A global centroid single force catalog of P-wave microseisms

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Abstract

A centroid location catalog of P-wave microseisms is crucial for understanding the origins of microseisms. Although a back-projection method is feasible for locating the centroids, the computational cost is still expensive for making a global catalog over ten years. Contrary, although the computational cost of beamforming is low, it cannot distinguish P from PP waves. To combine the advantages of both methods, we develop the auto-focusing method as a natural extension of beamforming. In the first step of this method, we estimated the slowness vector based on conventional beamforming and the epicentral distance inferred from the wavefront curvature by maximizing the beam power. In the second step, we iteratively update the values based on the perturbation theory. In the third step, based on the classified phase according to the estimated epicentral distance and the slowness, we infer the source location from the slowness vector with corrections for a global 3-D P-wave velocity structure. We also infer the centroid-single-force (CSF) from the beam power. We applied this method to the vertical components of seismic records at approximately 780 Hi-net stations in Japan from 2004 to 2020. We also compare the CSF catalog with a synthetic CSF catalog based on a numerical ocean wave model: WAVEWATCH III. Both catalogs generally show similar temporal-spatial patterns of centroids. The amplitudes of CSF are consistent with each other, although the seismic signal-to-noise ratio limits the detected events. Exceptionally, significant activities in the Gulf of Carpentaria cannot be explained by the ocean wave model.
A global centroid single force catalog of P-wave microseisms

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Key Points:

• We constructed a global centroid single force catalog of P-wave microseisms by auto-focusing method with a seismic array.
• An auto-focusing method, which utilizes the information both the slowness and the curvature of the wavefront, was developed.
• The catalog is consistent with a numerical ocean wave model except for the Gulf of Carpentaria.

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Abstract

A centroid location catalog of P-wave microseisms is crucial for understanding the origins of microseisms. Although a back-projection method is feasible for locating the centroids, the computational cost is still expensive for making a global catalog over ten years. Contrary, although the computational cost of beamforming is low, it cannot distinguish P from PP waves. To combine the advantages of both methods, we develop the autofocusing method as a natural extension of beamforming. In the first step of this method, we estimated the slowness vector based on conventional beamforming and the epicentral distance inferred from the wavefront curvature by maximizing the beam power. In the second step, we iteratively update the values based on the perturbation theory. In the third step, based on the classified phase according to the estimated epicentral distance and the slowness, we infer the source location from the slowness vector with corrections for a global 3-D P-wave velocity structure. We also infer the centroid-single-force (CSF) from the beam power. We applied this method to the vertical components of seismic records at approximately 780 Hi-net stations in Japan from 2004 to 2020. We also compare the CSF catalog with a synthetic CSF catalog based on a numerical ocean wave model: WAVEWATCH III. Both catalogs generally show similar temporal-spatial patterns of centroids. The amplitudes of CSF are consistent with each other, although the seismic signal-to-noise ratio limits the detected events. Exceptionally, significant activities in the Gulf of Carpentaria cannot be explained by the ocean wave model.

1 Introduction

The Earth oscillates persistently over a period range of around 10 s, even in the absence of earthquakes. Ambient seismic wavefields (Nishida, 2017) are also known as microseisms, which are caused by ocean swell activity. Based on the dominant frequency, microseisms can be categorized into two types. The first one is primary microseisms from 0.05–0.1 Hz, which correspond to the dominant frequency of ocean swell. The most probable excitation mechanism is a linear topographic coupling between the ocean swell and seismic waves (e.g., Hasselmann, 1963; Nishida et al., 2008; Fukao et al., 2010; Saito, 2010; Arduhin, 2018). The second one is secondary microseisms from 0.1 to 0.5 Hz. This frequency corresponds to the double frequency of the ocean swell, which is caused by second-order effects of the fluid motions (Longuet-Higgins, 1950; Hasselmann, 1963; Kedar et al., 2008; Arduhin et al., 2011). Secondary microseisms are the most powerful noise for
earthquake observations, even at inland stations. The observations of microseisms date back to the late 19th century (Bertelli, 1872) just after seismologists started to observe ground motions.

Most seismologists thought that surface waves are dominant, although pioneering works around 1970s (Vinnik, 1973; Toksöz & Lacoss, 1968) pointed out that body wave microseisms become important, particularly at inland stations. In the last ten years, a modern dense array dataset has enabled us to investigate the excitation in detail. Recent developments in seismic arrays have enabled us to track the center of tropical cyclones (Zhang et al., 2010; Retailleau & Gualtieri, 2019). All the observations showed that the squared amplitudes of P-wave microseisms are an order of magnitude larger than those of S-wave microseisms (Nishida & Takagi, 2016; Liu et al., 2016). As we focused on teleseismic P-wave microseisms, we analyzed the vertical components of seismic records in this study.

With the development of numerical wind-wave modeling (e.g., WAVEWATCH III, Tolman, 2008; Ardhuin et al., 2011), we can now quantitatively compare the seismic observations with the ocean wave model (Landès et al., 2010; Euler et al., 2014; Nishida & Takagi, 2016). When the comparison, Nishida and Takagi (2016) pointed out that a single force at a centroid location provides information on the excitation. This approach ("stormquake": Fan et al., 2019) is also feasible for primary microseisms. This study aims to construct a centroid-single-force (CSF) catalog of P-wave microseisms to reveal the excitation mechanisms. This catalog could also be feasible for seismic exploration of the deep Earth’s interior.

The back-projection method migrates seismic energy to a grid on the source area according to the travel time from the source to the receivers. Although a back-projection method is powerful for making a CSF catalog, the computational cost is still high for long-time data on a global scale (> 10 years). On the other hand, the beamforming method is a frequency-slowness analysis of array data with irregular spacing of the stations with an assumption of a plane wave. In the 2-D slowness domain, the slowness can be measured by taking the maximum value. The source is located using a slowness vector with a reference Earth’s model. Because the computation is efficient, the beamforming method is feasible for long-term monitoring. However, the lack of curvature information due to the plane wave assumption prevents accurate estimation of the epicentral distance. In
Figure 1. Spherical coordinates used in this study. The origin is located at the center of the array. \( \mathbf{x}_n \) is the location vector of the \( i \)th station, and \( \mathbf{x}_c \) is the centroid location. \( \Delta_n \) is the arc distance between \( \mathbf{x}_n \) and \( \mathbf{x}_c \), and \( \Delta_0 \) is the arc distance between \( \mathbf{x}_c \) and the origin \( \mathbf{x}_n \). The circles show the station locations, and the triangles show the source location.

In particular, in principle, a conventional beamforming method cannot distinguish PP waves from P waves. To create a global CSF catalog over ten years, we developed an auto-focusing method. This method is a natural excitation of the beamforming method that utilizes the curvature information of the wavefront. Although the auto-focusing method can also be recognized as a variant of the back-projection method, it has the advantage of (i) independent estimation of the slowness vector and epicentral distance inferred from the curvature of the wavefront, and (ii) efficient computational time.

We apply this method for the high-sensitivity seismograph network (Hinet) in Japan (Okada et al., 2004) from 2004 to 2020. For a comparison with ocean physics data, a synthetic CSF catalog of P-wave microseisms was calculated based on a numerical ocean wave model (WAVEWATCH III, Ardhuin et al., 2011). We will discuss the excitation mechanisms of the detected P-wave microseisms based on the temporal-spatial patterns of the centroids and the relationship between the CSF and the total number of events.

2 Auto-focusing method

This section formulates the auto-focusing method. Here, we consider array data, which are composed of vertical displacement \( u_i(\omega) \) at stations \( \mathbf{x}_i, (i = 1, \cdots, N) \), where \( N \) is the number of stations, and \( \omega \) is angular frequency. Because the equivalent body force for the excitation source of secondary microseisms can be represented by a verti-
cal single force (e.g., Ardhuin et al., 2011; Gualtieri et al., 2013), we assumed that the seismic source was written by a vertical point force at $x_c$. In this case, $u_n(\omega)$ depends only on the epicentral distance $\Delta_n$ between $x_c$ and $x_n$ for 1-D earth structure. $u_n$ can be written by

$$u_n(\omega) = A(\Delta_n)e^{-i\omega T(\Delta_n)}\Phi(\omega),$$  \hspace{1cm} (1)

where $A$ is amplitude, $\Phi$ is the source time function, and $T$ is the travel time from $x_c$ to $x_0$.

In this study, we used data from a seismic array around the center $x_0$ (Figure 1). With an assumption of the small spatial scale of the seismic array, the difference of epicentral distance $\Delta_n - \Delta_0$ is smaller than $\Delta_0$, where $\Delta_0$ is the epicentral distance between $x_c$ and the center of the array $x_0$ (Figure 1). We expanded the travel time $T$ in the form of a Taylor series with respect to $\Delta_n - \Delta_0$ up to the second order as

$$T(\Delta_n) \approx T(\Delta_0) + \frac{dT}{d\Delta}|_{\Delta_0}(\Delta_n - \Delta_0) + \frac{1}{2}\frac{d^2T}{d\Delta^2}|_{\Delta_0}(\Delta_n - \Delta_0)^2$$  \hspace{1cm} (2)

$$= T(\Delta_0) + p_0(\Delta_n - \Delta_0) + \frac{1}{2}\frac{dp}{d\Delta}|_{\Delta_0}(\Delta_n - \Delta_0)^2,$$  \hspace{1cm} (3)

where $p_0$ is the slowness. To simplify the problem, we assumed that the amplitude $A$ is constant inside the array. $\Delta_n - \Delta_0$ can also be approximated by the Taylor series $l(\phi_0, \Delta_0; x_n)$ with respect to $\eta_n$ and $\zeta_n$ (Figure 1) up to the third order:

$$\Delta_n - \Delta_0 \approx l(\phi_0, \Delta_0; x_n) \equiv -\eta_n + \frac{\zeta_n^2}{2\tan\Delta_0} + \eta_n\zeta_n\left(\frac{1}{6} + \frac{1}{2\tan^2\Delta_0}\right),$$  \hspace{1cm} (4)

where $\eta_n \equiv \theta_n \cos(\varphi_n - \phi_0)$, $\zeta_n \equiv \theta_n \sin(\varphi_n - \phi_0)$, $\theta_n$ is the arc distance from the center of the array to the station $x_n$, and $\varphi_n$ is the azimuth of the polar coordinates of the station at $x_n$. Thus, the travel time can be represented by a function of the model parameters $m$:

$$m \equiv \left(p_0, \phi_0, \Delta_0, \frac{dp_0}{d\Delta}\right),$$  \hspace{1cm} (5)

where $\phi_0$ is the azimuth of the polar coordinate of $x_c$. To simplify the notation we will write $dp/d\Delta|_{\Delta_0}$ as $dp_0/d\Delta$ later.

The above relations lead to a Taylor expansion of the travel time difference $\tau(m, x_n) \equiv T_n - T_0$ with respect to the ray coordinates $\eta_n$ and $\zeta_n$ (Figure 1) as

$$\tau(m; x_n) \approx p_0l(\phi_0, \Delta_0; x_n) + \frac{1}{2}\frac{dp_0}{d\Delta}l^2(\phi_0, \Delta_0; x_n).$$  \hspace{1cm} (6)
Since the first order term of $p_0 l$ is given by $\tau(m, x_n) \approx -p_0 \eta_n$, represents the plane wave propagation. In contrast, the second-order term of $p_0 l$ is $p_0 C_n^2 / (2 \tan \Delta_0)$, which represents the curvature of the wavefront.

To infer the centroid location $x_c$, we calculated the following squared amplitude

$$E(m) \equiv \frac{1}{2\pi} \int s(m; \omega) s^*(m; \omega) d\omega,$$  \hspace{1cm} (7)

where $s(\omega, m)$ is the sum of all records of the arrays defined by

$$s(\omega; m) \equiv \frac{1}{N} \sum_{n=1}^{N} u_n(\omega) e^{i \omega \tau(m, x_n)}. \hspace{1cm} (8)$$

Because the optimal value of $m$ maximizes $E$ without constraint, we can infer each parameter of $m$ independently from the array data, which are composed of vertical displacement $u_n(\omega)$. On the other hand, a back-projection method can be interpreted as a maximization problem of $E$ with a constraint by a travel time $T_{\text{model}}$ based on a reference Earth’s model as

$$p_0 = \frac{dT_{\text{model}}}{d\Delta} \bigg|_{\Delta=\Delta_0}. \hspace{1cm} (9)$$

This result suggests that the data can constrain $\Delta_0$ by utilizing curvature information. Although centroids can be located by $\Delta_0$ and $\varphi_0$ for 1-D Earth in principle, the lateral heterogeneities make $\Delta_0$ inaccurate, as discussed later (see Section 4.2 for details). For this reason, we used $\Delta_0$ only for phase associations between the P and PP waves. To improve the accuracy of the centroid locations, $p_0$ and $\Delta_0$ are corrected based on a global 3-D P-wave velocity structure. After the corrections, we locate the centroids from $p$ with a 1-D Earth model according to the phases.

Thus, the auto-focusing method infers $m$ by minimizing $E$ in three steps: (1) initial guess by grid search, (2) updates of $m$ by Newton’s iterative method, and (3) source locations with phase association after corrections of a global 3-D P-wave velocity structure. This study applied the auto-focusing method for vertical seismograms at approximately 780 Hi-net stations in Japan, as shown in later sections. To explain these steps, we present a synthetic example for Hi-net. The details of the instrumentation will be described in the Data section.
Figure 2. Beamforming analysis at 0.15 Hz for a synthetic data by ray theory. Here, we calculated travel times at all the Hi-net stations. For a 1-D model (AK135, Kennett et al., 1995), using TauP toolkit (Crotwell et al., 1999). The source was located at 170°E, 70°N, and the frequency is approximately 0.15 Hz.

2.1 Initial guess by grid search

We estimate the initial guess of the model parameters $p_0$, $\varphi_0$, $\Delta_0$, and $dp_0/d\Delta$ using the grid search method in the following three steps. To explain the grid search, we present a synthetic example for a point source at 170°E and 70°N in this subsection.

First, we measure the slowness vector ($p$ and $\varphi$) using the conventional beamforming method (e.g., Rost & Thomas, 2009) with the assumption of a plane wave incidence, which is equivalent to $\Delta = 90^\circ$ and $dp/d\Delta = 0$ in equations 4 and 6. Figure 2 shows a plain wave with a slowness of approximately 9 s/deg traveled from the northeast. We measure $p_0$ and $\varphi_0$ by looking for the maximum.

Next, we infer the angular distance $\Delta_0$ by maximizing $E$ with fixed $p_0$ and $\varphi_0$, as shown in Figure 3(a). Because we also assumed that $dp/d\Delta = 0$ in this procedure, Figure 3(a) corresponds to a cross-section along the horizontal dotted line in Figure 3(b). Because the panel shows a simple peak, we can measure $\Delta_0$ using the pick. The precision of $\Delta_0$ becomes worse as $\Delta_0$ approaches 90° because the peak becomes flat owing to the smaller curvature of the wavefront.

Finally, we infer $dp_0/d\Delta$ by maximizing $E$ with the inferred $p_0$, $\varphi_0$, and $\Delta_0$, as shown in Figure 3(c). Figure 3(c) corresponds to a cross-section along the vertical dotted line.
Figure 3. (a) Beam power as a function of arc distance, which is the cross-section along the horizontal dotted line of the panel (b). (b) Color map of beam power \( E \) as a function of arc distance \( \Delta \) and \( dp/d\Delta \) with the initial guess of slowness \( p \). Panel (a) shows the cross section along the horizontal solid line, whereas the open circle shows the final estimation of Newton’s method, and the open star shows the true value. The contour lines in black indicate the beam power with the final estimation of slowness \( p \). The final estimation of \( dp_0/d\Delta \) and \( \Delta_0 \) maximizes the beam power, as shown by the black contours. This result shows that the initial guess shown by the closed circle is close enough to the true value for Newton’s method. (c) Beam power as a function of \( dp/d\Delta \), which is the cross-section along the vertical dotted line in panel (b).

Figure 3(b) shows the color map of \( E \) as a function of \( \Delta \) and \( dp/d\Delta \). The black dot shows the measurement, which is located near the peak of the color map. The ray theoretical value is apart from the measurements. The difference clarifies the model parameter \( m \) should be improved by the next step of the auto-focusing method.
2.2 Iterative updates of model parameters by Newton’s method

From the estimated initial guess \( m^0 \), the model parameters \( m^q \) are updated iteratively as

\[
m^{q+1} = m^q + \Delta m^q, q = 0, 1, \cdots ,
\]

where \( q \) is the number of iterations and \( \Delta m^q \) is the \( q \)th update. To employ Newton’s method for the iteration, we expand the Taylor series of the mean square amplitude \( E(m^{q+1}) \) with respect to \( \Delta m^q \) up to the second order as

\[
E(m^{q+1}) = E(m^q) + D^q \Delta m^q + \frac{1}{2}(\Delta m^q)^\top H^q \Delta m^q + O((\Delta m^q)^3),
\]

where \( \top \) represents the transpose, \( D^q \) is the gradient row vector of \( E(m^q) \), and \( H^q \) is the Hessian matrix of \( E(m^q) \) defined by

\[
D^q \equiv \frac{\partial E(m^q)}{\partial m_j} = 2\Re \left[ \int \frac{\partial s(m^q; \omega)}{\partial m_j} s^*(m^q; \omega) d\omega \right],
\]

\[
H^q \equiv \frac{\partial^2 E(m^q)}{\partial m_j \partial m_k} = 2\Re \left[ \int \left( \frac{\partial s(m^q; \omega)}{\partial m_j} \frac{\partial s^*(m^q; \omega)}{\partial m_k} + \frac{\partial^2 s(m^q; \omega)}{\partial m_j \partial m_k} \right) s^*(m^q; \omega) d\omega \right],
\]

where \( \Re \) represents the real part, and the gradient and Hessian matrix of displacement \( s(m^q; \omega) \) are given by

\[
\frac{\partial s(m^q; \omega)}{\partial m_j} = \sum_{n=1}^{N} \eta_n \frac{\partial \tau(m^q; \eta_n, \zeta_n)}{\partial m_j} u_n(\omega) e^{i\omega \tau(m^q; \eta_n, \zeta_n)}
\]

\[
\frac{\partial^2 s(m^q; \omega)}{\partial m_j \partial m_k} = \sum_{n=1}^{N} \eta_n \frac{\partial^2 \tau(m^q; \eta_n, \zeta_n)}{\partial m_j \partial m_k} u_n(\omega) e^{i\omega \tau(m^q; \eta_n, \zeta_n)}
\]

\[
- \sum_{n=1}^{N} \omega^2 \eta_n \frac{\partial \tau(m^q; \eta_n, \zeta_n)}{\partial m_j} \frac{\partial \tau(m^q; \eta_n, \zeta_n)}{\partial m_k} u_n(\omega) e^{i\omega \tau(m^q; \eta_n, \zeta_n)}.
\]

The gradient vector of \( \tau(m) \) and the Hessian matrix of \( \tau(m) \) are given in Appendix A explicitly.

Because the Hessian matrix \( H^q \) is a Hermitian matrix, it is unitary diagonalizable as

\[
H^q = (Q^q)^\top \Lambda^q Q^q,
\]

where \( \Lambda^q \) is the diagonal eigenmatrix and \( Q^q \) is the unitary matrix. Equation 11 can be rewritten as

\[
E(\mu^{q+1}) - E(\mu^q) = G^q \Delta \mu^q + \frac{1}{2}(\Delta \mu^q)^\top \Lambda^q \Delta \mu^q = \sum_j G^q_j \Delta \mu_j^q + \frac{1}{2} \Lambda^q_j (\Delta \mu_j^q)^2,
\]

where \( \Delta \mu^q \equiv Q^q \Delta m^q \) and \( G^q \equiv D^q(Q^q)^\top \).
According to the convexity of $E(m^q)$, we choose the update method of the model parameter $\Delta \mu_j^q$ from Newton’s method and the steepest descent method. If $\lambda_j^q$ is negative, $\Delta S$ is convex upwards. In this case, we estimate $\Delta \mu_j^q$ by a Newton’s method as,

$$\Delta \mu_j^q = -\frac{G_j^q}{\lambda_j^q}. \tag{18}$$

If $\lambda_j^q$ is positive, $\lambda_j^q$ is convex downwards. In this case, we switched to the steepest descent method. To determine the step length, we compared the second-order term with the first-order term. If the ratio is smaller than $1/4$ as

$$\frac{1}{2} \lambda_j^q (\Delta \mu_j^q)^2 < \frac{1}{4}, \tag{19}$$

the first-order term is dominant in this range. Accordingly, we choose the following step length for the steepest descent method:

$$\Delta \mu_j^q = \frac{G_j^q}{2\lambda_j^q}. \tag{20}$$

We update the model parameters iteratively until the relative error $\Delta E/E$ becomes smaller than $10^{-9}$. Figure 4 presents a synthetic example of the iteration. Typically, after four or five iterations, the estimation converges. If an event does not meet the criterion, we reject the event. The contour lines in in Figure 3(b) shows $E$ with the final estimation of $p$. The update of $p$ sharpened the peak. The final estimation of $\Delta_0$ and $dp_0/d\Delta$ shown by the open circle approaches the ray theoretical values. The estimated model parameters still have errors (the difference between the open circle and the star symbol in Figure 3(b)) even for the synthetic test with a 1-D structure, because we truncated the Taylor series (e.g., $dp/d\Delta$ is not constant, even for a 1-D structure).

Under a realistic situation, the 3-D structure of the Earth causes a large bias in centroid locations (see Section 4 for details). To improve the accuracy of the centroid locations, we corrected the effects for a 3-D Earth structure, as shown in the next section.

2.3 Source locations with phase association after corrections for a 3-D structure

If the estimated $\Delta_0$ inferred from the curvature of the wavefront is sufficiently accurate, the centroid can be located with $\varphi_0$. However, practically, $\Delta_0$ is inaccurate because it is too sensitive to the lateral heterogeneities of the P-wave velocity structure (see Section 4 for details). For this reason, we used $\Delta_0$ only for the phase association (see Section 5 for actual phase associations). After the phase associations (e.g., P and PP), we
Figure 4. (a) Iterations of Newton’s method. The blue star symbols (⋆) show the beam power normalized by that of the conventional beamforming method (Figure 2). The red cross symbols (×) show perturbations of the back-azimuth ϕ₀ to the true value. The black dots show the perturbations of slowness (p₀) to the true value. (b) The red star symbols indicate perturbations of dp₀/dΔ to the true value. The blue dots show the perturbations of Δ₀ to the true value. (c) Estimated covariance matrix between the parameters for the synthetic data with Σₛ = 10⁻²E(mₑₚ). p₀ and ϕ₀ are normalized to the final value, whereas Δ₀ and dp₀/dΔ are normalized by 20 times the final value for the display. The standard deviation of p₀ is 0.57 %, that of ϕ₀ is 0.66 %, that of Δ₀ is 12 % and that of dp₀/dΔ is 24 %.

located the source from p₀ and ϕ₀, because slowness p₀ is less sensitive to lateral heterogeneities. Measurements of Δ₀ also improve the accuracy of the slowness measurements (p₀ and ϕ₀).

3 Corrections for 3-D structure

The location errors become significant when applying the auto-focusing method to real data because the lateral heterogeneities on a large scale bias the estimation. In particular, ray bending during P-wave propagation in a subducting slab produces large (> 2 s) travel time anomalies (Kendall & Thomson, 1993). Although a full 3-D calculation can suppress the bias with an appropriate 3-D model, the numerical calculations still cost a lot. Alternatively, we propose a simple method for correcting 3-D structure by 3-D ray tracing.

First, we assumed a vertical point source at surface location xₑ. We calculated the travel time Tₘ₃D at the nth Hi-net station with the given source using 3-D ray tracing.
Figure 5. (a) Corrections of slowness $p_x$ for P, PKP, and PKIKP waves. The correction becomes larger on the outside ($|p| > 9$), which is refracted in the upper mantle. (b) Corrections of slowness $p_y$ for P, PKP, and PKIKP waves. (c) Corrections of $\Delta$ for P, PKP, and PKIKP waves. The correction for P-waves from the southwestern direction becomes larger because the Pacific plate bends the ray path (Kendall & Thomson, 1993).

LLNL (Simmons et al., 2012; Fang et al., 2020). Assuming that the amplitudes are constant within the receiver array, we synthesize the record $u_n^{syn}(\omega)$ as

$$u_n^{syn}(\omega) = e^{-i\omega T_n^{3D}}.$$  \hfill (21)

To infer $m^{3D}$, we applied the auto-focusing method to the synthetic data. We also calculated $m^{1D}$ with the 1-D structure, which corresponds to the real data, by taup toolkit (Crotwell et al., 1999). The correction terms are evaluated by $m^{3D} - m^{1D}$. Figure 5 (a) and (b) show the corrections of slowness vectors $p$ for P, PKP, and PKIKP. When the slowness is larger than 9 s/deg, the corrections for slowness become larger. This slowness range corresponds to P-wave, which is sensitive to the upper mantle structure. The major origin of the large bias is the subducting Pacific plate beneath Japan (Kendall & Thomson, 1993). Figure 5 (c) shows corrections of $\Delta$. The larger anomalies traveled from the southwestern direction correspond to the source in Indonesia. Most of the ray paths from Indonesia to Japan travel in the Pacific slab. The defocusing effect due to the slab causes large anomalies (Kendall & Thomson, 1993). Because the curvature of the wavefront is more sensitive to lateral heterogeneities (Woodhouse & Wong, 1986), the correction of
Δ becomes larger. We did not correct \( dp_0/d\Delta \) because we did not use the information for the centroid locations in the 3rd step.

Figure 6. (a) Corrections of slowness \( p_x \) for PP waves. (b) Corrections of slowness \( p_y \) for PP waves. (c) Corrections of \( \Delta \) PP waves. Correction for P-wave from the southwester The correction for PP-waves from the southwestern direction becomes larger because the Pacific plate bends the ray path (Kendall & Thomson, 1993).

Figure 6 shows the corrections of \( p \) and \( \Delta \) for PP waves. We do not measure PP waves after the triplication of approximately 660 km (i.e., the slowness is smaller than approximately 8.87 s/deg) because the slab near the array complicates the PP wave propagations too much. All the corrections are larger than those for the P waves (Figure 6) because of the longer propagation distances. Because the travel time anomalies of the PP waves are larger than those of the P waves because of the longer paths, the measurements of PP waves are generally less precise.

4 Error estimations of model parameters

The errors of the estimated model parameters can be categorized into two categories (e.g., Billings et al., 1994; Thurber, 1992): (i) precision: how close estimations of the same event are to each other, and (ii) accuracy: how close estimations are to the true values.

The origin of precision is measurement error due to observation noise. Assuming that the estimation errors are subjected to a Gaussian distribution, the standard devi-
ation can evaluate the precision of the estimated model parameter. The bootstrap method (e.g., Efron & Tibshirani, 1994) is feasible for estimating the precision.

The accuracy represents the bias between the estimated and true values owing to the uncertainty of the modeling. Although we corrected the effects of the 3-D structure of the Earth in this study, the unmodeled lateral heterogeneity of the Earth caused the bias. Because nobody knows the true centroid locations of P-wave microseisms, evaluating the accuracy requires a well-established catalog as a reference. To evaluate the accuracy, we used the inferred centroid moment tensor (CMT) catalog of large earthquakes (Dziewonski et al., 1981; Ekström et al., 2012) as a reference. Because the CMT catalog used global seismic data, they suffered from fewer effects of the 3-D structure than a catalog constructed by a local array.

**4.1 Precision of centroid locations**

First, we estimate the precision using the covariance matrix of the model parameter $m^q$ from the Hessian matrix $H^q$ and the error of $E(m^q)$. If the estimation of the variance of $m^q$ is subjected to a Gaussian distribution, the covariance matrix $\Sigma^q$ is estimated by

$$
\Sigma^q = - \left( \frac{H^q}{\Sigma_s} \right)^{-1},
$$

(22)

where $\Sigma_s$ is the variance $E(m^q)$ estimated by 100 bootstrap resamplings of seismic stations (see Appendix B for details). Figure 4(c) shows a synthetic example of the covariance matrix normalized by the final value. Since the slowness vector is well-constrained, we only use the information on the slowness vector to locate the centroids in the final step. Although $\Delta_0$ and $d\phi_0/d\Delta$ have larger variances than $p_0$ and $\phi_0$, they can be constrained. The typical precision of location errors is approximately 100 km for large events of P-wave microseisms. The details of observed P-wave microseisms are evaluated in Section 5.

**4.2 Accuracy evaluated by large earthquakes**

To evaluate the accuracy, we compared the centroid locations of large earthquakes estimated by the auto-focusing method with those in a CMT catalog. We analyzed vertical components of Hi-net data from about 0.1 ～0.2 Hz, which include earthquakes with moments larger than $10^{18}$ Nm from 1997 to 2019 listed in the Global CMT catalog (Dziewonski...
et al., 1981; Ekström et al., 2012). We choose segments with a mean squared amplitude of P-wave 500 times larger than the noise level (see Section 5 for details). We estimated the parameter \( \overline{m} \) by applying the auto-focusing method with and without the 3-D corrections for the Hi-net data, which are exactly the same seismic data set for the P-wave microseisms (see section 5 for details). To estimate the accuracy, first we evaluated the differences between \( \overline{m}^{1D} \) and \( m^{\text{cat}} \), where \( \overline{m}^{1D} \) is the estimated model parameter without 3-D corrections, and \( m^{\text{cat}} \) is the 3-D ray theoretical value with centroid locations of the CMT catalog.

First let us compare the estimated slowness with that of the CMT catalog. The errors without 3-D correction \( (\delta x^{1D}, \delta y^{1D}) \) are estimated by

\[
\delta x^{1D} = \frac{p_x^{1D} - p_x^{\text{cat}}}{dp^{\text{cat}}_x/d\Delta}, \\
\delta y^{1D} = \frac{p_y^{1D} - p_y^{\text{cat}}}{dp^{\text{cat}}_y/d\Delta}.
\] (23)

Because the slowness measurement at one array cannot constrain the source depth, the source depth is fixed as the catalog value. The histograms of Figure 7 in red show the number density of the differences without 3-D corrections. They show that the 3-D structure without corrections biases the centroid locations up to 600 km.

To validate the effects of 3D corrections, we next evaluated the differences between \( m^{3D} \) and \( m^{\text{cat}} \), where the superscript 3D represents the estimation with 3D corrections. The longitude/latitude errors \( (\delta x^{3D}, \delta y^{3D}) \) were estimated as follows:

\[
\delta x^{3D} = \frac{p_x^{3D} - p_x^{\text{cat}}}{dp^{\text{cat}}_x/d\Delta}, \\
\delta y^{3D} = \frac{p_y^{3D} - p_y^{\text{cat}}}{dp^{\text{cat}}_y/d\Delta}.
\] (24)

The histograms of Figure 7 in blue show the number density of the differences. Because the 3-D corrections reduce the biases, the peaks of the distribution with the corrections shift around 0 km. Because the standard deviations are approximately 150 km, the accuracy is on the order of 1° after the corrections.

Let us discuss the accuracy of \( \Delta \) measured from the wavefront curvature. Figure 8(a) shows a comparison of anomalies of \( \Delta \) from the corresponding catalog value \( \Delta^{\text{cat}} \). The red triangle shows anomalies of \( \Delta^{1D} \) estimated from Hi-net data without 3D corrections, whereas the black circles show anomalies of \( \Delta^{\text{syn}} \) estimated from synthetic data based on 3-D ray tracing, which correspond to 3-D correction terms. The significant anomaly
Figure 7. (a) Number density of errors in longitude of centroid locations of the earthquakes, which represents the accuracy. The histogram in blue shows the number density of the differences between the estimation with 3-D corrections and the catalog values, whereas the histogram in red shows the number density without 3-D correction. (b) Number density of errors at the latitudes of centroid locations of the earthquakes. We chose high signal-to-noise ratio (SNR) events with a mean squared amplitude of P-wave 500 times larger than the noise level (see section for details).

from the back-azimuth of approximately $-140^\circ$ was caused by the defocusing effect of the Pacific plate. Figure 8(b) shows the number density of the anomalies of $\Delta^{3D}$ with the 3-D corrections (red), which show the accuracy of the estimations. Although the 3D model explains the measurement well (Figure 7(a)), the accuracy of $\Delta^{3D}$ ($\sim 10^\circ$) is larger than that estimated from the slowness vector (Figure 7). The measurements of $\Delta$ are more sensitive to lateral heterogeneities (e.g., Woodhouse & Wong, 1986) because they can be directly related to the curvature of the wavefront. To improve the accuracy and precision, we estimated the centroid locations from the slowness vector. Although $\Delta$ has larger errors than slowness $p$, it is helpful for the phase association (e.g., P and PP waves), as already explained.

5 Data analysis

We analyzed the seismograms of the high-sensitivity seismograph network (Hinet) in Japan, which is operated by the National Research Institute for Earth Science and Disaster Prevention (NIED) (Okada et al., 2004). Three-component velocity seismometers were installed at the bottom of boreholes. In this study, we used approximately 780
Figure 8. (a) Comparison of anomalies of measured $\Delta^{1D}$ with those of the corresponding 3-D prediction $\Delta^{syn}$. Red triangles show the estimated $\Delta^{1D} - \Delta^{cat}$ of the earthquakes without 3-D corrections. The black dots indicate the estimated $\Delta^{synth} - \Delta^{cat}$ for synthetic data based on the 3-D ray tracing, which represent the 3-D corrections. (b) Number density of anomalies of $\Delta^{3D}$ with the 3-D corrections (red), and those without the corrections $\Delta^{3D}$ (blue).

vertical components of Hi-net velocity seismometers from 2004 to 2020 (Figure 9). Because the natural frequency of the sensors is 1 Hz, the instrument response was deconvolved using a time-domain filter (Maeda et al., 2011). To subtract the common instrumental noise of the data logger (Takagi et al., 2015) below a single bit, we subtracted the template of the common data-logger noise. All the one-day records were divided into 1024 s segments with an overlap of 414 s to lay out the segments into 1-day data. Each segment was Fourier-transformed using the FFT algorithm. For analyzing microseisms, we discarded segments contaminated by large earthquakes or transient phenomena using two criteria.

First, we discarded segments that included earthquakes with moment magnitudes (Ekström et al., 2012) larger than 6. Assuming that the amplitude decays exponentially with time as $M \exp(-2\pi tf/2Q)$, where $f$ is the dominant frequency (approximately 0.1 Hz), $Q$ is the typical attenuation value of the P-wave (500), and $M$ is the moment. We define the typical duration $DT$ as:

$$DT = 2Q \frac{\ln(JM/10^{23})}{(2\pi f)}, \quad (25)$$
Figure 9. Station distribution. Black dots show stations of the high-sensitivity seismograph network (Hinet) in Japan, which is operated by the National Research Institute for Earth Science and Disaster Prevention (Okada et al., 2004).

where \( J \) represents a typical geometrical spreading given by

\[
J \approx \frac{\sin(60^\circ)}{\sin(\max(\Delta, 10^\circ))},
\]

(26)

We reject the data with duration \( DT \) after the arrival time. Figure 10 shows a typical example of the data selection. The orange dots represent the rejected segments based on this criterion for large earthquakes.

Next, we reject the segments contaminated by transients. To define the typical mean squared amplitude at the \( i \)th time-step, we calculate the median of the mean squared amplitude \( P_i \) from 0.1~0.25 Hz among all the stations at the \( i \)th time-step. Here, we consider that the \( i \)th time step is accepted. If \( P_{i+1} \) changes suddenly, we reject the \((i+1)\)th time-step. If the mean squared amplitude \( P_{i+1} \) meets the criterion:

\[
|\ln(P_{i+1}) - \ln(P_i)| > \ln(1.12),
\]

(27)
we reject all segments at time step $i+1$. Once we rejected a time step, we increased the rejection criterion with duration as follows: After we reject $n$ successive time-steps from the $i$th time-step, we evaluate the rejection by the following criterion:

$$|\ln(P_{i+n+1}) - \ln(P_i)| > n \ln(1.12),$$

(28)

where $P_i$ shows the mean squared amplitude of the previous accepted segment. We also reject segments with power normalized by NLNM (Peterson, 1993) larger than $4 \times 10^6$ (0.04-0.1 Hz) or $4 \times 10^5$ (0.1-0.2, 0.2-0.25 Hz). The dots in sky-blue represent rejected segments due to transient phenomena (Figure 10). The black dots in the figure show the segments analyzed in a later section.

**Figure 10.** Example of the data selection. The orange dots show data excluded using the global CMT catalog (Ekström et al., 2012), and the blue dots show data transients when the amplitude changes suddenly. The black dots represent selected segments. The vertical axis shows the relative power normalized by the Peterson NLNM (Peterson, 1993), which represents the lowest ground noise on Earth.

In the first step of the auto-focusing method, we conducted beamforming analysis averaged over a frequency range from 0.1 to 0.25 Hz for each selected segment. We stacked the beamforming results over every 46 segments (approximately 6 h). See (Nishida et al., 2005) for details of the beamforming analysis for stationary signals (e.g., the definition of the power spectral density). We selected local maxima (e.g., Figure 2), which is five times larger than the noise level of each beamforming result. Here, we define the
noise level by the median absolute deviation (MAD) of the power spectral density (PSD) of the beamforming result. The measured slowness values are the initial values of the auto-focusing method. We measured the angular distance $\Delta$ and $dp/d\Delta$ for each initial value using a grid search, as shown in Figure 3.

In the second step of the auto-focusing method, we infer $p$, $\theta$, $\Delta$, and $dp/d\Delta$ using Newton’s method. If the convex upward was broken or the iteration did not converge, the estimated value was rejected. Figure 11(a) shows the resultant events against the angular distance $\Delta$ and slowness $p$ without 3D corrections. Because the number of events is greater than $10^4$, we plot the probability density. For each slowness, the maximum probability was normalized to 1 for the display. This figure shows two P-wave branches (black lines). A branch can be explained by the synthetic line (the black solid line) based on the 1-D model (AK135, Kennett et al., 1995), whereas the other one (the black dashed line) significantly deviates from the synthetic line. The anomalous branch at approximately $\Delta = 100^\circ$ can be explained by the de-focusing effect of the Pacific plate, as shown in Figure 8. It also shows an ambiguous PP-wave branch below approximately 8 s/deg, although the lateral heterogeneities caused large anomalies in $\Delta$. This figure also shows clear core phases: PKP and PKIKP. The angular distance $\Delta$ is also biased by the large-scale P-wave velocity structure. Figure 12 shows the probability density as a function of $p$ and $dp/d\Delta$. The line in sky-blue shows the synthetic lines based on (AK135, Kennett et al., 1995), which can explain most of the observation. Because $dp/d\Delta$ is sensitive to lateral heterogeneity, it is difficult to constrain $dp/d\Delta$ by the observation.

In the third step of the auto-focusing method, we located the centroids from the estimated slowness vector $p$ after the 3D corrections. Figure 11 (b) shows the probability density in $\Delta-p$ domain after the corrections. Before the corrections shown by Figure 11 (a), the P-wave branch was split into two. Many events are outside the reference 1-D model. After the corrections, the two branches merged into one branch, which was consistent with the 1-D model. We can also identify the clear PP-wave branch in Figure 11 (b). The $\Delta$ and $p$ of the core phases is also improved by the corrections in this figure. With the phase associations between the P and PP waves in the $\Delta-p$ domain, we infer the centroid locations from the corrected $p$. The resultant centroid locations will be shown in the next section.
For the located centroids, we estimated the root mean square amplitude of the CSF. PSD of beam power \( \langle s \rangle \) can be represented by

\[
\langle s(\omega; m)s^*(\omega; m) \rangle = |G(x_c, x_0, \omega)|^2 P(\omega),
\]

where \( G(x_c, x_0, \omega) \) is the amplitude term of the ray theory Green’s function from the source at \( x_c \) to the center of the array \( x_0 \), and \( P \) is the PSD of the CSF. We also consider the source-side amplification of the Green’s function due to water reverberations (Gualtieri et al., 2014). We infer the mean squared amplitude of the centroid force as:

\[
\sqrt{\frac{1}{2\pi} \int_{2\pi f_1}^{2\pi f_2} P(\omega) d\omega} = \sqrt{\frac{1}{2\pi} \int_{2\pi f_1}^{2\pi f_2} \langle s(\omega; m)s^*(\omega; m) \rangle |G(x_c, x_0, \omega)|^2 d\omega},
\]

where \( f_1 = 0.1 \) Hz, and \( f_2 = 0.25 \) Hz. We also measured the maximum frequency \( f_{\text{max}} \)

\[
f_{\text{max}} = \frac{\arg \max \omega P(\omega)}{2\pi}.
\]

Although we estimated the single force at the centroid location represented by a monopole source, the source can be characterized by the sum of multipoles for distributed sources. If a global dense dataset is available, we can resolve all the amplitudes of the multipoles independently. However, because the array coverage is limited, we cannot distinguish the differences among the multipoles. For this reason, we assumed an apparent monopole source in an exact sense. In theory, the squared amplitude of the apparent monopole source can be represented by the sum of the squared amplitudes of the multipoles.

To verify the efficiency of 3D corrections, we compared the epicentral distance \( \Delta_p \) inferred from slowness \( p \) with \( \Delta_c \) inferred from the curvature of the wavefront. Figure 13 shows that the probability density with and without 3D corrections as a function of the difference between \( \Delta_p \) and \( \Delta_c \). Figure 13 (a) shows probability density with 3D corrections (blue) and that without 3D corrections (orange). Both the probability density without 3D corrections in the orange color show two peaks. The larger peaks are caused by defocusing effects due to the Pacific plate. After the corrections, the two peaks in both cases merge into one, shown by blue color. The standard deviation for P-wave events with the corrections is significantly smaller than that without the corrections. On the other hand, Figure 13(b) shows that the corrections reduce the standard deviation for PP-wave events slightly because the wave propagation becomes complex because of the lateral het-
Figure 11. (a) Probability density of events as a function of $\Delta_0$ and $p_0$ without correction for the 3D structure. The probability density was normalized to the maximum value for each slowness value. The solid lines represent the ray theoretical values for (AK135 Kennett et al., 1995). The black dashed line is a P-wave branch deviated from the theoretical values. We selected events with $\lambda_1$ values smaller than 500 km. (b) Probability density of selected events with 3D corrections. The panel shows that the correction improves the estimations of $\Delta$, which is more consistent with the ray theoretical values.

erogeneities. The modifications of the distribution suggest the 3D corrections still improve the accuracy of PP-wave events.

To check the precision of the centroid locations, we calculated the estimation errors by the broadness of $E(m)$, which was evaluated by the Hessian matrix (equation 22) and the standard deviation of $E(m)$ by a bootstrap sampling of 100 times (see Appendix B for details). First, we plot the semi-major length of the error ellipse $\lambda_1$ as a function of the epicentral distance $\Delta$. Because the errors strongly depend on the SNRs, the color shows the ratios between the peak amplitude of $E$ and the MAD. The figure shows that events with high SNRs had smaller errors. We note that the P-wave events with epicentral distances between 90° and 100° have a larger $\lambda_1$ because $dp_0/d\Delta$ is significantly smaller than other distance ranges of the P-wave (see the ray-theoretical value in Figure 11). PP waves cover events with epicentral distances of approximately 100–150°. The worse SNRs and the smaller $dp_0/d\Delta$ cause larger errors of PP-wave events. Core
Figure 12. Probability density as a function of \( p_0 \) and \( dp_0/d\Delta \). The probability density was normalized by the maximum value for each slowness value.

Figure 13. (a) Probability density of \( \Delta_c - \Delta_p \) without the 3D corrections for P wave (red) and that with the corrections (blue), where \( \Delta_p \) is epicentral distance inferred from slowness, and \( \Delta_c \) is that from curvature of the wavefront. (b) Probability density of \( \Delta_c - \Delta_p \) without corrections for the PP wave (red) and those with the corrections (blue).

phases cover events with epicentral distances of approximately 150-180°. The SNRs are again high because of the antipodal focusing.

To clarify the relationship between the precision and the SNR, Figure 15 plots events against SNRs and the estimated errors (semi-major and semi-minor length of the error ellipse \( \lambda_1 \) and \( \lambda_2 \)). For all phases (P, PP, and core phases), the plots show similar behaviors: the errors are proportional to \( \sqrt{MAD/\text{MAX}} \), where \( MAD \) is the MAD of the beam power, and \( MAX \) is the corresponding peak value of the beam. \( \lambda_1 \) is approximately –23–
Figure 14. The semi-major length of the error ellipse $\lambda_1$ is a function of the arc distance. The color represents the ratio of the peak power $MAX$ to the $MAD$. Events with a good SNR (i.e., larger $MAX/MAD$) have smaller estimation errors.

three times larger than $\lambda_2$. This ratio can also be explained by the shape of the array response functions (Rost & Thomas, 2009). This figure also shows that the precision of events with high SNRs ($\sqrt{MAD/MAX} < 0.1$) is smaller than approximately 100 km.

6 A centroid single force catalog

Figure 16(a) shows resultant centroid locations with slowness. They are located in the north Pacific ocean, the northern North Atlantic ocean, Tasman sea, the southern south Atlantic, and the southern ocean. These features are consistent with past studies (e.g., Landès et al., 2010; Euler et al., 2014; Wang et al., 2018). The figure shows significant activities at low latitudes, although the number of the detected events is small. The events tend to be located along depth contours of the bathymetry because of the ocean acoustic resonance (Euler et al., 2014; Gualtieri et al., 2014), which will be discussed later.

Within the epicentral distance of approximately $90^\circ$, most of the events were detected as P-waves. Behind the shadow zone of the P wave, the figure also shows events detected as PP waves (see the slowness in Figure 16(a)). For example, in the northern North Atlantic Ocean, red dots were detected as P-waves. Behind the shadow zone (south
Figure 15. (a) Number density of semi-major length of the error ellipse $\lambda_1$ for P waves. The vertical axis shows the SNR characterized by $\sqrt{MAD/MAX}$, where $MAD$ is the MAD of the beam power, and $MAX$ is the corresponding peak value of the beam. (b) Number density of the semi-major length of the error ellipse $\lambda_1$ for PP waves. (c) Number density of the semi-major length of the error ellipse $\lambda_1$ for P, PKP, and PKIKP waves. (d) Number density of semi-minor length of the error ellipse $\lambda_2$ for P waves. (e) Number density of the semi-minor length of the error ellipse $\lambda_2$ for PP waves. (f) Number density of the semi-minor length of the error ellipse $\lambda_2$ for P, PKP, and PKIKP waves.

of the P events), the figure shows fewer PP events (orange dots). The figure shows a similar overlap between P and PP events south to Australia. Figure 17 is helpful to identify the overlap between P and PP-wave events since the figure shows them separately. Near the antipodes of the array, the figure shows core-phase events (black dots in Figure 16(a)). Amplification due to the focusing effect enhanced the detection of events. The figure also shows an overlap between PP events (orange dots) and core-phase events in an area south of Madagascar.
Figure 16. (a) Centroid locations of P-wave microseisms with slowness. The color shows the slowness of P waves (P, PP, PKP, and PKIKP in a precise sense). The color of the map shows bathymetry and topography. We selected events according to the semi-major length of the error ellipse $\lambda_1$ smaller than 1000 km. (b) Centroid locations of P-wave microseisms with semi-major lengths of the error ellipse $\lambda_1$. Events with smaller $\lambda_1$ are plotted on the top. This figure exhibits the best $\lambda_1$ for each area. $\lambda_1$ of the PP waves were larger than those of the other phases. In particular, centroids in the southeastern region had larger estimation errors.

Figure 16(b) shows semi-major length of the error ellipse $\lambda_1$. Because we plot events with smaller errors on top, they show the minimum errors in each region. In most regions, they were smaller than 200 km. PP-wave events in the southeastern Pacific have larger errors than 500 km, whereas PP-wave events in the Indian Ocean (approximately $40^\circ$S, $30^\circ$SE) have smaller errors of approximately 200 km. The worse precision of events located perpendicular to the Japanese arc is due to the poor resolution in slowness domain, which reflects the array response functions (e.g., Rost & Thomas, 2009).

Figure 17 shows the angle between the semi-major axis and the great circle path between the centroid and the array. The color pattern in Figure 17 reflects the array response functions. With decreasing $|dp/d\Delta|$, the error ellipses are elongated along the great circle path from the source to the array. For this reason, the semi-major axes of the PP wave are smaller than those of the P-waves, as shown in Figure 17(b).
Figure 17. (a) Angles of semi-major axes $\varphi_e$ for P-wave events. The angle $\varphi_e$ is measured from the great circle path between the centroid and the array counterclockwise. The definitions of $\lambda_1$, $\lambda_2$, and $\varphi_e$ are illustrated in the figure. (b) Angles of semi-major axes $\varphi$ for PP-wave events. The smaller angles of the PP-wave show that the error ellipses are elongated along the great circle paths.

The map in Figure 18 shows the amplitudes of the estimated single forces at the centroid locations. The large events were located between 30° and 70° in the latitudes. In particular, the largest events with amplitudes larger than $10^{11}$ N were in the northern North Atlantic Ocean (area 1 on the map), and the CSFs are consistent with past studies (Nishida & Takagi, 2016). The figure also shows significant events with a CSF of approximately $10^{11}$ N in the Northern Pacific (area 2 on the map) and South Pacific (area 5 on the map). The lower detection limit of CSFs is approximately $10^{10}$ N because of the SNRs. Figure 18 (1) to (9) show the temporal variations in different areas: (1) Northern Atlantic, (2) Northern Pacific, (3) Indian ocean, (4) Southern Atlantic, (5) Southern Pacific, (6) the Southern Ocean, (7) Eastern Pacific, (8) Off Madagascar and (9) Gulf of Carpentaria Australia. They show clear seasonal variations. In the northern hemisphere, an estimated single force took maximums in the winter months (from Nov. to Feb.). On the other hand, in the southern hemisphere, the centroids were located only in the summer months (from May to Jul.). The activities in the southern hemisphere lacked in the
Figure 18. Centroid locations of P-wave microseisms with the CSFs. The color shows the CSFs. The nine boxes on the map show areas of significant activity. Panels (1) to (9) show temporal evaluations of the CSF with time. The red dots show the observed CSFs, whereas the blue dots show the modeled CSFs. We selected events according to the semi-major length of the error ellipse $\lambda_1$ smaller than 1000 km.
winter months. Since the lower limit of CSF is about $10^{10}$ N in most regions, the typical CSFs in winter months could be smaller than the detection limit.

Figure 19. (a) Probability density of P-wave centroids against the dominant frequency and the ocean depth below the centroids. The probability densities were normalized to the maximum for each frequency. To estimate the probability density, we conduct kernel density estimation using a Gaussian kernel with a bandwidth of 0.15 (the stats subpackage of SciPy Virtanen et al., 2020). The solid line shows the fundamental resonant frequency of the ocean acoustic wave, whereas the dotted line shows the first overtone. (b) Probability density of PP wave. (c) Probability density of core phases.

Figure 16 shows the lineation of centroids, which are located along depth contours. Nishida and Takagi (2016) reported that an acoustic resonance of the ocean explains centroids migration along iso-depth in the northern Atlantic ocean. When the dominance frequency of the P-wave microseisms matches the resonance frequency of the water layer, the P-wave microseisms are amplified by the ocean site effect (e.g., Gualtieri et al., 2014). To verify this hypothesis using the global catalog, we plotted centroids against the dominant frequency and water depth below the centroid locations (Figure 19). The solid line in the figure shows the fundamental acoustic mode (the frequency $f$ is given by $c_s/(4D)$) for vertically propagating P waves, and the dotted lines show the first overtone (the frequency $f$ is given by $3c_s/(4D)$), where $c_s$ is the sound speed of the ocean, and $D$ is the ocean depth. The acoustic resonance explains the dominant frequency well. Because the site amplification factor emphasizes the excitations that meet the condition of ocean acoustic resonance, the centroids tend to be located along a depth contour (e.g. Euler et al.,
2014; Gualtieri et al., 2014; Meschede et al., 2017). Figure 19 (c) shows many centroids fall outside of the lines because they cover the limited water depths near the antipodes of the array. When we discuss the source characteristics based on the dominant frequency, we must take care of the water resonance.

7 Comparison with a wave action model WAVEWATCH III

We have made a global CSF catalog for 17 years. This catalog is not consistent with past studies, but also enables a comparison with ocean physics data. Recent developments in numerical ocean wave models have enabled us to quantitatively compare seismic observations with the models (e.g., Ardhuin et al., 2011; Farra et al., 2016; Nishida & Takagi, 2016). Although the ocean wave model is feasible for discussing the activities of P-wave microseisms, the raw data are too complex to compare directly. For a systematic comparison between the CSF catalog with a numerical ocean wave mode for the entire time period, we calculated a CSF catalog based on WAVEWATCH III (Ardhuin et al., 2011). The PSD for teleseismic P-wave microseisms observed at a small-aperture array is formulated by Farra et al. (2016) as follows:

\[
B(\omega, p) = \int \int |G(p', f)|^2 4\pi^2 F_p(p', f) R(p - p', \omega) |J(p')| dp',
\]

where \(p\) is the P-wave slowness vector pointing from the array center, and \(B\) [m²/Hz] is the beam PSD at the slowness \(p\) and angular frequency \(\omega\), \(G(p', \omega)\) [m/N] is the amplitude term of the ray theoretical Green’s function in a 1-D medium, \(R(p, \omega)\) is the array response function, \(F_p(p, \omega)\) [Pa·m²/s] is the PSD of the random pressure field based on WAVEWATCH III (Ardhuin et al., 2011), and \(|J(p)|\) is the Jacobian. The convolution is done with the two-dimensional fast Fourier transform. The beam power \(E(p)\) is obtained by the frequency integral of the beam power spectral density as follows:

\[
E(p) = \frac{1}{2\pi} \int B(x_c, \omega) d\omega.
\]

The centroid of the P-wave source \(x_c\) is located by the slowness that maximizes \(E(p)\) as follows:

\[
p_{\text{max}} = \arg \max_p E(p).
\]

The centroid location \(x_x\) is inferred from \(p_{\text{max}}\) according to the phase. We consider the point-source representation of P-wave sources (Nishida & Takagi, 2016). The PSD of the effective point force that reproduces the beam PSD at the centroid \(x_\phi\) corresponding to
the slowness $p_{\text{max}}$ is given by

$$F_c(x_\phi, f) \equiv \frac{B(p_{\text{max}}, f)}{|G(p_{\text{max}}, f)|^2},$$  \hspace{0.5cm} (35)$$

where $F_c(x_\phi, f)$ [N$^2$ s] is the PSD of the effective point force. The root-mean-squared
amplitude of the point force $\phi(x_c)$ N is given by:

$$\phi(x_c) = \sqrt{\frac{1}{2\pi} \int F_c(x_c, f) d\omega}.$$  \hspace{0.5cm} (36)$$

For the calculations of CSFs for WAVEWATCH III, we calculated the within the epicentral distance of about 30° to avoid the complexity of the triplications.

Figure 18 also show temporal variations predicted by WAVEWATCH III by blue
dots. The model explains the observations well in most regions. Although the detection
limit of CSF amplitudes depends on the SNR, it is approximately $10^{10}$ N in all regions.
Because centroids in the eastern Pacific and off Madagascar were located by PP waves,
the differences between the observations and the model predictions were larger. In the
Gulf of Carpentaria, the model did not predict any events, as discussed later. Most of
the observed P-wave microseisms were observed only during local winter months. In the
northern Atlantic and the northern Pacific, the observations and predictions show cen-
troids even in the local summer months because the amplitude is still larger than $10^{10}$ N.

![Figure 20](image-url)  \hspace{0.5cm} \text{Figure 20.} \hspace{0.5cm} (a) Probability densities of observed centroid locations, and (b) those of WAVE-
WATCH III model. We use a cosine kernel with bandwidth=0.05 for the estimation.

Figure 20 shows a comparison between the probability density of observed centroid
locations and those of the models. Although they are consistent with each other in most
regions, differences exist in the Indian Ocean. In the southern part of the Indian Ocean,
the observations lack activities shown by WAVEWATCH III because the corresponding
PP-wave events are less accurate than the P-wave, as shown below. These observations
suggest that the detection capability of the PP-wave is lower than that of the P-wave
and core phases.

Figure 21. Probability densities of observed centroid locations, and those of WAVEWATCH
III. The upper four panels show the probability densities of the observations every three months
throughout the entire period, whereas the lower four panels show those of WAVEWATCH III. We
use a cosine kernel with bandwidth=0.07 for the estimation.

To compare the seasonal variations, we calculated the probability density every three
months throughout the entire period (Figure 21). Seasonal variations in the centroid lo-
cations based on the WAVEWATCH III model were consistent with the observations.
Activities in the northern Pacific and the northern Atlantic dominated from October to
March, whereas activities in the south Pacific and the South Atlantic dominated from
April to September. Centroids in the Indian Ocean were located from April to Septem-
ber. The observed centroids around 130°W near the equator from July to September are
shifted eastward to those of WAVEWATCH III, and they are isolated to centroids in the
northern Pacific. The westward migration may be a bias due to the lateral heterogeneities
because PP-waves suffer from ray bending by the Pacific plate (Kendall & Thomson, 1993)
(see details later).
Figure 22. (a) Month of maximum probability density of observed centroids. The color represents the month in which activities took the maximum. (b) Month of the maximum probability density of modeled centroids with WaveWatch III. We estimate the probability density by kernel density estimation on a sphere with a cosine kernel with bandwidth=0.1 using scikit-learn.

Figure 22 shows the month when the probability density took the maximum at each point. The map of the observations is consistent with the prediction by the WaveWatch III model. The activity took the maximum from November to February in the northern hemisphere, whereas it took from April to July. In the smaller scales (e.g., in the northern North Pacific) both results show similar spatial patterns consistent with each other.

There are two exceptions of the seasonality: (i) Tonga-Fiji and (ii) the Southern Ocean. In the Tonga-Fiji region (from 30°S to 0°S and approximately 180°E), both the observation and model predictions reached a maximum in January. Although the Southern Ocean near Antarctica is in the Southern Hemisphere, it also reached its maximum in January. In the Southern Ocean, the observations lack centroids because centroids inferred by the PP wave were smaller, as already discussed.

We note the significant activity of the observed P-wave microseisms in the Gulf of Carpentaria Australia, where the WaveWatch III model lacks. Figure 18 (9) shows the centroid locations with the CSF and temporal evaluations, which show clear seasonality. Although the activity in the Gulf of Carpentaria is significant, smaller events are also located along the coast of northern Australia (Figures 18 and 21). This lack may be attributed to the ambiguity of some physical parameters of WaveWatch III (e.g., coastal reflection). However, because the gulf generally experiences low (approximately
<1.5 m significant wave height) ocean swell activities (Porter-Smith et al., 2004), it is difficult to explain the observed excitations. Another possibility of excitations could originate from the shallow depth (around 50 m). When the estimation of CSFs is based on the Longuet-Higgins–Hasselmann theory (Longuet-Higgins, 1950; Hasselmann, 1963; Ardhuin et al., 2011), the theory assumes that the wavelength of the ocean swell is much shorter than the ocean depth. In this situation, pressure fluctuations cannot practically reach the ocean floor. However, in the Gulf of Carpentaria, pressure fluctuations of ocean swells can reach the ocean floor because of the shallow depth of around 50 m. A different nonlinear interaction of the ocean (e.g., tidal current) from the Longuet-Higgins–Hasselmann theory may be effective. This mechanism should be addressed in future studies. Another possible origin is enigmatic tremors, which do not originate from ocean swells. For example, some groups reported 26-s tremor activities in the Gulf of Guinea (Oliver, 1962; Shapiro et al., 2006; Xia et al., 2013) with maximum amplitudes during the southern hemisphere winter. Although the most probable mechanism is a volcanic process, the details remain unclear. Horologic tremors have also been reported in northeast Japan (Nishida & Shiomi, 2012). Because the sources were located at a fixed point in both cases, it was difficult to explain the observed centroids distributed in the Gulf of Carpentaria.

To compare the modeled centroid locations with the observation, we measured the separation distance. The separation distance of an observed event is defined as the shortest distance between the observed event and the modeled event every 6 h. Figure 23 shows the two-dimensional histogram of relative locations of the modeled events to the observed ones for P-wave, PP-wave and core-phase events. We can classify the P-wave and core-phase events with an accuracy of about 150 km. On the other hand, PP-wave events are more scattered, and the peak is slightly biased. The PP-wave propagation is more complex in general because it suffers more from strong lateral heterogeneity near the surface.

Here, we classify the same events when the separation distance is shorter than 500 km. Table 1 shows the number of the classified events and the others. About 30% of P-wave and core-phase events can be related to the modeled events. On the other hand, only about 10% of PP-wave events were classified because the lateral heterogeneities are too strong to model the data with our method.
Figure 23. (a) Probability density of separation distance of P-wave events between the observations and the WAVEWATCH III. The vertical axis shows the radial distance from the center of the array to the event. The transverse direction is perpendicular to the radial direction. (b) Probability density of separation distance of PP-wave events. (c) Probability density of the separation distance of core phase events. \( \lambda_1 < 500 \text{ [km]} \).

Table 1. The number of detected events. We counted the number with slowness smaller than 8.87 s/deg from 2004/4/1 to 2019/12/31. We also chose events with an error \( \lambda_1 \) smaller than 500 km. The total number of events in the entire catalog is 66597.

<table>
<thead>
<tr>
<th></th>
<th>P-wave</th>
<th>PP-wave</th>
<th>Core-phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classified events</td>
<td>9383</td>
<td>440</td>
<td>2316</td>
</tr>
<tr>
<td>Other events</td>
<td>15112</td>
<td>2845</td>
<td>3951</td>
</tr>
<tr>
<td>Total number</td>
<td>24495</td>
<td>3285</td>
<td>6267</td>
</tr>
</tbody>
</table>

To compare the modeled CSF given by equation 36 with the observation, Figure 24 shows the scatter plot against the observed and modeled CSF amplitudes for P-wave, PP-wave, and core-phases, respectively. Both CSFs of P-wave and core-phase events are consistent with each other, although they are scattered. On the other hand, the modeled single force of PP-wave is slightly larger than the observed ones because the observed beam powers are smaller than the model prediction systematically. This over-prediction suggests that large travel time anomalies of the PP-wave distort the beam powers by destructive interference. The assumption of the corrections in the slowness domain for lateral heterogeneities works for P-wave and core-phases because the travel time anomalies are smaller than the dominant period of around 8s. On the other hand, the assump-
tion does not work for PP-waves because travel time anomalies become larger than the dominant period. The resultant destructive interference caused underestimation. Although the travel time correction for all waveforms may solve this problem, the computations are still expensive. Such corrections should be addressed in future studies.

Figure 24. (a) Plots of P-wave events against observed and modeled CSF for all the events. The vertical axis shows the modeled CSF and the horizontal axis shows the observed one. (b) Plot of PP-wave events (c) Plot of core-phase events We choose events as $\lambda < 500$ [km].

To characterize the activities of the P-wave microseisms, we calculated the relationship between CSFs and the number of detected events. A number of observed events approach that of WAVEWATCH III with increasing CSFs, as shown in Figure 25 (a). Although they are consistent with each other above the amplitude of $10^{11}$ N, the slope of the observed events is slightly gentler than that of WAVEWATCH III. One possible origin of this difference is the bias due to the SNR. To verify this hypothesis, we calculated the relationship between the SNRs (MAX/MAD) and the number of detected events (Figure 25 (b)). Here, we again evaluate the SNR by the ratio of the maximum beam power to the MAD (section 5). For detection, we selected events with an SNR threshold of 5 in this study. Above the threshold, the number distribution of the detected events was consistent with that of the model predictions (Figure 25 (b)). The modeled events below the threshold distort the distribution shown in Figure 25 (a). This figure suggests that our catalog covers most of the events predicted by WAVEWATCH III above the threshold.

In this frequency range, the noise level in Japan strongly depends on local ocean swell activity, which excites background Rayleigh and Love waves (Nishida et al., 2008).
Figure 25. (a) Relation between the CSFs and the number of events. The number was counted in equally spaced bins on a logarithmic scale of the CSF. The solid line shows the relation of the observed events, and the dashed line shows that of WAVEWATCH III. (b) Relationship between SNR and number of events. The SNR is defined as the ratio between the mean squared amplitude and MAD. The number was counted in equally spaced bins on a logarithmic scale of SNRs. The solid line shows the relation of the observed events, and the dashed line shows that of WAVEWATCH III.

Reduction of the noise is, therefore, difficult. For better completeness, several inland seismic arrays could be feasible in further studies.

8 Migration events: typhoons and cyclones

When a severe storm migrates, a storm can be tracked using a seismic array. Zhang et al. (2010) reported that seismic arrays could track Typhoon Ioke in 2006. The centroids of P-wave microseisms migrated behind the typhoon because of the development of standing waves. Retailleau and Gualtieri (2019) showed that the seismic sources followed the typhoon, and the seismic amplitude increased with the typhoon propagation speed (Haubrich & McCamy, 1969).

For a systematic search for such migration events, we classify migration events from our CSF catalog as follows: First, we consider a P-wave event at the $i$th time step and search the nearest event to the event among all events at the $i+1$ time step. If the distance between the events is smaller than 600 km (i.e., the migration speed is approximately 100 km/h), we merge them into one group. If we cannot find the neighbor at the $i+1$ time step, we stop the grouping procedure to classify migration events. Figure 26(a) shows the migration events with a duration longer than two days. In the northern At-
lantic, we can track severe storms, including a weather bomb in 2014 (Nishida & Takagi, 2016) throughout the whole time period. In the Pacific ocean, the figure also shows migration events consistent with tracks of tropical cyclones.

Figure 26(b) compares a track of typhoon Ioke (2006) and two cyclones (LIN 2009, and PAM 2015) with the migration events. The centroids follow the best track data, which is consistent with previous studies (e.g., Zhang et al., 2010; Retailleau & Gualtieri, 2019). Typhoon Ioke lacked observed events in the middle part of the track because the corresponding events were too close to the array to be located. This result suggests that this dataset is feasible for future studies on tropical cyclones. The classification of migration events unable us to compare the corresponding best track data systematically.

Figure 26. (a) Migration events in P-wave microseisms The migration speed was slower than 100 km/h, and the duration was longer than two days. The colors of lines show the year of the events. (b) Migration events of typhoon Ioke 2006, cyclone LIN 2009, and cyclone PAM 2015. The dotted lines show the best track data of typhoon Ioke taken from the Regional Specialized Meteorological Center (RSMC), Tokyo, Japan, and the two best track data of cyclones (LIN and PAM) taken from the Joint Typhoon Warning Center (JTWC) of the U.S. Naval Pacific Meteorology Oceanography Center in Hawaii (Guard et al., 1992).

9 Conclusions

In this study, we developed a new technique that can extract information, both the slowness and curvature of the wavefront, as a natural extension of beamforming. To improve the accuracy, we corrected the slowness and curvature based on 3D ray tracing.
To discuss the accuracy, we applied this method to earthquake data. After corrections for the 3D P-wave velocity structure, we can reduce the estimation errors by the order of 1°.

We applied this method to vertical components of velocity meters at approximately 780 Hi-net stations in Japan from 2004 to 2020. We succeeded in creating a new CSF catalog of global P-wave microseisms. To verify the quality of the catalog, we compared the observed CSFs with CSFs based on WAVEWATCH III, a wave action model. P-wave and core-phase events were consistent with the locations, CSF amplitudes, and seasonality. On the other hand, observed PP-wave events were missing because PP-waves suffer more from lateral heterogeneities at shallow depths. The resultant larger travel time anomalies of the PP-wave cause larger location errors. The estimated CSFs of PP-wave events are also systematically smaller than those of the WAVEWATCH III model because the larger travel time anomalies distorted the beam power.

Centroids in the North Atlantic and North Pacific were dominant in the winter months, whereas centroids in the south Pacific and southern ocean were dominant in the summer months. Centroids in the southern Indian Ocean were lacking because the PP wave covered them. Although the WAVEWATCH III model can explain these features, observed centroids in the Gulf of Carpentaria Australia were missing in the centroids of WAVEWATCH III. The difference may originate from the unmodeled coastal reflection of ocean swells or different types of nonlinear interactions of ocean waves in the shallow sea of around 50 m.

We also compared observed migration events with track data from tropical cyclones (Typhoon Ioke 2006, cyclone LIN 2009, and cyclone PAM 2015). These are consistent with each other. Since the other migration events in the catalog may have information on severe storms, they will be fundamental data for further studies on P-wave microseisms excited by severe storms.

The estimated CSF catalog could provide basic information for further studies on the origin of microseisms. If we consider the CSF catalog as an analogy with the CMT catalog, it may be a new source for exploring the deep Earth structure for future studies.
Appendix A  Partial derivative of $\tau(m)$

In this section we derive the gradient vector and the Hessian matrix $\tau(m)$ (equations 14 and 15) explicitly. Partial derivatives of $l$ are given by

$$\frac{\partial l}{\partial \phi} = \left(-\zeta \left(1 + \frac{\eta}{\tan \Delta}\right) + \frac{1}{6} \left(\frac{1}{2} + \frac{1}{2\tan^2 \Delta}\right) \zeta (\zeta^2 - 2\eta^2)\right)$$

$$\frac{\partial l}{\partial \Delta} = -\zeta \left(\frac{1}{2} + \frac{\eta}{\tan \Delta}\right)$$

$$\frac{\partial^2 l}{\partial \phi^2} = \left(\eta + \frac{\eta^2 - \zeta^2}{\tan \Delta} + \frac{1}{6} \left(\frac{1}{2} + \frac{3}{2\tan^2 \Delta}\right) (2\eta^3 - 7\eta\zeta^2)\right)$$

$$\frac{\partial^2 l}{\partial \phi \partial \Delta} = \zeta \sin^2 \Delta \left(\eta + \frac{2\eta^2 - \zeta^2}{\sin^2 \Delta}\right)$$

$$\frac{\partial^2 l}{\partial \Delta^2} = \zeta^2 \sin^2 \Delta \left(\frac{1}{2\tan \Delta} + \eta \left(\frac{3}{\sin^2 \Delta} - 2\right)\right).$$  (A1)

With the partial derivatives, the gradient vector of $\tau(m)$ is given by

$$\begin{pmatrix} l \\ p \frac{\partial l}{\partial \phi} + \frac{dp}{\partial \phi} \frac{\partial l}{\partial \Delta} \\ p \frac{\partial l}{\partial \Delta} + \frac{dp}{\partial \Delta} \frac{\partial l}{\partial \phi} \\ \frac{\partial^2 l}{\partial \phi \partial \Delta} \end{pmatrix}.$$  (A2)

The Hessian matrix of $\tau(m)$ is given by

$$\begin{pmatrix} 0 & \frac{\partial l}{\partial \phi} & \frac{\partial l}{\partial \Delta} & 0 \\ \frac{\partial l}{\partial \phi} & p \frac{\partial^2 l}{\partial \phi^2} + \frac{dp}{\partial \phi} \left(\frac{\partial l}{\partial \phi}\right)^2 + \frac{\partial l}{\partial \Delta} \left(\frac{\partial \phi}{\partial \phi}\right)^2 + \frac{\partial l}{\partial \Delta} \left(\frac{\partial \phi}{\partial \Delta}\right) & \frac{dp}{\partial \phi} \left(\frac{\partial l}{\partial \phi}\right) + \frac{\partial l}{\partial \Delta} \left(\frac{\partial \phi}{\partial \Delta}\right) + p \frac{\partial^2 l}{\partial \phi \partial \Delta} + \frac{\partial l}{\partial \phi} \left(\frac{\partial \phi}{\partial \phi}\right)^2 + \frac{\partial l}{\partial \Delta} \left(\frac{\partial \phi}{\partial \Delta}\right) & \frac{\partial l}{\partial \phi} \left(\frac{\partial \phi}{\partial \phi}\right)^2 + \frac{\partial l}{\partial \Delta} \left(\frac{\partial \phi}{\partial \Delta}\right) \\ \frac{\partial l}{\partial \Delta} & \frac{dp}{\partial \phi} \left(\frac{\partial l}{\partial \phi}\right) + \frac{\partial l}{\partial \Delta} \left(\frac{\partial \phi}{\partial \phi}\right)^2 + \frac{\partial l}{\partial \Delta} \left(\frac{\partial \phi}{\partial \Delta}\right) & p \frac{\partial^2 l}{\partial \phi \partial \Delta} + \frac{dp}{\partial \phi} \left(\frac{\partial l}{\partial \phi}\right) + \frac{\partial l}{\partial \Delta} \left(\frac{\partial \phi}{\partial \phi}\right)^2 + \frac{\partial l}{\partial \Delta} \left(\frac{\partial \phi}{\partial \Delta}\right) & \frac{\partial l}{\partial \phi} \left(\frac{\partial \phi}{\partial \phi}\right)^2 + \frac{\partial l}{\partial \Delta} \left(\frac{\partial \phi}{\partial \Delta}\right) + \frac{\partial l}{\partial \phi} \left(\frac{\partial \phi}{\partial \phi}\right)^2 + \frac{\partial l}{\partial \Delta} \left(\frac{\partial \phi}{\partial \Delta}\right) \\ 0 & \frac{\partial l}{\partial \phi} & \frac{\partial l}{\partial \Delta} & 0 \end{pmatrix}.$$  (A3)

Appendix B  Reduction of noise power for bootstrap sampling

To estimate the standard deviation of the beam power $E$ by bootstrap sampling,

we evaluate the beam power $E^{\alpha}$ at the $\alpha$th time step. We consider the observed displacement $u^{\alpha}(\omega)$ at angular frequency $\omega$ as

$$u^{\alpha}_i = \phi^{\alpha}(\omega) + n^{\alpha}_i,$$  (B1)

where $i$ represents the station number, $\phi^{\alpha}(\omega)$ is a common signal, and $n^{\alpha}_i(\omega)$ is the site noise. For simplicity, we drop the dependency on $\omega$ later. $E^{\alpha}$ can be represented by

$$E^{\alpha} = |\phi^{\alpha}|^2 + \frac{2}{N} \Re \left[ \phi^{\alpha} \sum_i^{N} n^{\alpha*}_i \right] + \frac{1}{N^2} \sum_i^{N} |n^{\alpha}_i|^2.$$  (B2)
To improve the SNR, we average \(E^\alpha\) for every \(M\) steps. To clarify the problem we consider here, we assume that \(M\) is sufficiently large as \(M \rightarrow \infty\). We also assume that \(n^\alpha_i\) is subject to \(\mathcal{N}(0, \sigma_i^2)\), and that \(\phi^\alpha(\omega)\) is subjected to \(\mathcal{N}(0, P_\phi)\). If \(\phi^\alpha\) does not correlate with \(n^\alpha_i\). The time average of the beam power \(\langle E \rangle\) can be simplified as:

\[
\langle E \rangle \equiv \lim_{M \rightarrow \infty} \frac{1}{M} \sum_{\alpha=1}^{M} E^\alpha = P_\phi + \frac{1}{N^2} \sum_{i=1}^{N} \sigma_i^2 ,
\]

(B3)

where \(\langle \rangle\) represents the time average over \(M\) steps. Because \(\langle E \rangle\) converges to the value, the standard deviation of \(\langle E \rangle\) becomes 0.

Below let us estimate the standard deviation by bootstrap sampling method. The bootstrap samplings were constructed by random selection of stations with replacement among all stations (e.g., Efron & Tibshirani, 1994). We constructed 100 bootstrap sampling sets for this study. We calculated the beam power \(E^\alpha_{bst}\) for the \(\beta\)th set of the bootstrap samplings, which can also be interpreted as a weighted average as

\[
E^\alpha_{bst} = \left| \sum_{i=1}^{N} w_{\beta}^i n_i^\alpha \right|^2 = \frac{1}{N^2} \left| \sum_{i=1}^{N} w_{\beta}^i n_i^\alpha \right|^2 ,
\]

(B4)

where \(w_{\beta}, i = 1, \cdots, N\) are positive integers that satisfy \(\sum_{i=1}^{N} w_{\beta}^i = N\).

Again we consider the average of \(E^\alpha_{bst}\) over time as \(M \rightarrow \infty\) as

\[
\langle E^\beta_{bst} \rangle \equiv \lim_{M \rightarrow \infty} \frac{1}{M} \sum_{\alpha=1}^{M} E^\alpha_{bst} = P_\phi + \frac{1}{N^2} \sum_{i=1}^{N} (w_{\beta}^i)^2 \sigma_i^2.
\]

(B5)

\(\langle E^\beta_{bst} \rangle\) changes with \(\beta\), even if \(\langle E \rangle\) converges to the value. If the noise variances \(\sigma_i^2\) are constant at all stations, the variance of the bootstrap becomes 0. Under a realistic situation, because \(\sigma_i^2\) (i.e., the noise level at a station) strongly depends on the stations in this study (Obara et al., 2005), the variations cause a large bias. The variance of the bootstrap samplings became significantly larger than expected.

To reduce the bias we define a reduced beam power \(E^\alpha_{red}\) as

\[
E^\alpha_{red} = \left| \sum_{i=1}^{N} w_{\alpha}^i n_i^\alpha \right|^2 - \frac{1}{N^2} \sum_{i=1}^{N} (w_{\alpha}^i)^2 |n_i^\alpha|^2.
\]

(B6)

After the subtraction of the auto-correlation part, the reduced beam power converges to the true value for \(M \rightarrow \infty\) as

\[
\langle E^\beta_{red} \rangle \equiv \lim_{M \rightarrow \infty} \frac{1}{M} \sum_{\alpha=1}^{M} E^\alpha_{red} = P_\phi .
\]

(B7)

The above value converges to the value, which does not depend on bootstrap sampling.

When we estimated the standard deviation of the peak beam power by bootstrap sam-
pling, we calculated the reduced beam power. Without the corrections, the typical location errors become larger than 30° in many cases. This correction led to more realistic estimations as shown in this paper.

Acknowledgments

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Data and materials availability:

We used data from Hi-net (doi.org/10.17598/nied.0003) managed by the National Research Institute for Earth Science and Disaster Prevention (NIED), Japan.


The CSF catalog in this study will be available at https://github.com/qnishida/CSF_supplement after the acceptance of this paper. The test python code of the autofocus method will be available at the cite.

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