Comparison of techniques for coupled earthquake and tsunami modeling

Lauren S. Abrahams¹,¹,¹, Eric M. Dunham¹,¹,¹, Lukas Krenz²,²,², Tatsuhiko Saito³,³,³, and Alice-Agnes Gabriel⁴,⁴,⁴

¹Stanford University
²Technical University of Munich (TUM)
³National Research Institute for Earth Science and Disaster Resilience
⁴Ludwig Maximilians University of Munich

November 30, 2022

Abstract

From interpreting data to scenario modeling of subduction events, numerical modeling has been crucial for studying tsunami generation by earthquakes. Seafloor instruments in the source region feature complex signals containing a superposition of seismic, ocean acoustic, and tsunami waves. Rigorous modeling is required to interpret these data and use them for tsunami early warning. However, previous studies utilize separate earthquake and tsunami models, with one-way coupling between them and approximations that might limit the applicability of the modeling technique. In this study, we compare four earthquake-tsunami modeling techniques, highlighting assumptions that affect the results, and discuss which techniques are appropriate for various applications. Most techniques couple a 3D Earth model with a 2D depth-averaged shallow water tsunami model. Assuming the ocean is incompressible and that tsunami propagation is negligible over the earthquake duration leads to technique (1), which equates earthquake seafloor uplift to initial tsunami sea surface height. For longer duration earthquakes, it is appropriate to follow technique (2), which uses time-dependent earthquake seafloor velocity as a time-dependent forcing in the tsunami mass balance. Neither technique captures ocean acoustic waves, motivating newer techniques that capture the seismic and ocean acoustic response as well as tsunamis. Saito et al. (2019) propose technique (3), which solves the 3D elastic and acoustic equations to model the earthquake rupture, seismic wavefield, and response of a compressible ocean without gravity. Then, sea surface height is used as a forcing term in a tsunami simulation. A superposition of the earthquake and tsunami solutions provides the complete wavefield, with one-way coupling. The complete wavefield is also captured in technique (4), which utilizes a fully-coupled solid Earth and ocean model with gravity (Lotto & Dunham, 2015). This technique, recently incorporated into the 3D code SeisSol, simultaneously solves earthquake rupture, seismic waves, and ocean response (including gravity). Furthermore, we show how technique (3) follows from (4) subject to well-justified approximations.
Comparison of techniques for coupled earthquake and tsunami modeling

Lauren S. Abrahams¹, Eric M. Dunham¹, Lukas Krenz², Tatsuhiko Saito³, and Alice-Agnes Gabriel⁴

¹ Stanford University,
² Technical University of Munich (TUM),
³ National Research Institute for Earth Science and Disaster Resilience,
⁴ Ludwig Maximilians University of Munich
One-Way Coupled Techniques

Pass information from an earthquake simulation to separate tsunami simulation.
Coupled earthquake and tsunami modeling
Comparison of Model Techniques

Static Initial Conditions

- Final earthquake seafloor or sea surface uplift recorded
- Set initial tsunami sea surface height

Static Technique

Pass information from an earthquake simulation

one-way information flow

To separate tsunami simulation

Citations

(Kajiura, 1963, 1970)
(Tanioka & Satake, 1996)
(Saito, 2019)
Coupled earthquake and tsunami modeling
Comparison of Model Techniques

- **Earthquake Simulation**
  - Static Initial Conditions
    - Final earthquake seafloor or sea surface uplift recorded
    - Set initial tsunami sea surface height
  - Time-dependent Seafloor Velocity as Forcing
    - Record earthquake seafloor velocity
    - Set time-dependent forcing in the tsunami mass balance

- **Tsunami Simulation**

- **Citations**
  - Kajiura, 1963, 1970
  - Tanioka & Satake, 1996
  - Saito, 2019
  - Saito & Furumura, 2009
  - Saito & Tsushima, 2016
  - Saito, 2019
  - Lotto & Dunham, 2015

- Time-dependent forcing

Pass information from an earthquake simulation

one-way information flow

To separate tsunami simulation
Coupled earthquake and tsunami modeling
Comparison of Model Techniques

**Earthquake Simulation**
- Static Initial Conditions
  - Final earthquake seafloor or sea surface uplift recorded
  - Set initial tsunami sea surface height
- Time-dependent Seafloor Velocity as Forcing
  - Record earthquake seafloor velocity
  - Set time-dependent forcing in the tsunami mass balance
- Time-dependent Sea Surface Velocity as Forcing
  - Solves earth and ocean response without gravity
  - Use sea surface velocity as a forcing term in a tsunami simulation

**Tsunami Simulation**
- Static Initial Conditions
- Time-dependent Seafloor Velocity as Forcing
- Time-dependent Sea Surface Velocity as Forcing

**Citations**
- (Kajiura, 1963, 1970)
- (Tanioka & Satake, 1996)
- (Saito & Furumura, 2009)
- (Saito & Tsushima, 2016)
- (Saito, 2019)
- (Saito et al., 2019)
- (Saito, 2019)
- (Lotto & Dunham, 2015)

Pass information from an earthquake simulation one-way information flow To separate tsunami simulation

Time-dependent forcing
## Coupled earthquake and tsunami modeling

### Comparison of Model Techniques

<table>
<thead>
<tr>
<th>Earthquake Simulation</th>
<th>Tsunami Simulation</th>
<th>Citations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final earthquake seafloor or sea surface uplift recorded</td>
<td>Set initial tsunami sea surface height</td>
<td></td>
</tr>
</tbody>
</table>

| **Time-dependent Seafloor Velocity as Forcing** | | (Saito & Furumura, 2009) (Saito & Tsushima, 2016) (Saito, 2019) |
| Record earthquake seafloor velocity | Set time-dependent forcing in the tsunami mass balance | |

| **Time-dependent Sea Surface Velocity as Forcing** | | (Saito et al., 2019) (Saito, 2019) |
| Solves earth and ocean response without gravity | Use sea surface velocity as a forcing term in a tsunami simulation | |

---

**Pass information from an earthquake simulation**

**Do not account for compressibility effects**

---

**one-way information flow**

**To separate tsunami simulation**
**Coupled earthquake and tsunami modeling**

**Comparison of Model Techniques**

<table>
<thead>
<tr>
<th>Earthquake Simulation</th>
<th>Tsunami Simulation</th>
<th>Citations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Static Initial Conditions</strong></td>
<td></td>
<td>(Kajiura, 1963, 1970)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Tanioka &amp; Satake, 1996)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Saito, 2019)</td>
</tr>
<tr>
<td></td>
<td><strong>Final earthquake seafloor or sea surface uplift recorded</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Set initial tsunami sea surface height</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Time-dependent Seafloor Velocity as Forcing</strong></td>
<td></td>
<td>(Saito &amp; Furumura, 2009)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Saito &amp; Tsushima, 2016)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Saito, 2019)</td>
</tr>
<tr>
<td></td>
<td><strong>Record earthquake seafloor velocity</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Set time-dependent forcing in the tsunami mass balance</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Time-dependent Sea Surface Velocity as Forcing</strong></td>
<td></td>
<td>(Saito et al., 2019)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Saito, 2019)</td>
</tr>
<tr>
<td></td>
<td><strong>Solves earth and ocean response without gravity</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Use sea surface velocity as a forcing term in a tsunami simulation</strong></td>
<td></td>
</tr>
</tbody>
</table>

Approximate approaches

Pass information from an earthquake simulation

one-way information flow

To separate tsunami simulation
Coupled earthquake and tsunami modeling
Comparison of Model Techniques

Earthquake Simulation
- Static Initial Conditions
  - Final earthquake seafloor or sea surface uplift recorded
  - Set initial tsunami sea surface height

Tsunami Simulation
- Time-dependent Seafloor Velocity as Forcing
  - Record earthquake seafloor velocity
  - Set time-dependent forcing in the tsunami mass balance
- Time-dependent Sea Surface Velocity as Forcing
  - Solves earth and ocean response without gravity
  - Use sea surface velocity as a forcing term in a tsunami simulation

Fully Coupled Method (SeisSol)
- Simultaneously solves earthquake rupture, seismic waves, and ocean response (including gravity)
- http://www.seissol.org/

Citations
- (Kajiura, 1963, 1970)
- (Tanioka & Satake, 1996)
- (Saito, 2019)
- (Saito & Furumura, 2009)
- (Saito & Tsushima, 2016)
- (Saito, 2019)
- (Saito et al., 2019)
- (Saito, 2019)
- (Lotto & Dunham, 2015)

Considered “ground truth”
Newly implemented in 3D

Approximate approaches
Coupled earthquake and tsunami modeling
Comparison of Model Techniques

Earthquake Simulation
- Static Initial Conditions
  - Final earthquake seafloor or sea surface uplift recorded
- Set initial tsunami sea surface height

Tsunami Simulation
- Time-dependent Seafloor Velocity as Forcing
  - Record earthquake seafloor velocity
- Set time-dependent forcing in the tsunami mass balance

- Time-dependent Sea Surface Velocity as Forcing
  - Solves earth and ocean response without gravity
  - Use sea surface velocity as a forcing term in a tsunami simulation

- Fully Coupled Method (SeisSol)
  - Simultaneously solves earthquake rupture, seismic waves, and ocean response (including gravity)
    - New in 3D

Citations
- (Kajiura, 1963, 1970)
- (Tanioka & Satake, 1996)
- (Saito, 2019)
- (Saito & Furumura, 2009)
- (Saito & Tsushima, 2016)
- (Saito et al., 2019)
- (Saito, 2019)
- (Lotto & Dunham, 2015)

Model the full-wave field
One-Way Coupled Techniques

Pass information from an earthquake simulation

to separate tsunami simulation

Model techniques:
- Static initial conditions vs time-dependent forcing
- Incompressible vs compressible ocean

How is modeled data affected by:
1. long rupture duration
2. acoustic wave generation
Problem setup

- We want to examine how long duration rupture and compressibility affect wave generation in the four modeling techniques
Problem setup

- We want to examine how long duration rupture and compressibility affect wave generation in the four modeling techniques.
Problem setup

- We want to examine how long duration rupture and compressibility affect wave generation in the four modeling techniques.
Problem setup

• We want to examine how long duration rupture and compressibility affect wave generation in the four modeling techniques.
Problem setup

• We want to examine how long duration rupture and compressibility affect wave generation in the four modeling techniques.
Problem setup

- We want to examine how long duration rupture and compressibility affect wave generation in the four modeling techniques.

![Seafloor velocity graph](image)
Problem setup

• We want to examine how long duration rupture and compressibility affect wave generation in the four modeling techniques.
• We vary source width ($\sigma_r$) and duration ($\sigma_t$) in the earthquake simulation to test different scenarios setups.
Problem setup

- We want to examine how long duration rupture and compressibility affect wave generation in the four modeling techniques.
- We vary source width ($\sigma_r$) and duration ($\sigma_t$) in the earthquake simulation to test different scenarios setups.

Short rupture duration $\sigma_t$

Long rupture duration $\sigma_t$
Problem setup

- We want to examine how long duration rupture and compressibility affect wave generation in the four modeling techniques.
- We vary source width ($\sigma_r$) and duration ($\sigma_t$) in the earthquake simulation to test different scenarios setups.
Problem setup

- We want to examine how long duration rupture and compressibility affect wave generation in the four modeling techniques.
- We vary source width ($\sigma_r$) and duration ($\sigma_t$) in the earthquake simulation to test different scenarios setups.

Within shallow water limit?  
No if:  
$$\frac{H}{\sigma_r} \gg 1$$

Tsunami propagates over source duration?  
No if:  
$$\frac{\sigma_r}{\sigma_t \sqrt{gH}} \gg 1$$

Acoustic waves significant?  
No if:  
$$\frac{c \sigma_t}{H} \gg 1$$

Where, the ocean depth is $H = 4\text{km}$, tsunami wave speed is $\sqrt{gH}$, and acoustic wavespeed $c$.  

Problem setup

Source

(modified from Saito et al., 2019)
Problem setup

Source

Earthquake Simulation

no gravity (leaves static sea surface deformation)

Applied as a boundary condition

(modified from Saito et al., 2019)
Problem setup

Source

Earthquake Simulation

Tsunami Simulation

no gravity (leaves static sea surface deformation)

one-way information flow

(modified from Saito et al., 2019)
**Problem setup**

- **Source**
  - no gravity (leaves static sea surface deformation)

- **Earthquake Simulation**
  - one-way information flow
  - Static Initial condition

- **Tsunami Simulation**
  - (modified from Saito et al., 2019)
Problem setup

Source

no gravity (leaves static sea surface deformation)

Earthquake Simulation

Time-dependent Seafloor Velocity as Forcing

Static Initial condition

Tsunami Simulation

(modified from Saito et al., 2019)

one-way information flow
Problem setup

Source

Earthquake Simulation

time-dependent
Sea Surface Velocity as Forcing

time-dependent
Seafloor Velocity as Forcing

no gravity (leaves static sea surface deformation)

Tsunami Simulation

no gravity (permanent deformation)

one-way information flow

(modified from Saito et al., 2019)
Problem setup

- Source
  - no gravity (leaves static sea surface deformation)

- Earthquake Simulation
  - one-way information flow

- Tsunami Simulation
  - (modified from Saito et al., 2019)

- Fully Coupled Method
  - gravity on, tsunami wave propagates
Revised Slides Begin here

Research is always ongoing, this talk has been revised to show new exciting results!
Example 1:

Source Width : \( \sigma_r = 12.5 \text{ km} \)
Source Duration : \( \sigma_t = 500 \text{ s} \)

Within shallow water limit?
Yes: \( \frac{H}{\sigma_r} = 0.32 < 1 \)

Tsunami propagates over source duration?
Yes: \( \frac{\sigma_r}{\sigma_t \sqrt{gH}} = 0.13 < 1 \)

Acoustic waves significant?
No: \( \frac{c\sigma_t}{H} = 187.5 > 1 \)

We can anticipate method 1 (using a static initial condition) will be incorrect
Seafloor displacement as initial conditions

Source Width: $\sigma_r = 12.5$ km
Source Duration: $\sigma_t = 500$ s
Yes: Within shallow water limit?
Yes: Tsunami propagates over source duration?
No: Acoustic waves significant?

Method 1. Static Initial condition
Method 2. Time-dependent Seafloor Velocity as Forcing
Method 3. Time-dependent Sea Surface Velocity as Forcing
Method 4. Fully Coupled Method
Seafloor displacement as initial conditions

Seafloor velocity as forcing

Method 1. Static Initial condition
Method 2. Time-dependent Seafloor Velocity as Forcing
Method 3. Time-dependent Sea Surface Velocity as Forcing
Method 4. Fully Coupled Method

Source Width: $\sigma_r = 12.5$ km
Source Duration: $\sigma_t = 500$ s
Yes: Within shallow water limit?
Yes: Tsunami propagates over source duration?
No: Acoustic waves significant?
Source Width: \( \sigma_T = 12.5 \text{ km} \)
Source Duration: \( \sigma_T = 500 \text{ s} \)
Yes: Within shallow water limit?
Yes: Tsunami propagates over source duration?
No: Acoustic waves significant?

**Method 1.** Static Initial condition
**Method 2.** Time-dependent Seafloor Velocity as Forcing
**Method 3.** Time-dependent Sea Surface Velocity as Forcing
**Method 4.** Fully Coupled Method

**Sea surface velocity as forcing**

**Seafloor velocity as forcing**

Tsunami wave speed \( gH \)
Source Width: $\sigma_r = 12.5$ km  
Source Duration: $\sigma_t = 500$ s  
Yes: Within shallow water limit?  
Yes: Tsunami propagates over source duration?  
No: Acoustic waves significant?  

**Method 1.** Static Initial condition  
**Method 2.** Time-dependent Seafloor Velocity as Forcing  
**Method 3.** Time-dependent Sea Surface Velocity as Forcing  
**Method 4.** Fully Coupled Method
Example 2:

Source Width : \( \sigma_r = 12.5 \text{ km} \)
Source Duration : \( \sigma_t = 1.25 \text{ s} \)

Within shallow water limit?
  Yes: \( \frac{H}{\sigma_r} = 0.32 < 1 \)
Tsunami propagates over source duration?
  No: \( \frac{\sigma_r}{\sigma_t \sqrt{gH}} = 50.5 > 1 \)
Acoustic waves significant?
  Yes: \( \frac{c \sigma_t}{H} = 0.47 < 1 \)

Method 1 and 2 do not model acoustic wave generation, we anticipate the results will differ compared to methods 3 and 4.
Source Width: $\sigma_r = 12.5$ km
Source Duration: $\sigma_t = 1.25$ s
Yes: Within shallow water limit?
No: Tsunami propagates over source duration?
Yes: Acoustic waves significant?

Method 1. Static Initial condition
Method 2. Time-dependent Seafloor Velocity as Forcing
Method 3. Time-dependent Sea Surface Velocity as Forcing
Method 4. Fully Coupled Method

Seafloor displacement as initial conditions
Seafloor velocity as forcing
Source Width: $\sigma_r = 12.5$ km
Source Duration: $\sigma_t = 1.25$ s
Yes: Within shallow water limit?
No: Tsunami propagates over source duration?
Yes: Acoustic waves significant?

Method 1. Static Initial condition
Method 2. Time-dependent Seafloor Velocity as Forcing
Method 3. Time-dependent Sea Surface Velocity as Forcing
Method 4. Fully Coupled Method

**Full wavefield**

**No acoustic waves**
Source Width: \( \sigma_r = 12.5 \, \text{km} \)
Source Duration: \( \sigma_t = 1.25 \, \text{s} \)
Yes: Within shallow water limit?
No: Tsunami propagates over source duration?
Yes: Acoustic waves significant?

Full wavefield (one-way coupling)

| Method 1. Static Initial condition |
| Method 2. Time-dependent Seafloor Velocity as Forcing |
| Method 3. Time-dependent Sea Surface Velocity as Forcing |
| Method 4. Fully Coupled Method |

Full wavefield (fully coupled)
Source Width: \( \sigma_r = 12.5 \text{ km} \)
Source Duration: \( \sigma_t = 1.25 \text{ s} \)
Yes: Within shallow water limit?  
No: Tsunami propagates over source duration?  
Yes: Acoustic waves significant?

Method 1. Static Initial condition  
Method 2. Time-dependent Seafloor Velocity as Forcing  
Method 3. Time-dependent Sea Surface Velocity as Forcing  
Method 4. Fully Coupled Method

---

**Source**: [Image of tsunami simulation graphs for different methods.]
Source Width: $\sigma_r = 12.5$ km
Source Duration: $\sigma_c = 1.25$ s
Yes: Within shallow water limit?
No: Tsunami propagates over source duration?
Yes: Acoustic waves significant?

Method 1. Static Initial condition
Method 2. Time-dependent Seafloor Velocity as Forcing
Method 3. Time-dependent Sea Surface Velocity as Forcing
Method 4. Fully Coupled Method
Example 3:

Source Width : \( \sigma_r = 1.25 \text{ km} \)
Source Duration : \( \sigma_t = 1.25 \text{ s} \)

Within shallow water limit?
   No: \( \frac{H}{\sigma_r} = 3.20 > 1 \)

Tsunami propagates over source duration?
   No: \( \frac{\sigma_r}{\sigma_t \sqrt{gH}} = 5.05 > 1 \)

Acoustic waves significant?
   Yes: \( \frac{c \sigma_t}{H} = 0.47 < 1 \)

Method 1 and 2 do not model acoustic wave generation, we anticipate the results will differ compared to methods 3 and 4

*Note, in this study Method 1, 2, and 3 all use a linear shallow water solver not accounting for dispersion affects
Seafloor displacement as initial conditions

Source Width: $\sigma_x = 1.25$ km
Source Duration: $\sigma_t = 1.25$ s
No: Within shallow water limit?
No: Tsunami propagates over source duration?
Yes: Acoustic waves significant?

Method 1. Static Initial condition
Method 2. Time-dependent Seafloor Velocity as Forcing
Method 3. Time-dependent Sea Surface Velocity as Forcing
Method 4. Fully Coupled Method

Seafloor velocity as forcing
Source Width: $\sigma_r = 1.25$ km
Source Duration: $\sigma_t = 1.25$ s
No: Within shallow water limit?
No: Tsunami propagates over source duration?
Yes: Acoustic waves significant?

### Method 1.
Static Initial condition
Method 2.
Time-dependent Seafloor Velocity as Forcing
Method 3.
Time-dependent Sea Surface Velocity as Forcing
Method 4.
Fully Coupled Method

**With short wavelengths**

![Method 1 (initial cond. seafloor displacement)](

**Without short wavelengths**

![Method 1 (initial cond. sea surface displacement)](

- Time (s)
- Horizontal distance (km)
- Sea surface displacement (m)
Source Width: $\sigma_r = 12.5$ km
Source Duration: $\sigma_t = 1.25$ s
Yes: Within shallow water limit?
No: Tsunami propagates over source duration?
No: Acoustic waves significant?

**Sea surface velocity as forcing**

**Sea surface displacement as initial condition, without short wavelengths**

**Methods**

1. Static Initial condition
2. Time-dependent Seafloor Velocity as Forcing
3. Time-dependent Sea Surface Velocity as Forcing
4. Fully Coupled Method
Source Width: $\sigma_r = 12.5$ km
Source Duration: $\sigma_t = 1.25$ s
Yes: Within shallow water limit?
No: Tsunami propagates over source duration?
No: Acoustic waves significant?

**Method 1.** Static Initial condition

**Method 2.** Time-dependent Seafloor Velocity as Forcing

**Method 3.** Time-dependent Sea Surface Velocity as Forcing

**Method 4.** Fully Coupled Method
Source Width: $\sigma_r = 1.25$ km
Source Duration: $\sigma_c = 1.25$ s
No: Within shallow water limit?
No: Tsunami propagates over source duration?
Yes: Acoustic waves significant?

Method 1. Static Initial condition
Method 2. Time-dependent Seafloor Velocity as Forcing
Method 3. Time-dependent Sea Surface Velocity as Forcing
Method 4. Fully Coupled Method
**Static Initial Conditions**
- Final earthquake seafloor or sea surface uplift recorded
- Set initial tsunami sea surface height

**Time-dependent Seafloor Velocity as Forcing**
- Record earthquake seafloor velocity
- Set time-dependent forcing in the tsunami mass balance

**Time-dependent Sea Surface Velocity as Forcing**
- Solves earth and ocean response without gravity
- Use sea surface velocity as a forcing term in a tsunami simulation

**Fully Coupled Method (SeisSol)**
- Simultaneously solves earthquake rupture, seismic waves, and ocean response (including gravity)

http://www.seissol.org/

**Requires:**
- Shallow water limit (if short wavelength are not filtered)
  \[
  \frac{H}{\sigma_r} \ll 1
  \]
- Tsunami waves do not propagate over source duration
  \[
  \frac{\sigma_r}{\sigma_t \sqrt{gH}} \gg 1
  \]
- And acoustic waves are not generated
  \[
  \frac{c\sigma_t}{H} \gg 1
  \]

**Time-dependent Sea Surface Velocity as Forcing**
- Solves earth and ocean response without gravity
- Use sea surface velocity as a forcing term in a tsunami simulation

**Fully Coupled Method (SeisSol)**
- Simultaneously solves earthquake rupture, seismic waves, and ocean response (including gravity)

http://www.seissol.org/

**In Summary**

<table>
<thead>
<tr>
<th>Static Initial Conditions</th>
<th>Time-dependent Seafloor Velocity as Forcing</th>
<th>Time-dependent Sea Surface Velocity as Forcing</th>
<th>Fully Coupled Method (SeisSol)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final earthquake seafloor or sea surface uplift recorded</td>
<td>Record earthquake seafloor velocity</td>
<td>Solves earth and ocean response without gravity</td>
<td>Simultaneously solves earthquake rupture, seismic waves, and ocean response (including gravity)</td>
</tr>
<tr>
<td>Set initial tsunami sea surface height</td>
<td>Set time-dependent forcing in the tsunami mass balance</td>
<td>Use sea surface velocity as a forcing term in a tsunami simulation</td>
<td><a href="http://www.seissol.org/">http://www.seissol.org/</a></td>
</tr>
</tbody>
</table>

**Requires:**
- Shallow water limit (if short wavelength are not filtered)
  \[
  \frac{H}{\sigma_r} \ll 1
  \]
- Tsunami waves do not propagate over source duration
  \[
  \frac{\sigma_r}{\sigma_t \sqrt{gH}} \gg 1
  \]
- And acoustic waves are not generated
  \[
  \frac{c\sigma_t}{H} \gg 1
  \]

**Valid in all cases**

**In Summary**
### Static Initial Conditions
- Final earthquake seafloor or sea surface uplift recorded
- Set initial tsunami sea surface height

<table>
<thead>
<tr>
<th>Requires:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shallow water limit (if short wavelength are not filtered)</td>
</tr>
<tr>
<td>$\frac{H}{\sigma_r} \ll 1$</td>
</tr>
<tr>
<td>Tsunamis waves do not propagate over source duration</td>
</tr>
<tr>
<td>$\frac{\sigma_r}{\sigma_t \sqrt{gH}} \gg 1$</td>
</tr>
<tr>
<td>And acoustic waves are not generated</td>
</tr>
<tr>
<td>$\frac{c_\sigma_t}{H} \gg 1$</td>
</tr>
</tbody>
</table>

### Time-dependent Seafloor Velocity as Forcing
- Record earthquake seafloor velocity
- Set time-dependent forcing in the tsunami mass balance

<table>
<thead>
<tr>
<th>Requires:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shallow water limit</td>
</tr>
<tr>
<td>$\frac{H}{\sigma_r} \ll 1$</td>
</tr>
<tr>
<td>And acoustic waves are not generated</td>
</tr>
<tr>
<td>$\frac{c_\sigma_t}{H} \gg 1$</td>
</tr>
</tbody>
</table>

### Time-dependent Sea Surface Velocity as Forcing
- Solves earth and ocean response without gravity
- Use sea surface velocity as a forcing term in a tsunami simulation

| If non-dispersive shallow water solver, requires: |
| Shallow water limit |
| $\frac{H}{\sigma_r} \ll 1$ |
| And acoustic waves are not generated |
| $\frac{c_\sigma_t}{H} \gg 1$ |

### Fully Coupled Method (SeisSol)
- Simultaneously solves earthquake rupture, seismic waves, and ocean response (including gravity)

Valid in all cases

http://www.seissol.org/

**In Summary**
Static Initial Conditions

Final earthquake seafloor or sea surface uplift recorded

Set initial tsunami sea surface height

Time-dependent Seafloor Velocity as Forcing

Record earthquake seafloor velocity

Set time-dependent forcing in the tsunami mass balance

Time-dependent Sea Surface Velocity as Forcing

Solves earth and ocean response without gravity

Use sea surface velocity as a forcing term in a tsunami simulation

Fully Coupled Method (SeisSol)

Simultaneously solves earthquake rupture, seismic waves, and ocean response (including gravity)

http://www.seissol.org/

Requires:
Shallow water limit (if short wavelength are not filtered)
\[
\frac{H}{\sigma_r} \ll 1
\]
Tsunami waves do not propagate over source duration
\[
\frac{\sigma_r}{\sigma_t \sqrt{gH}} \gg 1
\]
And acoustic waves are not generated
\[
\frac{c\sigma_t}{H} \gg 1
\]

In Summary

If non-dispersive shallow water solver, requires:
Shallow water limit
\[
\frac{H}{\sigma_r} \ll 1
\]
And acoustic waves are not generated
\[
\frac{c\sigma_t}{H} \gg 1
\]
If Boussinesq tsunami solver: We expect valid in more cases

Valid in all cases
Citation


