Error and Uncertainty Degrade Topographic Corrections of Remotely Sensed Data

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Abstract

Chemical and biological composition of surface materials and physical structure and arrangement of those materials determine the intrinsic spectral reflectance of Earth’s land surface at the plot scale. As measured by a spaceborne or airborne sensor, the apparent reflectance depends on the intrinsic reflectance, the surface texture, the contribution and attenuation by the atmosphere, and the topography. Compensation or correction for the topographic effect requires information in digital elevation models (DEMs). Available DEMs with global coverage at ~30 m spatial resolution are derived from interferometric radar and stereo-photogrammetry. Locally or regionally, airborne lidar altimetry, airborne interferometric radar, or stereo-photogrammetry from airborne or fine-resolution satellite imagery produces DEMs with finer spatial resolutions. Characterization of the quality of DEMs typically expresses the root-mean-square (RMS) error of the elevation, but the accuracy of remote sensing retrievals is acutely sensitive to uncertainties in the topographic properties that affect the illumination geometry. The essential variables are the cosine of the local illumination angle and the shadows cast by neighboring terrain. We show that calculations with globally available DEMs underrepresent shadows and consistently underestimate the values of the cosine of illumination angle; the RMS error increases with solar zenith angle and in more rugged terrain. Analyzing imagery of Earth’s mountains from current and future missions requires addressing the uncertainty introduced by errors in DEMs on algorithms that estimate surface properties from retrievals of the apparent spectral reflectance. Intriguing potential improvements lie in novel methods to gain information about topography from the imagery itself.
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Key Points:

• Mountain topography causes apparent remotely sensed reflectance to differ from the intrinsic reflectance of the surface.

• Errors in solar geometry derived from globally available digital elevation models introduce substantial uncertainty into analyses.

• Retrieval of the intrinsic reflectance and surface biogeophysical properties requires assessment of and correction for topographic effects.
Abstract

Chemical and biological composition of surface materials and physical structure and arrangement of those materials determine the *intrinsic* reflectance of Earth’s land surface. The *apparent* reflectance—as measured by a spaceborne or airborne sensor that has been corrected for atmospheric attenuation—depends also on topography, surface roughness, and the atmosphere. Especially in Earth’s mountains, estimating properties of scientific interest from remotely sensed data requires compensation for topography. Doing so requires information from digital elevation models (DEMs). Available DEMs with global coverage are derived from spaceborne interferometric radar and stereo-photogrammetry at ~30 m spatial resolution. Locally or regionally, lidar altimetry, interferometric radar, or stereo-photogrammetry produces DEMs with finer resolutions. Characterization of their quality typically expresses the root-mean-square (RMS) error of the elevation, but the accuracy of remotely sensed retrievals is sensitive to uncertainties in topographic properties that affect incoming and reflected radiation and that are inadequately represented by the RMS error of the elevation. The most essential variables are the cosine of the local solar illumination angle on a slope, the shadows cast by neighboring terrain, and the view factor, the fraction of the overlying hemisphere open to the sky. Comparison of global DEMs with locally available fine-scale DEMs shows that calculations with the global products consistently underestimate the cosine of the solar angle and underrepresent shadows. Analyzing imagery of Earth’s mountains from current and future spaceborne missions requires addressing the uncertainty introduced by errors in DEMs on algorithms that analyze remotely sensed data to produce information about Earth’s surface.

Plain Language Summary

Earth’s mountain regions significantly influence the planet’s climate, hydrology, ecology, and geology. Studying them with remote sensing requires that we compensate for the influence of topography on the reflection of solar radiation. Digital Elevation Models (DEMs) are used across scientific disciplines to understand topography’s effect on the remotely sensed signal. Small errors in the estimates of elevation lead to larger errors in calculations of the solar illumination on the terrain and portions that are in shadow, thereby leading to misinterpretation of remotely sensed imagery from satellites and airplanes. Here, we present estimates of the errors and uncertainty in DEM retrievals, and we identify some outright mistakes. Compensating for uncertainty will inform algorithms that consider the effect of Earth’s topography, improving the characterization from satellite missions of attributes of the planet’s surface.

1 Introduction

We use remotely sensed data to derive geophysical and biological properties of importance to the study of Earth and other planets. On Earth these analyses must include mountains, which play a key role in the planet’s climate, hydrology, ecology, and geology. For example, mountains drive orographic enhancement of precipitation and lead to their function as the world’s water towers, resources at risk in a warming climate (Immerzeel et al., 2020; Viviroli et al., 2007). About a quarter of Earth’s land surface is
mountainous (Wrzesien et al., 2019, 12% to 39% depending on the definition of "mountainous"), but mountain snowmelt supplies water resources for more than one billion people (Mankin et al., 2015), serving an important water storage role as climate warming transitions some snow to rain (Barros, 2013). Further, vegetation changes in high mountains indicate carbon-dioxide fertilization in areas where the partial pressure of all gases is lower (Shugart et al., 2001). Combinations of drought and fire affect mountain forests and sources of water (Moody & Martin, 2001). The critical role that mountains serve as water towers and vegetation hotspots may change under climate change, contributing to hazards to people living in or relying on mountain resources (Kirschbaum et al., 2020).

The recent National Academies’ Decadal Survey for Earth science and applications, Thriving on our Changing Planet, reflects these multiple concerns, with recommendations calling for observations “at scales driven by topographic variability” to reflect the heterogeneity of ecological, hydrological, and geological dynamics in Earth’s mountains (National Academies of Sciences, Engineering, & Medicine, 2018). Investigating these processes via remote sensing requires spatial resolutions fine enough to characterize the variability, recognizing that the topography affects the reflected signals, thereby affecting the retrieval algorithms that interpret the state variables and fluxes of energy and mass.

Analysis of the topographic effect requires information in digital elevation models of the bare surface, usually but not universally meaning DEMs, as distinct from digital surface models (DSMs) that include vegetation, buildings, or other features. We consider two globally available DEM datasets: the NASADEM (Buckley, 2020) and the Copernicus DEM (European Space Agency, 2021), both distributed at a resolution of 1 arcsecond (~30 m at the Equator). Locally or regionally, finer-resolution DEMs are available, so we consider three of those, which were derived by lidar, interferometric synthetic aperture radar, and structure-from-motion stereo photogrammetry from fine-resolution images. Our analysis considers the fine-resolution DEMs, in three different terrains, to provide the best assessment of the topographic effects on solar illumination geometry, and we compare those assessments to those derived from the two globally available datasets.

Characterization of the quality of DEMs typically assesses the vertical accuracy of the elevation. Uuemaa et al. (2020), through comparison of globally available products with fine-resolution lidar elevations, estimated root-mean-square (RMS) errors of 8-10 m for the NASADEM and TanDEM-X datasets (TanDEM-X is the primary source of data for the Copernicus DEM). Guth and Geoffroy (2021) compared several datasets with airborne lidar and ICESat-2 data and preferred the Copernicus DEM based on its ability to penetrate vegetation canopies and retrieve bare-Earth elevations.

However, the focus on elevation errors misses the effect of the topography on remotely sensed information in the wavelengths of the solar spectrum, which lies with the solar illumination geometry. The cosine of the local solar angle and the shadows cast by neighboring terrain are the most important variables for remote sensing of Earth’s surface in the reflective domain. On clear days, most of the irradiance is direct, but the diffuse component is significant (~30%) at the blue end of the solar spectrum. The surrounding landscape causes multiple reflections, which can be represented by the sky view factor—
the fraction of the overlying hemisphere open to the sky. The topographic variables that affect the solar angles also affect the viewing angles from the sensor.

We therefore assess the DEMs based on their ability to provide insight into the ways that topography affects our ability to retrieve properties of the surface important to the study of Earth science. Fundamentally, retrievals that are sensitive to the magnitude of the spectral reflectance will be most affected. Examples include snow albedo (Bair et al., 2021; Painter et al., 2013) and ecosystem composition (Bogan et al., 2019). Retrievals that utilize the shape of the reflectance spectrum, characterized for example by the spectral angle (Kruse et al., 1993), will be less affected but not entirely immune because the fraction of the incident irradiance that is diffuse vs. direct is sensitive to the topography. Finally, retrievals that depend on the wavelength of absorption features do not depend on the magnitude of the reflectance. The primary example is mineral identification in soils and vegetation, which requires enough illumination to identify spectral features (Clark et al., 2003; Mulder et al., 2013).

2 Data and Methods

2.1 Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>ASO</td>
<td>Airborne Snow Observatories</td>
</tr>
<tr>
<td>ASTER</td>
<td>Advanced Spaceborne Thermal Emission and Reflection Radiometer</td>
</tr>
<tr>
<td>AVIRIS-NG</td>
<td>Airborne Visible and Infrared Imaging Spectrometer – Next Generation</td>
</tr>
<tr>
<td>CHIME</td>
<td>Copernicus Hyperspectral Imaging Mission for the Environment</td>
</tr>
<tr>
<td>DEM</td>
<td>Digital elevation model of the bare Earth surface</td>
</tr>
<tr>
<td>DSM</td>
<td>Digital surface model including vegetation, buildings, etc.</td>
</tr>
<tr>
<td>DTM</td>
<td>Same as DEM</td>
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<tr>
<td>EMIT</td>
<td>Earth Surface Mineral Dust Source Investigation</td>
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<tr>
<td>EnMAP</td>
<td>Environmental Mapping and Analysis Program</td>
</tr>
<tr>
<td>EROS</td>
<td>Earth Resources Observation and Science</td>
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<td>HMA</td>
<td>High Mountain Asia</td>
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<td>IFSAR</td>
<td>Interferometric synthetic aperture radar</td>
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<td>InSAR</td>
<td>Same as IFSAR</td>
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<tr>
<td>ISRO</td>
<td>Indian Space Research Organization</td>
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<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
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<tr>
<td>NOAA</td>
<td>National Oceanic and Atmospheric Administration</td>
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<tr>
<td>OLI</td>
<td>Operational Land Imager</td>
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<tr>
<td>SBG</td>
<td>Surface Biology and Geology mission</td>
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<tr>
<td>SRTM</td>
<td>Shuttle Radar Topography Mission</td>
</tr>
<tr>
<td>USGS</td>
<td>US Geological Survey</td>
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<tr>
<td>UTC</td>
<td>Coordinated Universal Time</td>
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Ellipsoids and Geoids

<table>
<thead>
<tr>
<th>Ellipsoid</th>
<th>Description</th>
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<tbody>
<tr>
<td>EGM2008</td>
<td>Earth Gravitational Model 2008</td>
</tr>
<tr>
<td>EGM96</td>
<td>Earth Gravitational Model 1996</td>
</tr>
<tr>
<td>GR80</td>
<td>Geodetic Reference System 1980</td>
</tr>
<tr>
<td>NAD83</td>
<td>North American Datum of 1983</td>
</tr>
</tbody>
</table>
NAVD88 North American Vertical Datum of 1988
WGS84 World Geodetic System 1984

2.2 Elevation data

We consider two spatial resolutions of digital elevation models: fine and coarse. The coarse-resolution datasets are available globally, whereas the fine-resolution data are available selectively in specific locations. Table 1 summarizes the information sources for three fine-resolution and two global coarse-resolution datasets.

Insert Table 1 near here

For the fine-resolution imagery, data are derived from three different methods: lidar altimetry, interferometric synthetic aperture radar, and structure-from-motion using fine-resolution commercial satellite imagery.

1. Airborne Snow Observatories Inc. (Painter et al., 2016) maps snow depth with lidar altimetry over drainage basins in the Western U.S., Switzerland, and Norway. The operation acquires elevation data during the snow-free summer and then periodically measures the snow-on elevation during the winter and derives snow depth by subtraction. The company provided a 3 m DEM of the Carson River Watershed in the Sierra Nevada of California/Nevada, covering 2052 km².

2. The U.S. Geological Survey’s Alaska Mapping Initiative acquired airborne interferometric synthetic aperture radar (InSAR) data over much of Alaska in 2010 and 2012 (USGS EROS Archive, 2018). InSAR acquisitions can take place even in cloudy weather, and the data from a high latitude provide a broad range of solar illumination angles during the year. We downloaded and spliced tiles at 5 m resolution for a 2582 km² area in the Wrangell Mountains in Southeast Alaska.

3. Shean et al. (2016) employ structure-from-motion to measure elevation using commercial fine-resolution satellite imagery. From the National Snow and Ice Data Center, we downloaded part of the High Mountain Asia 8 m DEM for a 3514 km² area in the Himachal Pradesh state in the Indian Himalaya that covers 9 flight lines of the 2016 NASA-ISRO AVIRIS-NG campaign (Space Applications Centre, 2017).

For the coarse resolution imagery, we used two global data sources at one arcsecond resolution distributed in geographic (latitude-longitude) format. In cropping to the boundaries of each fine-resolution area, we added 5 km to each edge to minimize edge effects in calculating topographic parameters.

1. We spliced 1° × 1° tiles from the NASADEM (Buckley, 2020) together because our areas of interest crossed latitude or longitude tile boundaries. The NASADEM combines information from the Shuttle Radar Topography Mission (Farr et al., 2007) and stereo-photogrammetry from ASTER imagery (NASA & METI, 2019).

2. We downloaded Copernicus DEMs (European Space Agency, 2021) that were spliced and distributed by Open Topography. The Copernicus DEM is derived from TanDEM-X imagery.

Figure 1 shows the Copernicus DEM on the left and the ASO DEM on the right for the Carson River Watershed. The red rectangle in the left-hand image shows the area that the ASO DEM illustrates the detail of the topographic data at 3 m spatial resolution.
2.3 Notation

We selected or calculated the following variables for each grid point in each elevation dataset. \( \theta_0, \phi_0, \mu_s, F_{\text{diff}}, \text{ and } I \) vary with date; the other variables are independent of date and thus the solar illumination. Deep snow can smooth the topography, but our comparisons of snow-off with snow-on elevations find only a few grid cells with significantly different slope and azimuth. At the 3 m spatial resolution of the Carson River DEM, the RMS difference in slopes between the snow-on and snow-off elevations is 1.8°, and the 99th percentile absolute difference is 6.8°. Resampled to 10 m spatial resolution, the RMS difference is 1.3°, and the 99th percentile absolute difference is 5.3°.

- \( \theta_0, \phi_0 \) Solar zenith and azimuth angles, \( \mu_0 = \cos \theta_0 \)
- \( \mu_s \) Cosine of solar illumination angle on a slope, set to zero for slopes that are in shadow, either by adjacent terrain or when \( \mu_s \) is negative
- \( \rho \) Spectral directional-hemispherical or bihemispherical reflectance, depending on subscripts (Schaepman-Strub et al., 2006)
- \( F_{\text{diff}} \) Fraction of incoming spectral irradiance that is diffuse
- \( H_\phi \) Angle to the topographic horizon, upward from horizontal, in azimuth direction \( \phi \)
- \( I \) Spectral irradiance, incoming or reflected depending on subscript
- \( \text{RMS} \) Root-mean-square value \( \text{RMS}(x) = \sqrt{\frac{1}{N} \sum_{n=1}^{N} |x_n|^2} \)
- \( S, A \) Slope angle, upward from horizontal, and slope azimuth, south at 0°, eastward positive and westward negative, consistent with a right-hand coordinate system
- \( V_\Omega \) Sky view factor, hereafter just view factor, the fraction of the upward hemisphere open to the sky
- \( Z \) Elevation of the surface

2.4 Methods

We compared topographic variables by reprojecting both fine- and coarse-resolution data to an intermediate resolution approximating the geometric mean of the two resolutions, thereby to include the range and distribution of topographic values in the landscape. The one-arcsecond resolution of the NASADEM and Copernicus DEM translate to about 30 m. For the Carson River Watershed, the intermediate resolution between the 3 m ASO lidar and the globally available data is 10 m. For the InSAR data at 5 m over the Wrangell Mountains in Alaska, the intermediate resolution is 12 m. For the 8 m data in the HMA DEM, the intermediate resolution is 15 m. We assume the fine DEM is more accurate, particularly because variables derived over multiple points are compared to those derived from an individual location in the coarse DEM; therefore, the RMS of the difference between the coarse and fine estimates of a variable is considered the RMS error in the coarse-resolution data.
We calculated $\mu$ for seven dates between the winter and summer solstices, spaced so that the intervals between the solar declinations were equal (Figure 2). For every date, we chose 10:45 in the local time zone to match typical mid-morning acquisition times of satellites: Pacific Standard (UTC–8:00) for the Carson River, Alaska Standard (UTC–9:00) for the Wrangell Mountains, and India Standard (UTC+5:30) for the Himachal Pradesh. 

Figure 3 shows cosine of solar illumination angles for the Himachal Pradesh on the seven dates in Figure 2. The pixels are more illuminated as the solar illumination angle gets closer to zenith, and the fraction of diffuse illumination decreases, thereby affecting the relationship between intrinsic and apparent reflectance.

3 An Illustration of the Problem

Figure 4 shows two images and two graphs. The upper left (Figure 4a) shows band 5 (center wavelength 865 nm) of a Landsat 8 OLI image of the Indian Himalaya, acquired on 22 February 2016 over the Himachal Pradesh state of India. We chose band 5 because of the small fraction of diffuse illumination in the solar spectrum in those wavelengths. Figure 4b shows a calculation of $\mu$, the cosine of the solar illumination angle at the same date and time as the Landsat image, using elevation data from NASADEM (Buckley, 2020). The cosines are calculated from the slope and aspect of the terrain and the solar zenith and azimuth angles on a flat surface. Where shaded by local horizons or by the slope itself, the cosines are set to zero. Superficially, the two images appear to match, allowing that some illuminated areas are dark because that surface material is dark, whereas shadows are dark even if the surface material is bright. The bright areas in the Landsat image correspond to highly illuminated pixels. However, the scatter density plot (Figure 4c) indicates some problematic values. The high reflectance values in the upper left corner of Figure 4c correspond to pixels either in shadow or with highly oblique solar illumination, indicating that the solar angle calculated from the DEM is wrong. The low reflectance values in the lower right corner of the scatter plot tell a similar but more ambiguous story. These dark pixels are well illuminated; they could represent a dark surface, or they might not truly be well illuminated. Figure 4d shows probability density functions (pdf) of the reflectance values in areas of low ($\mu < 0.2$) and high ($\mu > 0.87$) illumination (each threshold represents 14% of the image). Each pdf has a long tail. Those in the tail of the low illumination category indicate that $\mu$ is not correctly estimated and is too small. With a correct DEM, we would not see such high reflectance values at highly oblique solar illumination, because of the low values of incident irradiance at those locations.

Algorithms that retrieve land surface properties analyze the spectral “reflectance,” broadly defined to cover the several possible angular reflectance configurations that Schaeppman-Strub et al. (2006) articulate. In their study of the effects of surface roughness on snow albedo, Bair et al. (2022) defined intrinsic reflectance of a substance independent of effects of roughness or topography. The corresponding apparent value, as one might measure at a plot, incorporates artifacts caused by roughness or topography. For example, the intrinsic reflectance of clean snow in the visible wavelengths cannot drop to 0.2, but the apparent reflectance of shadowed snow can reach such low values. Corrected for
atmospheric effects, measurements of spectral top-of-atmosphere radiance by a satellite sensor can be converted to apparent values of, for example, bihemispherical reflectance or bidirectional reflectance (Schaepman-Strub et al., 2006). Retrieval of a parameter of interest at Earth’s surface using these data requires an estimate of the intrinsic spectral reflectance, interpreted by a combination of topographic information and the apparent value derived from the satellite sensor data (Brodrick et al., 2021). For some pixels, however, incorrect or imprecise topographic information could cause those retrievals of the surface properties to produce incorrect interpretations, an issue addressed in Section 5, Discussion. This study characterizes the errors in the solar angles in globally available digital elevation models and recommends steps to mitigate these uncertainties in retrieval of Earth’s properties in mountainous terrain.

4 Results

Tables 2 and 3 summarize results for all fine- and coarse-resolution datasets analyzed. Figures 5 and 6 illustrate examples of the results, comparing pairs of variables derived from a fine- and a coarse-resolution image. We include examples from each of the three study sites: Carson River Watershed, Himachal Pradesh in the Indian Himalaya, and Wrangell Mountains in Southeast Alaska.

4.1 Topographic variables independent of solar illumination

Variations in elevation across topography create sloping terrain, so we characterize each pixel by its slope $S$ upward from the horizontal and its aspect $A$ as the direction the slope faces. Slope and aspect combine with the solar angles to create variability in local illumination. The varying terrain also creates the view factors $V$, the fraction of the sky hemisphere open above a point. The view factor controls the re-reflection of solar radiation that strikes the surface and the fraction of the diffuse irradiance that reaches the surface. The view factor is also important in modeling the thermal infrared radiation in the mountains (Robledano et al., 2022). The terrain geometry affects the incoming irradiance and the reflected radiation, so the errors in elevation itself are less important than errors in slope, aspect, and view factor. Based on the differences between the fine-resolution and coarse-resolution DEMs, Table 2 shows the RMS error for elevation, differences in elevation between neighbors, slope, aspect, and view factor. Because the differences between the datum sources (Table 1) for elevation exceed 25 m and because we are mostly interested in the internal differences within an elevation grid, we subtract the mean elevation of each grid from that grid’s values before calculating the RMS errors for elevation.

Errors in elevation are small fractions of the elevation values themselves, but the errors in slope and aspect indicate significant errors in the differences between elevations of neighboring points. Calculation of the slope $S$ and aspect $A$ of a topographic pixel considers the spatial derivative of elevation $Z$ in two or more directions $x$ and $y$ (Dozier & Frew, 1990), which could be projection coordinates or longitude and latitude distances computed from the coordinates and the dimensions of the ellipsoid:

$$\tan S \equiv |\nabla Z| = \sqrt{(\partial Z/\partial x)^2 + (\partial Z/\partial y)^2}$$ (1)
\[ \tan A = -\frac{\partial Z / \partial y}{\partial Z / \partial x} \]

Including the signs for the numerator and denominator separately enables calculation of aspect \( A \) over the full circle. From topographic data, the derivatives are calculated numerically. In a matrix \( Z \) with grid spacing \( \Delta h \) representing topography and the rows running west-east and columns north-south, the derivatives at point \([i, j]\) as calculated by a central difference method, are:

\[
\begin{align*}
\frac{\partial z}{\partial x} &= \frac{Z_{i,j+1} - Z_{i,j-1}}{2\Delta h} \\
\frac{\partial z}{\partial y} &= \frac{Z_{i+1,j} - Z_{i-1,j}}{2\Delta h}
\end{align*}
\]

The grid spacing \( \Delta h \) is usually known accurately, so assessment of errors that affect topographic radiation depends on the error distribution of the differences between neighboring elevations. We estimated the RMS error of the differences by calculating the numerators of Equation (2) in each direction and then the hypotenuse of the \( x \)- and \( y \)-direction differences in each pixel. Table 2 shows that the RMS errors in the differences between neighboring elevations are smaller than the RMS errors in the elevations themselves, thereby indicating some spatial coherence in the elevation errors. Otherwise, if the RMS errors of the elevations were indeed independent, then the variance of the differences would be the sum of the variances in the elevations themselves (Weisstein, 2021) and the RMS error of the differences would be \( \sqrt{2} \times \) the RMS error of the elevations. However, the RMS errors of the differences are much smaller.

Results for the NASADEM and the Copernicus DEM are similar, but both show outliers that translate into outliers in calculating illumination angles.
datasets do not use the same datum (Table 1), so the regression includes an intercept. For
the view factor (Figure 5c) all values are above about 0.6, so that regression is constrained
to go through (1,1) instead of (0,0). In all cases except elevation, the slopes of the
regression lines that characterize the relationship between the coarse- and fine-resolution
variables are less than 1.0, indicating generally that the Copernicus DEM and NASADEM
slightly underestimate the magnitudes. The section on Bias in Table 2 therefore indicates a
negative bias in the coarser-resolution datasets (i.e., a regression slope of 0.90 corresponds
to a bias of ~10%).

Aspect values and their RMS errors must be treated with caution, because aspect
has negligible effect on solar radiation when the slope is small but a huge effect when the
slope is steep. In our formulation, we follow the right-hand convention that 0° aspect
represents south, from which eastward aspects are positive and westward aspects are
negative.

4.2 Effect of topography on illumination and reflection

The two crucial topographic variables in order of importance are $\mu_S$, the cosine of
the local illumination angle measured from normal to the slope, and $V_\Omega$, the fraction of the
hemisphere over a point that is open to the sky. The equation for $V_\Omega$ uses the horizon
angles $H_\phi$ for all directions $\phi$ (Dozier, 2022b):

$$V_\Omega = \frac{1}{2\pi} \int_{\phi_1}^{\phi_2} \left[ \cos S \cos^2 H_\phi + \sin S \cos (A - \phi) \left( \frac{\pi}{2} - H_\phi - \sin H_\phi \cos H_\phi \right) \right] d\phi$$

For the slopes where $\cos (A - \phi) < 0$, the slope itself might obscure the horizon,
so in integrating across those values with the limits of integration corresponding
to those azimuths, for each azimuth $\phi$, $H_\phi$ is set to

$$\max \left[ H_\phi, \sin^{-1} \left( \frac{1}{1 + \cos^2 (A - \phi) \tan^2 S} \right) \right]$$

Over a flat unobstructed surface, $V_\Omega = 1$.

The local illumination angle is related to the topography and the solar illumination
geometry as:

$$\mu_S = \max[0, \mu_0 \cos S + \sin \theta_0 \sin S \cos (\phi_0 - A)]$$

The max function accounts for slopes facing away from the sun by setting $\mu_S = 0$ in
situations where the equation would yield $\mu_S < 0$. To account for points where neighboring
horizons block the Sun, we also set $\mu_S = 0$ where $\sin H_{\phi_0} \geq \mu_0$.

The variables $\mu_S$ and $V_\Omega$ affect the relationship between the apparent reflectance of
the surface and its intrinsic reflectance that would be measured independent of any
topographic effects (Bair et al., 2022). The apparent reflectance of a topographic surface
involves multiple reflections, especially for bright surfaces such as snow. Let \( \rho \) indicate spectral reflectance, omitting a wavelength identifier, and \( F_{\text{diff}} \) as the fraction of the spectral irradiance that is diffuse. Set the initial irradiance on a horizontal surface to \( I \). The spectral radiation that initially escapes into the overlying hemisphere without being re-reflected is:

\[
I_{\text{esc}}^{(0)} = IV_\Omega \left[ \frac{\mu_S}{\mu_0} (1 - F_{\text{diff}}) \rho_{\text{intrinsic}}^{\text{(direct)}} + F_{\text{diff}} \rho_{\text{intrinsic}}^{\text{(diffuse)}} + (1 - V_\Omega) \left( \rho_{\text{intrinsic}}^{\text{(diffuse)}} \right)^2 \right]
\]

(5)

The equation assumes that \( \rho_{\text{intrinsic}} \) is approximately isotropic averaged over the field of view, it neglects atmospheric attenuation within the topography, and it ignores variation in albedo and irradiance within the neighborhood. The superscripts designate the reflectance to direct vs. diffuse irradiance. The right-most term inside the brackets accounts for reflected radiation within a point’s field of view impinging on the point. The direct and diffuse spectral albedos might differ slightly, for example for snow.

Not all the initially reflected radiation escapes into the overlying hemisphere. Instead, some of it re-reflects and eventually escapes or is trapped by the topography, in which case it is subject to internal reflection. At the first iteration, its value is:

\[
I_{\text{internal}}^{(0)} = I_{\text{esc}}^{(0)} \left( \frac{1 - V_\Omega}{V_\Omega} \right).
\]

(6)

To account for multiple reflections, at each reflection the value of the incident radiation is multiplied by the fraction \( (1 - V_\Omega) \) that accounts for the reflection remaining within the topography, the fraction \( V_\Omega \) that escapes, and the intrinsic spectral reflectance. An orders-of-scattering approach to the multiple reflections lets some reflected radiation escape at each iteration \( n \) and some remains available for re-reflection:

\[
\begin{align*}
\text{escaped } I_{\text{esc}}^{(n)} &= I_{\text{internal}}^{(n-1)} \rho_{\text{intrinsic}}^{\text{(diffuse)}} V_\Omega \\
\text{remaining } I_{\text{internal}}^{(n)} &= I_{\text{internal}}^{(n-1)} \rho_{\text{intrinsic}}^{\text{(diffuse)}} (1 - V_\Omega)
\end{align*}
\]

(7)

This series converges in a half dozen iterations because \( I_{\text{internal}}^{(n)} \) declines in proportion to \( (1 - V_\Omega)^n \). The apparent reflectance for the pixel is \( \rho_{\text{apparent}} = \sum I_{\text{esc}} / I \).

4.3 Errors in estimating \( \mu_S \), the cosine of local illumination

RMS errors and outliers in the topographic variables combine with the solar illumination geometry to propagate into the calculation of each pixel's illumination. The most important variable whose accuracy affects the interpretation of the remotely sensed signal is the cosine of the local illumination angle. The ratio \( \mu_S / \mu_0 \) appears in Equation (5), but \( \mu_0 \) is usually known accurately. The view factor \( V_\Omega \) affects the diffuse irradiance from the sky and the internal reflections within the topography.

Therefore, the accuracy of the cosine of illumination from the DEM affects our ability to calculate or correct for the topographic effects. For example, attempting to invert Equation (5) to solve for \( \rho_{\text{intrinsic}} \) would involve the ratio \( \mu_0 / \mu_S \); uncertainty in the denominator of a fraction often has significant consequences, especially if the denominator is small (Richter & Schläpfer, 2021, chapter 7). Table 3 shows the RMS errors for the cosine of illumination, along with the fraction of the terrain that is shadowed, for the dates in
Figure 2 that extend from the winter to the summer solstice in equal changes of the solar declination. The RMS error for $\mu_S$ varies inversely with the value of $\mu_0$; the errors in slope $S$ and aspect $A$ have a greater effect when $\mu_0$ is smaller.

The full extent of errors in the results indicates issues with outliers that the RMS errors do not reveal. Figure 6 shows scatter diagrams of $\mu_s$ calculated from the Copernicus DEM vs $\mu_s$ calculated from the Alaska IFSAR DEM. On all dates but particularly early in the year, some pixels that are illuminated ($\mu_s > 0$) in the Copernicus DEM are in the dark ($\mu_s < 0.1$) in the Alaska IFSAR DEM. Similarly, some pixels that the Alaska IFSAR DEM shows to be illuminated are dark in the Copernicus DEM. A popular text on surveying published six decades ago (Davis et al., 1966) calls these kinds of mistakes “blunders” rather than “errors,” because they cannot be characterized by an error distribution.

### 5 Discussion

Although errors or blunders in the NASADEM and Copernicus DEM are minor compared to the elevation values, their impact on remote sensing can be large. Thus, the small dispersion around the 1:1 line in the scatter diagram for elevation in Figure 5a translates to much greater dispersion in the slope, aspect, and view factor (Figure 5cde), which in turn translates to large dispersion in the illumination angles that Figure 6 shows. Therefore, small errors in slope or aspect can then significantly affect estimated reflectance, especially wherever $\mu_s$ is small.

Algorithms to retrieve surface properties differ in their sensitivities to topographic uncertainty. The effect is mostly a shift in spectral reflectance magnitude, so algorithms that rely on relative spectral shapes may escape significant harm. These include detection of materials based on diagnostic spectral absorptions, as in mineral identification (Clark et al., 2003). On the other hand, studies that rely on absolute radiometry, such as surface energy balance investigations (Wang et al., 2015) or retrieval of snow properties (Bohn et al., 2021), could be more severely affected. Moreover, errors in $\mu_s$ change the estimated balance between diffuse and direct illumination onto the surface. Therefore, they can distort the estimated reflectance spectrum in visible wavelengths, harming snow or vegetation studies that rely on features in this spectral range.

Solar illumination geometry in mountains affects current satellite imagery from Landsat 8/9 and Sentinel-2A/B, it affects data from imaging spectrometers EnMAP (Chabrillat et al., 2020) and EMIT (Connelly et al., 2021), and it will affect data from future missions SBG (Cawse-Nicholson et al., 2021; Stavros et al., 2022) and CHIME (Rast et al., 2021). Locally, fine-resolution DEMs will be available from lidar, InSAR, or structure-from-motion deployed from drones or aircraft, and slightly coarser DEMs will be available using structure-from-motion from spaceborne data. However, the prospect is unlikely for globally available data to accurately estimate the solar illumination geometry for these imaging satellites. A chapter in *Thriving on our Changing Planet* (National Academies of Science, Engineering, & Medicine, 2018, p. 513) identifies applications that “would benefit from multibeam, space-based lidar to obtain global coverage of bare-earth topography and of the biomass/canopy at $<<5$ m spatial and 0.1 m vertical resolutions.” However, no such
recommendation carried through to that report’s Executive Summary, and no future NASA mission is in the planning stages.

Therefore, we face a future where the globally available DEMs at ~30 m resolution are what we have now, at least through the launches and initial few years of the spectrometers SBG and CHIME and future versions of Landsat and Sentinel. If we could trust the variables calculated from DEMs and consider only the RMS errors, we could implement topographic correction algorithms that estimate $\rho_{\text{intrinsic}}$ from measurements of atmospherically corrected $\rho_{\text{apparent}}$ and thereby recover the geophysical and biological properties of the surface that govern spectral reflectance, with known uncertainty.

However, we face the problem of outliers in the calculations of $\mu_S$ and less crucially $V_\Omega$, so applying any correction algorithm globally on entire images would produce some incorrect, thus misleading, retrievals.

Strategies to mitigate the impact of topographic errors in processing and distributing image data and products must be considered. The list is deliberately terse; any bullet point could be expanded to a whole journal article:

- In the basis documents for algorithms for geophysical and biological products, assess their sensitivity to uncertainty in illumination geometry and distinguish between topographic effects that change the spectral shape of the signal vs. those that change the magnitude only (Lamare et al., 2020).
- Gain a better understanding of the use of shade endmembers (Adams et al., 1986) in spectral mixture analysis, which implicitly acknowledge the limitations of available DEMs by solving for an illumination adjustment on modeled values of a pixel’s reflectance.
- Understand the relative magnitudes of topographic effects on angular properties of the reflectance vs. the effects of illumination and viewing geometry on the intrinsic reflectance (Roupioz et al., 2014; Schaepman-Strub et al., 2006).
- Develop and validate image processing methods that identify pixels where errors in the underlying DEM would lead to incorrect calculations of the illumination geometry, for example detection of shadowed terrain (Hagolle et al., 2017; Hollstein et al., 2016; Shahtahmassebi et al., 2013).
- Avoid exclusively prescribing global topographic correction solutions. Preserve the flexibility, within the mission science data system, for investigators to apply new regional DEMs of higher accuracy as these become available, or to ignore topography.

In the longer term, future research may reduce DEM-induced reflectance errors through strategies such as the following:

- Implement topographic corrections in superpixels, thereby smoothing out the errors in individual pixels (Gilmore et al., 2011).
- Continue efforts to improve DEMs globally, especially in mountainous areas, for example the USGS 3D elevation program in the U.S. (Stoker & Miller, 2022).
- Examine and validate novel methods to estimate illumination geometry directly from images, for example by simultaneously solving for unknown atmospheric and...
topographic properties in retrieval of surface reflectance from top-of-atmosphere radiances.

6 Conclusions

Our analyses show that calculations in the globally available DEMs miss shadows and consistently underestimate cosines of solar illumination angles, RMS error increasing with solar zenith angle. Analyzing imagery of Earth’s mountains from current and future missions requires addressing the uncertainty introduced by errors and outliers in the DEMs on algorithms that retrieve surface properties from measurements of the apparent spectral reflectance. Intriguing potential improvements lie in assessing the uncertainties in retrievals of geophysical and biological properties and in novel methods to gain information about topography from the imagery itself.

Acknowledgments

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Open Research

We have assembled all elevation data used in this research in Dryad (Dozier et al., 2022) under the Creative Commons Zero Waiver. Those elevation files include splicing and cropping to match areas of fine and coarse resolution, so within each region (Carson River, Himachal Pradesh, Wrangell Mountains) the DEMs at different resolutions cover the same area, thereby enabling the comparison of the same topographic variables calculated from different data sources.

Public sources of the data are:

- NASADEM tiles are available from the U.S. Geological Survey Land Processes DAAC Data Pool (NASA JPL, 2020). Registration is required but is free.
- Copernicus DEMs customized to specific latitude-longitude quadrilaterals are available from Open Topography (European Space Agency, 2021).
- Airborne Snow Observatories Inc. provided the snow-off elevation data at 3 m spatial resolution for the Carson River Watershed. The data are available in Dozier et al. (2022).
- The Alaska elevation data, acquired by airborne interferometric synthetic aperture radar, are available from the U.S. Geological Survey (USGS EROS Archive, 2018).
- Tiles for the High Mountain Asia 8 m DEM are available at the National Snow and Ice Data Center (Shean, 2017).
- Global grids of the EGM96 and EGM2008 Geoids are available from Agisoft (2008).
Computer codes for calculating solar illumination geometry (Dozier, 2020) and topographic horizons and other terrain parameters (Dozier, 2022a) are available from the MATLAB Central file exchange. Code for reprojecting raster data is on GitHub (Dozier, 2021). All codes are published and copyrighted under a free re-use license, even for commercial purposes.

References


Dozier, J., Baskaran, L., & Painter, T. H. (2022). Topographic data to support the analysis of error and uncertainty that degrade topographic corrections of remotely sensed data [Distributed by Dryad]. https://doi.org/10.25349/D98896


Table Captions

Table 1. Information sources for digital elevation models used in the analysis.

Table 2. RMS error statistics for topographic variables that are independent of solar illumination.

Table 3. Shadowed fraction and RMS error of $\mu_S$ (cosine solar illumination) for each date in each dataset, varying monotonically with the solar zenith angle $\mu_0 = \cos \theta_0$.

Figure Captions

Figure 1. Example of the elevation sources for the Carson River Watershed. The left image shows the Copernicus DEM, whose spatial resolution is 1 arcsecond; the right image shows a segment of the ASO 3 m DEM, corresponding to the red rectangle in the left image. Both images are in a UTM projection, Zone 11N.

Figure 2. Dates and their solar declinations (degrees) used in the analysis, spaced in equal latitude intervals from the winter solstice to the summer solstice (NOAA, no date, solar calculator).

Figure 3. Values of $\mu_S$ (cosine of local solar illumination angle, including shadowing by horizons) over the Indian Himalaya at 10:45 am on the dates shown in Figure 2, from the winter to the summer solstice. Solar zenith angles varied from winter to summer: 60°, 55°, 48°, 41°, 33°, 27°, 23°. The area coincides with 9 flight lines by AVIRIS-NG during the 2016 ISRO-NASA campaign. The solar illumination values are calculated from the High Mountain Asia 8 m DEM, whose tiles are in an Albers Equal Area Projection, an equal area projection with origin 36°N 85°E and standard parallels 25°N and 47°N.

Figure 4. (a) Top-of-atmosphere reflectance ($\pi \times$ radiance/irradiance) in Landsat 8 OLI band 5 (851-879 nm) in the Indian Himalaya acquired on 22 February 2016 at UTC 05:24. (b) Cosine $\mu_S$ of the solar illumination, including cast shadows, at the same time over a NASADEM matching the Landsat image. The solar zenith angle on a flat surface was 49.3°. (c) Scatter density diagram with the Landsat reflectance on the vertical axis and $\mu_S$ on the horizontal axis. The colors show density of points, with bright yellow indicating high concentrations. The blank area eliminates the values within 1 RMS error of the linear regression $f(x) = ax + b$. (d) Probability density functions (pdf) of the reflectance values in two illumination categories, $\mu_S < 0.2$ and $\mu_S > 0.87$, covering the same fractions (14%) of the image’s values.

Figure 5. Detailed illustration supporting one row in Table 2 for the Copernicus DEM in the Carson River Watershed in the Sierra Nevada. The x-axes show data for the ASO 3 m DEM; the y-axes show the same information derived from the Copernicus DEM, with both DEMs reprojected to a common size and projection. Aspect angles represent south as 0°, eastward positive, westward negative, and therefore consistent with a right-hand coordinate system. Regression lines in the figure and statistics in Table 2 are based on the whole topographic grid, but just 100,000 points are randomly selected for the illustrative scatter plots.
**Figure 6.** Detailed illustration supporting the Wrangell Mountains group in Table 3 for the Copernicus DEM. All axes show values of $\mu_s$, the cosine of local illumination, varying with the dates that Figure 2 shows. Points along either the x- or y-axis identify locations that are shadowed in one DEM and illuminated in the other. Regression lines in the figure and statistics in Table 3 are based on all pixels in the data, but just 100,000 points are randomly selected for the illustrative scatter density plots. Note that the yellow (bright) values in the scatter density plots migrate to higher values of $\mu_s$ as the solar declination moves northward.
Figure 1.
Figure 2.
Figure 4.
Figure 5.
x-axes represent values from the fine-resolution (3 m) data, sampled to the common 10 m resolution. y-axes represent the coarse-resolution data (1 arcsec), sampled to the same 10 m resolution.
21-Dec 10:45  05-Feb 10:45  28-Feb 10:45  20-Mar 10:45

µₚ calculated from Copernicus DEM at 1 arcsec resolution

µₛ calculated from Alaska IFSAR DEM at 5 m resolution

09-Apr 10:45  02-May 10:45  21-Jun 10:45

µₛ calculated from Copernicus DEM at 1 arcsec resolution

µₛ calculated from Alaska IFSAR DEM at 5 m resolution
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*Albers equaconic projection.

Alaska origin 50°N, 154°W, standard parallels 55°N and 65°N
High Mountain Asia origin 36°N, 85°E, standard parallels 25°N and 47°N
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**Bias (%), based on regression slope**

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**K, from regression**

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**HMA DEM, Himachal Pradesh, India**

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**Alaska IFSAR DEM, Wrangell Mountains**

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