. $T_{500}$ data are from the ERA5 reanalysis. $T_s$ data are from Hadley Center Global Historical Climatology Network Daily data (HadGHCND) ground observations (red contours) and the ERA5 reanalysis (grey shading). b, Relative vorticity at 500 hPa as a function of $T_{500}$ and $T_s$ for June, July, and August (JJA) of 2001-2021. c, Same as b but for the surface-layer (0-7 mm) volumetric soil water content. d-f, $T_s$ and $T_{500}$ relationship over the three regions within the green boxes in Fig. 1b-d for JJA of 2001-2021.
Figure 4: **Trends of annual hottest daily maximum temperature (TXx) in agreement with theory.** a, Time series of the global mean surface air temperature (GMST) from HadCRUT5 (gray), and the 40°N-65°N land average of TXx from ERA5 (red solid) and from HadEX3 (red dashed), and $T_{500}$ on the annual hottest days from ERA5. b, Trends of GMST, $T_{500}$ on annual hottest days, the upper bound of $T_s$, and TXx from ERA5 from 1979 to 2021. Confidence intervals for the linear trends represent 95% significance. Ratios of these trends to the GMST trend over the same period are annotated. c, Location-specific trends of TXx from 1979 to 2021 based on ERA5. d, Same as c but for the calculated trends in the upper bound of $T_s$ from theory.
Extended Data Figure 1: **Time series of three heatwaves.** a Daily maximum tropospheric temperature anomalies (color shading), 500-hPa relative vorticity (black line), and precipitation (cyan line) during the 2021 Western North America heatwave. The same time series are shown for the 2019 Western European heatwave in b and the 2010 Russian heatwave in c. Average daily-maximum temperature for each vertical level over the shown time periods are subtracted to emphasize the anomalies. Borders of the three regions are the green boxes in Fig. 1b,c,d. Temperature and relative vorticity data are from ERA5, and precipitation data are from GPM.
Extended Data Figure 2: The surface-layer (0-7 mm) volumetric soil water content for June-August from 1979 to 2021 as a function of $T_{500}$ and $T_s$ for (a) Western North America, (b) Western Europe, and (c) Russia. Dashes lines are the theoretical upper bound of $T_s$ as in equation (3) and dotted lines are the upper bound subtracted by the minimum 2-m specific humidity for these regions.
Extended Data Figure 3: The increase of the upper bound of $T_s$ per unit warming of $T_{500}$. The distribution of $T_{500}$ on annual hottest days from 1979 to 2021 based on ERA5 reanalysis is shown.
Extended Data Figure 4: **Trends in the surface (2-meter) air specific humidity.**

The time series of **a**, 2-m specific humidity on TXx days and **b**, annual-mean 2-m specific humidity averaged over land between 40°N and 65°N. The location-specific trends of **c**, 2-m specific humidity on TXx days and **d**, annual-mean 2-m specific humidity. Hatched regions are those where the local null hypothesis can not be rejected on a 0.05 significance level.
<table>
<thead>
<tr>
<th>Data set</th>
<th>Time period</th>
<th>Trend normalized by global warming</th>
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<tr>
<td>ERA5 $T_{500}$ on TXx days (land)</td>
<td>1979-2021</td>
<td>1.0 ± 0.3</td>
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<td>ERA5 annual mean $T_{500}$ (land&amp;ocean)</td>
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<td>1.0 ± 0.2</td>
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<tr>
<td>IUKv2 annual mean $T_{500}$ (land)</td>
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<td>1.2 ± 0.4</td>
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<td>MSU/AMSU TMT channel (land&amp;ocean)</td>
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<td>0.9 ± 0.1</td>
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<td>MSU/AMSU TTT channel (land&amp;ocean)</td>
<td>1979-2020</td>
<td>1.1 ± 0.1</td>
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</tbody>
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Table 1: Average trends of $T_{500}$ between 40°N and 65°N normalized by global warming for multiple data sets. For IUKv2 radiosondes, only the 123 sites with more than 80% of data available during 1979-2012 are included. Error bars are calculated as propagation of uncertainties using the 95% confidence interval of linear trends of $T_{500}$ and global mean surface air temperature assuming the independence of the two variables.